High-performance millimeter-scale silicon grating emitters for beam steering applications

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A 2 mm-long silicon-on-insulator grating emitter with a narrow angular full width at half maximum (FWHM) and a high sideband suppression ratio (SSR) is proposed and designed. It consists of a Si3N4/Si grating with an approximate Gaussian emission profile along the grating length, which aims to reduce the sidelobe intensity of the scanning light in the far-field, thereby improving the resolution of longitudinal steering resolution of the lidar. Numerical analysis shows that the angular FWHM of the emitted beam could be as low as 0.026º for a grating length of 2.247 mm for the input TE-like waveguide mode at 1550 nm, and the SSR could be more than 32.622 dB. Moreover, this Si3N4/Si grating exhibits a favorable fabrication error tolerance when considering the width and length variation of the Si3N4 overlayer in practice. Our design offers a promising platform for realizing integrated optical phased arrays for the long-distance solid-state lidar.

Keywords: Grating emitter, CMT, genetic algorithm

1. Introduction

With the rise of autonomous vehicles[1–3] and unmanned drones[4], light detection and ranging (lidar) has become an indispensable device for them. So far, the mechanical lidar[5] is still one of the most mature solutions, but its high cost and difficult assembly have been plagued by researchers. Besides, the short detection distance limits the wide application of flash Lidars[6]. The integrated on-chip silicon optical phased array (OPA)[7–11] as an advanced solid-state beam steering device, can overcome the above defects and has gained significant interest for its energy-saving and miniaturization. In application, the grating-emitter-based OPA is considered a viable candidate to achieve two-dimensional optical steering, i.e., phase steering in one direction and wavelength steering in the other direction. For phase steering, the OPA with a pitch close to a half wavelength along the lateral direction is constructed to realize a wide beam steering range with low crosstalk, which is usually built upon metamaterial waveguides[12, 13], corrugated waveguides[14], or nano-structured silicon waveguide array[15], etc. Notably, OPAs with phase mismatched unequal width waveguide distribution has been applied, implementing a steering range of 110º and a maximum peak power of 720 mW[16]. Wavelength steering is often enabled by the grating dispersion when the wavelength of the input laser light is scanned. A macroscopic emitting aperture with its size > 1 mm is typically required for ranging distances of interest for autonomous vehicles[17] because a larger aperture would generally enable a narrower width of the main beam lobe and thus a high angular imaging resolution. The silicon waveguide grating with shallow etching is one possible approach to realizing long-length emitting for its weak emission rate[18]. But in practice, they are challenging to fabricate. A more promising approach of integrating silicon nitride (Si3N4) overlay on silicon waveguide has gained significant interest because of its low nonlinearity, broad transparency range, low propagation loss, and low index contrast characteristics[4]. On this basis, several surface gratings with novel structures, such as strip-line grating[19] and fishbone grating[20], have been realized in the modulation of emission profile and the improvement of radiation beam quality. In addition, the downward radiant power of the grating always introduces destructive interference, which reduces the performance of the grating emitter. An effective approach of dual-layer grating misalignment has been proposed, consequently achieving more than 95% unidirectional radiation[21].

In this letter, we propose a Si3N4/Si grating emitter structure with a specific variation in widths and duty cycles of Si3N4 overlayers, This structure provides an approximate Gaussian emission profile and long effective coupling length of 2.247 mm. The design is mainly centered around two targets, 1) to increase the far-field SSR, thus reducing the interference of the scanning light sidelobes in the far-field to the steering detection, which is realized by constructing the Gaussian near-field emission profile, and 2) to increase the steering accuracy, which is directly governed by the angular FWHM beamwidth related to the effective grating length. The coupled-mode theory (CMT) is employed to analyze the coupling between the guided mode and radiation mode. In addition, a genetic algorithm (GA)[22–24] drives the coupled-mode model to search a large parameter space containing...
the two geometric freedoms of $\text{Si}_3\text{N}_4$ overlayers (with fixed height) to produce the large-scale approximate Gaussian emission profile on the grating surface. Our theoretical investigation suggests that when the working wavelength is 1550 nm, the designed grating emitter can obtain a far-field angular FWHM beamwidth of less than 0.026° and an SSR of larger than 32.622 dB. In addition, the proposed grating emitter can realize longitudinal beam steering of 3.94° by wavelength tuning within the wavelength range of 1530 ~ 1570 nm. Our design here provides a practical and feasible approach for beam steering, shedding light on the possibilities of realizing high-performance solid-state lidars with a high signal-to-noise ratio (SNR) and detection accuracy.

2. Theory and design

![Diagram](image.png)

Fig. 1 (a) Partial schematic diagram of our proposed $\text{Si}_3\text{N}_4$/Si grating with a varying duty cycle and width of the $\text{Si}_3\text{N}_4$ overlayer. (b) 3D view of our proposed grating without SiO$_2$ cladding.

Purely numerical methods such as finite difference time domain (FDTD) are theoretically feasible for the grating design. However, accurate calculation results can only be obtained by the ultra-high-precision meshing of the mm-length rectangular structure, which undoubtedly consumes a significant amount of computing resources. By contrast, the CMT avoids the tedious meshing process and can give quantitative predictions of the coupling between the guided mode and the radiation mode and a physical understanding of grating radiation. In the coupled-mode model employed here, by modulating the widths and duty cycles of $\text{Si}_3\text{N}_4$ overlayers, we can precisely control the coupling between the guided mode and radiation mode. In prior studies, radiation modes can be constructed in relatively simple forms for two-dimensional (2-D) slab waveguides and optical fibers. They cannot apply to our design of the $\text{Si}_3\text{N}_4$/Si grating emitter with a rectangular structure. We adopt the semi-analytical method proposed by Christopher G. Poulton to construct fully 3-D radiation modes of the ideal rectangular silicon waveguide (see Appendix for details), where the radiation modes are deduced from the response of the waveguide to an incoming extended field with a given symmetry and polarization to calculate power density functions of the radiation modes more explicitly and conveniently.

Shown schematically in Fig. 1, the customized grating is designed on a silicon-on-insulator (SOI) platform. The silicon waveguide has a height of 220 nm and a width $W_{0}$ of 500 nm. A $\text{Si}_3\text{N}_4$ overlayer with a height $h_{0}$ of 50 nm is deposited above the Si waveguide with SiO$_2$ cladding. The 2.247-mm grating contains 2500 grating periods totally with a period $A$ of 899.07 nm. The coupling between the guided mode and the radiation mode is constrained by the (quasi-) phase matching condition, and the radiation field consequently emerges at an angle of 30° (referred to the grating facet normal). As a traveling wave with a fixed radiation angle, the radiation field has a planar phase front. In practice, 2500 independently parametrized $\text{Si}_3\text{N}_4$ overlayers undoubtedly increase the difficulty of actual fabrication and reduce the robustness of the device. An effective optimization strategy is proposed here by 1) grouping a series of 25 neighboring grating periods as an optimization group, where the widths and duty cycles of the $\text{Si}_3\text{N}_4$ overlayers are set to be identical; 2) constructing two fitting functions $g_1$ and $g_2$, representing respectively the mapping relations from the group order $S = 1$ ~ 100 to the common widths and duty cycles of the $\text{Si}_3\text{N}_4$ overlayers in each group, to reduce the number of optimal variables. In particular, $g_1$ represents the percentage of the $\text{Si}_3\text{N}_4$ overlayer width to the silicon core width. The fitting functions $g_1$ and $g_2$ can be expressed as

$$
g_1(S) = \sum_{j=0}^{40} a_j \left[ (S-50)/a_1 \right]^j + a_2 e^{-j(S-a_4)/b_1} 
$$

$$
g_2(S) = \sum_{j=0}^{40} b_j \left[ (S-50)/b_1 \right]^j + b_2 e^{-j(S-b_4)/b_1} 
$$

where $a_j$ and $b_j$ ($j = 0, 1, ..., 44$) are the independent fitting parameters to be optimized. So the width of the $\text{Si}_3\text{N}_4$ overlayer can be given by

$$
W(z) = \begin{cases} 
   g_1(S) & 0 < z \leq A(n-1) + Ag_2(S) \\
   0, & \text{other} 
\end{cases} 
$$

where $n = 1 + 25(S-1), ..., 25S$. Along the propagation direction, the $z$-component of the coupling coefficient can be ignored, so it can be approximately expressed as

$$
\kappa_{\mu\nu}(z) = \frac{j\omega\varepsilon_0}{4P_{\xi}^2} \int_0^W dx \int_0^b dy \left( n^2 - n_0^2 \right) E_{\mu\nu}^* \cdot E_{\nu\lambda} 
$$

where $P_{\xi}$ ($\xi$, a label indicating radiation modes) is the power spectral density, $n$ and $n_0$ are the refractive index of $\text{Si}_3\text{N}_4$ and cladding medium, respectively. $E_{\mu\nu}$ and $E_{\nu\lambda}$ are the transverse components of the electric field of the guided mode and radiation mode,
corresponding optimized emission profile at the normalized fundamental guided mode is 1 (a.u.), the line, respectively. Assuming that the amplitude of the emission profile and the results simulated by FDTD in the wavelength of 1550 nm along the 2.247-mm grating length is plotted in Fig. 2(b). The solid black line represents the calculation result, agreeing well with a fitted Gaussian curve, having expected value \( \mu = 0.001 \) and variance \( \sigma^2 = 9 \times 10^{-4} \), plotted with the solid red line. Their quantitative similarity can be expressed using the following cross-correlation coefficient formula, where \( F_1 \) and \( F_2 \) represent our optimized distribution of emission profile and the corresponding fitted Gaussian profile, respectively.

\[
C = \frac{\int_0^L F_1 \cdot F_2 \, dz}{\sqrt{\int_0^L |F_1|^2 \, dz \cdot \int_0^L |F_2|^2 \, dz}} = 0.9992 \quad (8)
\]

here, the integral variable \( z \) is the propagation distance of the light in the longitudinal direction, and \( L \) indicates the total propagation distance. The cross-correlation coefficient of 0.9992 strongly suggests that our designed grating has an emission profile nearly approaching an ideal Gaussian profile. The emission profile in the grating periods of 1145 ~ 1365 is investigated using a 3D FDTD method. The corresponding major electric field distribution of the guided mode is depicted in Fig. 2(c), and the calculated emission profile, shown in Fig. 2(d), approaches the results simulated by 3D FDTD, which further illustrates the effectiveness and accuracy of our design. Unfortunately, through the simulated result, the unidirectionality of our device only exceeds 46.2\%, which can be improved by dual-layer grating misalignment\(^{[21]}\).

3. Result and analysis

The optimized width and duty cycle distribution of the Si\(_3\)N\(_4\) overlayers in each grating period are plotted in Fig. 2(a) as the solid black line and the solid red line, respectively. Assuming that the amplitude of the normalized fundamental guided mode is 1 (a.u.), the corresponding optimized emission profile at the same grating region.

Fig. 2. (a) The optimized width and duty cycle distribution of the Si\(_3\)N\(_4\) overlayer as a function of the grating period order. (b) The corresponding emission profile for our designed grating with a length of 2.247 mm at the wavelength of 1550 nm. (c) The major electric field distribution of the guided mode during the grating periods of 1145 ~ 1365. (d) The comparison of the calculated emission profile and the results simulated by FDTD in the same grating region.
Fig. 3. (a) The far-field intensity distribution for our proposed grating at 1550 nm wavelength. (b) The far-field intensity distribution for beam steering within 1530 ~ 1570 nm wavelength range with a 5 nm step.

At the wavelength of 1550 nm, the far-field beam profile for our proposed grating with the approximate Gaussian near-field emission intensity is shown in Fig. 3(a). The simulated FWHM beamwidth is as low as 0.026º, and the SSR is as high as 32.622 dB. Fig. 3(b) illustrates the far-field intensity distribution for beam steering within the 1530 ~ 1570 nm wavelength range with a 5 nm step. A steering angle of 3.94º centered at 30º (working wavelength of 1550 nm) is achieved with approximately 0.1º/nm angular steering dispersion. Moreover, the far-field SSR for the 1530 ~ 1570 nm wavelength range is greater than 32 dB, suggesting that high-performance beam steering can be maintained during wavelength tuning.

Table 1. Effect of constant changes in Si$_3$N$_4$ overlayer width and length on the SSR of our proposed grating.

<table>
<thead>
<tr>
<th>Monte-Carlo model</th>
<th>Average SSR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 10$ nm</td>
<td>32.227</td>
</tr>
<tr>
<td>$\sigma = 20$ nm</td>
<td>30.766</td>
</tr>
<tr>
<td>$\sigma = 30$ nm</td>
<td>29.110</td>
</tr>
<tr>
<td>$\sigma = 40$ nm</td>
<td>26.130</td>
</tr>
<tr>
<td>$\sigma = 50$ nm</td>
<td>17.707</td>
</tr>
<tr>
<td>$\sigma = 60$ nm</td>
<td>15.379</td>
</tr>
</tbody>
</table>

In practice, deviations in fabricated dimensions of the Si$_3$N$_4$ overlayers from designed dimensions can lead to the radiation pattern change in our proposed grating antenna. To simulate the impact of fabrication errors as accurately as possible, we apply the Monte Carlo method to characterize the radiation fluctuations in the far field. Assuming that the fabrication errors of the width and length (that is duty cycle) of Si$_3$N$_4$ overlayer satisfy the Gaussian random distribution with mean value $\mu = 0$. Table 1 presents the averaged far-field SSR with the standard deviation $\sigma$ varying from 10 nm to 60 nm, and Fig 4 shows the error maps at $\sigma = 30$ nm, 40 nm, 50 nm, and 60 nm, respectively. The results show that the grating antenna can still maintain good performance with reasonable fabrication errors, e.g., $\sigma$ below 30 nm, which is a quite exaggerated error for any state-of-the-art CMOS photonics foundry. Hence, our design could hold considerable merit and value in developing practical large aperture solid-state phase arrays.

4. Conclusion

In conclusion, we have proposed a practical design of a Si$_3$N$_4$ overlay assisted silicon grating emitter with an approximate Gaussian emission profile over > 2 mm length which has a narrow angular FWHM of 0.026º and a large far-field SSR of 32.622 dB. The theory clearly reveals the general way to design a grating with a special emission profile by using CMT and GA. Further analysis shows that our emitter can also function well with almost undeteriorated angular divergence and SSR performance for the wavelength range from 1530 nm to 1570 nm, while achieving a longitudinal steering angle of about 3.94º. Our theoretical analysis indicates that our design is robust to the practical fabrication errors for the width and length of the Si$_3$N$_4$ overlay. We believe that the high-performance solid-state lidar systems can benefit from our demonstrated device.

Appendix

Radiation Mode

As shown in Fig. 5, in the upper half of coordinate space, visualizing that a plane wave from infinity impinges onto the surface of the ideal silicon waveguide, a portion of this wave is scattered (or reflected) at the core boundary, and the remaining wave is refracted into the waveguide. The total field outside the silicon core consists of the incoming field and the response field of the boundary to it. The resulting radiation field (e.g., the total field) is the eigen-solutions of Maxwell’s equations under specific boundary conditions. The propagation constant $q$ of the radiation mode forms a continuum because it is related to the incident direction of the incoming field, which indicates that radiation modes are also continuous in the $q$ space. Confusion may arise in the previous discussion of setting up the theoretical model, e.g., the radiation mode in our model is a standing wave, which someone may confuse with the fact that the radiation field is an outgoing traveling wave. The explanation is that the guide mode in the core of the
perturbed waveguide would excite a series of infinite continuous radiation modes, in which the incoming fields are eliminated by destructive interference\cite{25}, while the response fields superimpose themselves into an outgoing traveling wave.

![Fig. 5 Profile diagram of a rectangular silicon waveguide surrounded by SiO$_2$. $n_s$ and $n_c$ represent the refractive index of the waveguide and the cladding, respectively. $E_z$ and $H_z$ are the longitudinal (z-axis) parts of the electric field and magnitude field located at the boundary. $E_T$ and $H_T$ are the transverse components. The discrete boundary grid points are represented by $(r_m, \theta_m)$.]

In the internal region of the silicon core, the transverse propagation constant can be defined as $\beta = (n_s^2 k^2 - \beta_c^2)^{1/2}$, and in the external region, $\rho = (n_s^2 k^2 - \beta_c^2)^{1/2}$, where $n_s k$ and $n_c k$ are the wavenumbers in the silicon waveguide and SiO$_2$ cladding, respectively, and $\beta_c$ ($n_s k \leq \beta_c \leq n_c k$) is the longitudinal propagation constant of radiation mode $\nu$ in the SiO$_2$ cladding. In the radiation model, the longitudinal components of electric and magnetic fields can be expanded in terms of cylindrical harmonic functions, which are expressed as\cite{26,27}

$$
\begin{align*}
E_z^m (r, \theta) &= \sum_{m=0}^{\infty} A_m J_m (\eta r) \sin (m \theta + \varphi) \\
H_z^m (r, \theta) &= \sum_{m=0}^{\infty} B_m J_m (\eta r) \cos (m \theta + \varphi)
\end{align*}
$$

(9)

in the internal region, and

$$
\begin{align*}
E_z^m (r, \theta) &= \sum_{m=0}^{\infty} C_m H_m^{(1)} (\rho r) \sin (m \theta + \varphi) \\
&\quad + P_N J_m (\rho r) \sin (N \theta + \varphi) \\
H_z^m (r, \theta) &= \sum_{m=0}^{\infty} D_m H_m^{(1)} (\rho r) \cos (m \theta + \varphi) \\
&\quad + Q_N J_m (\rho r) \cos (N \theta + \varphi)
\end{align*}
$$

(10)

in the external region. Here, the terms $J_m$ and $H_m^{(1)}$ are the $m$th-order Bessel functions and Hankel functions of the first kind, respectively. Hankel functions of the first kind possess the characteristics of actual outgoing traveling-wave, solving the seeming paradox. The coefficients $P_N$ and $Q_N$, satisfying $P_N \cdot Q_N = 0$, represent the magnitudes of the incoming field and can directly determine the power spectral density $P_L (\xi, \nu)$, a label indicating radiation modes) for the normalization of radiation modes. When $P_N = 0$, $Q_N \neq 0$, the incoming field exhibits $E$-polarization, and when $P_N \neq 0$, $Q_N = 0$, it shows $H$-polarization. Furthermore, the parity of the order $N$ and $m$, together with the phase $\varphi (\varphi = 0$ or $\pi/2$) factor, identifies the symmetry of a certain radiation mode. The selection rules of these parameters are listed in Table 1.

To some extent, the radiation modes can thus be defined as resulting from the response of the waveguide to an incoming field possessing a given polarization and symmetry.

**Table 2. The selection rules of the parameter $N$, $m$, and $\varphi$, and the corresponding symmetries of incoming fields**

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\varphi$</th>
<th>$m$ (E)</th>
<th>$m$ (H)</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odd ($&gt; 0$)</td>
<td>0</td>
<td>Odd</td>
<td>Odd</td>
<td>Symmetric</td>
</tr>
<tr>
<td>Even ($&gt; 0$)</td>
<td>0</td>
<td>Even</td>
<td>Even</td>
<td>Anti-symmetric</td>
</tr>
</tbody>
</table>

*When $N = 0$ and $\varphi = 0$, incident wave is $E$-polarization, so $Q_N \neq 0$. And, when $N = 0$ and $\varphi = \pi/2$ and $P_N \neq 0$.|

![Fig. 6 (a-d) Distribution of the z-component of the electric field of the radiation mode outside the waveguide, corresponding to $N = 1$ ~ 4, respectively. We assume that the incident wavelength $\lambda = 1550$ nm, the longitudinal propagation constant $\beta_c = 2.926 \mu m^{-1}$, the phase factor $\varphi = 0$, and the incoming polarization is $H$-polarization. The tangential components (represented by $E_T$ and $H_T$) and longitudinal components (represented by $E_z$ and $H_z$) of the radiation model must be continuous at the boundary of the silicon waveguide. Deriving from the symmetry of the fields, we choose the core boundary in the first quadrant in Fig. 5 and discretize it into $M$ boundary points $(r_m, \theta_m)$ where $r_m$ is the length from the origin to a boundary point and $\theta_m = \pi/2 \cdot (m - 0.5) / M (m = 1, ..., M)$.

Furthermore, a series of cylindrical harmonics in Eq. (9) and (10) must be appropriately truncated, which ensures that the extension coefficients $A_m, B_m, C_m, D_m$ occupy the unique solution. Combining these matching equations, we finally get a $4M \times 4M$ matrix equation system about expansion coefficients (see reference\cite{27} for...
The ideal silicon waveguide is then given by the radiation modes. The real $z$-components of the electric field of different radiation modes are depicted in Fig. 6, with $N$ taking values in the range of $1 \sim 4$, which meets the symmetry requirements of the radiation mode shown in Table 2.

Given the longitudinal propagation constant $\beta_z$, the coefficient $N$, the phase factor $\varphi$, and the polarization of the incoming field, the total radiation field of an ideal silicon waveguide is then given by

$$
\begin{align*}
E_{\text{rad}} &= \sum_{j=0}^{N} a_j(\beta_z) e_j(x, y, \beta_z) \exp(\beta_z z) d\beta_z, \\
H_{\text{rad}} &= \sum_{j=0}^{N} a_j(\beta_z) h_j(x, y, \beta_z) \exp(\beta_z z) d\beta_z.
\end{align*}
$$

(11)

Here the $a_j$ is the model amplitudes. The integration indicates that the radiation mode is continuous in terms of the propagation constant, and the summation shows that the radiation modes are discrete in terms of spatial symmetry.

Acknowledgments

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