Magnetic-Field-Induced Deflection of Nonlocal Light Bullets in a Rydberg Atomic Gas

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Light bullets (LBs) are localized nonlinear waves propagating in high spatial dimensions. Finding stable LBs and realizing their control are desirable due to the interesting physics and potential applications. Here, we show that nonlocal LBs generated in a cold Rydberg atomic gas via the balance between the dispersion, diffraction, and giant nonlocal Kerr nonlinearity contributed by long-range Rydberg-Rydberg interaction can be actively manipulated by using a weak gradient magnetic field. Nonlocal LBs are generated by a balance between dispersion, diffraction, and large non-local Kerr nonlinearities contributed by long-range Rydberg-Rydberg interactions. Here we find that active manipulation can be achieved by weak gradient magnetic fields in cold Rydberg atomic gases. Especially, the LBs may undergo significant Stern-Gerlach deflections, and their motion trajectories can be controlled by adjusting magnetic-field gradient. The results reported here are helpful not only for understanding unique properties of LBs in nonlocal optical media but also for finding ways for precision measurements of magnetic fields. DOI: 10.3788/COL202220.041902

INTRODUCTION

In recent years, much attention has been paid to the investigation on the electromagnetically induced transparency (EIT) in cold Rydberg atomic gases [1–24, 32, 33]. This is rooted in the fact that atomic Rydberg states have long coherent lifetimes and strong long-range interaction (called Rydberg-Rydberg interaction) between remote atoms. The Rydberg-Rydberg interaction makes the atomic gases be nonlocal optical media and they can be effectively mapped to strong photon-photon interactions via EIT. As a result, strong nonlinearities at very low light intensity can be realized [5, 6, 14], which opens up a new avenue to study nonlinear and quantum optics and realize novel photon devices, such as single-photon switches [25–29], optical transistors [4, 30], photon memories [10, 11], and single-photon source [31].

Light bullets (LBs) [34] are solitary nonlinear waves localized in \(m\) spatial dimensions and one time dimension \((m+1)D; m = 1, 2, 3\). In recent years, the study of LBs has attracted intensive theoretical and experimental interests [35] because of their rich nonlinear physics and technological applications [36, 37]. However, the generation of stable high-dimensional LBs is a topic not solved for a long time. It has been shown recently that stable \((3+1)D\) nonlocal LBs can be realized in Rydberg atomic gases; such LBs have extremely low generation power and ultra slow propagation velocity [17]. Different from non-interaction system, the central element is that the co-existence of giant nonlocal and local optical Kerr nonlinearities. The former features a fast (sub-microsecond) response [38], which is complemented by the latter, whose response is relatively slow (in the order of microsecond). In conjunction with tunable dispersion and diffraction, this allows us to precisely control dynamics of LBs.

In this work, we propose a scheme to realize the active control of the nonlocal LBs in a Rydberg atomic gas. We show that the \((3+1)D\) LBs generated in such a system via EIT can be manipulated by using a gradient magnetic field. In particular, the LBs can undergo significant Stern-Gerlach deflections even when the magnetic-field gradient is weak, and their motion trajectories can be adjusted through the changing of the magnetic-field gradient. Our work contributes to the efforts for understanding the unique properties and realizing the active controls of high-dimensional LBs, and also for finding new techniques for precision measurements of magnetic fields.

MODEL

The system under study is a cold three-level atomic gas working with a Rydberg-EIT scheme, shown in Fig. 1(a). Here, the level \(|1\rangle, |2\rangle,\) and \(|3\rangle\) are ground, intermediate, and Rydberg state, respectively; a weak, pulsed \((\text{with time duration } \tau_0)\) probe laser field \((\text{center angular frequency } \omega_p, \text{ center wavenumber } k_p = \omega_p/c, \text{ half Rabi frequency } \Omega_p)\) couples to the transition between \(|1\rangle \text{ and } |2\rangle\); a strong, continuous-wave control laser field \((\text{angular frequency } \omega_c, \text{ wavenumber } k_c = \omega_c/c, \text{ half Rabi frequency } \Omega_c)\) couples to the transition between \(|2\rangle \text{ and } |3\rangle\). The total electric field of the system reads \(E = E_p + E_c\), with \(E_{\alpha} = e_{\alpha} E_0 \exp[i(k_{\alpha} \cdot r - \omega_0 t)] + \text{h.c.}\). Here, with \(r = (x, y, z), e_{\alpha}, \) and \(E_{\alpha}\) are unit polarization vector and field amplitude for \(\alpha \text{ field } (\alpha = p, c, \text{ respectively). To suppress Doppler effect, the probe and the control fields are assumed to counter-propagate along } z \text{ direction. Fig. 1(b) shows the geometry of the system.}

For realizing the active control on the LBs, a weak gradient magnetic field is assumed to act on the atomic gas, with the form

\[
B(x, y) = \hat{z} B z(x, y) = \hat{z} (B_1 x + B_2 y), \tag{1}
\]

where \(r_z = (x, y, 0), \hat{z} = (0, 0, 1)\) is the unit vector in the \(z\) direction, \(B_1\) and \(B_2\) characterize the gradients of the magnetic field in the \(x-y\) plane. Due to the presence of the magnetic field, each level of the atoms is split into a series of Zeeman sub-levels with energy \(\Delta E_{\alpha, m} = \mu_B g_{\alpha} m_B^* B,\) where \(\mu_B, g_{\alpha}, \) and \(m_B^*\) are the Bohr magneton, gyromagnetic factor, and magnetic quantum number of level \(|\alpha\rangle\), respectively.
As a result, the one- and two-photon detunings $\Delta_2$ and $\Delta_3$ are changed into $\Delta_2(r, t) = (\omega_p - \omega_3 + \omega_1) + \mu_{32} B(r, t)$ and $\Delta_3(r, t) = (\omega_p - \omega_3) + \mu_{31} B(r, t)$, with $\mu_{32} = \mu_B (g_{32}^2 m_r^2 - g_{31}^2 m_r^2) / \hbar$.

Under electric-dipole and rotating-wave approximations, the Hamiltonian of the atomic gas including the Rydberg-Rydberg interaction is given by $\hat{H} = N_a \int d^3r \hat{H}(r, t)$, with $\hat{H}(r, t)$ the Hamiltonian density, given by

$$\hat{H} = \sum_{\alpha \in \{\text{p}, \text{d}\}} \hbar \omega_\alpha \hat{S}_{\alpha} S(\mathbf{r}, t) - \hbar \Omega_p \hat{S}_{12}(\mathbf{r}, t) + \Omega_c \hat{S}_{23}(\mathbf{r}, t) + \text{h.c.} + N_a \int d^3r' \hat{S}_{33}(\mathbf{r}', t) hV(\mathbf{r} - \mathbf{r}') \hat{S}_{33}(\mathbf{r}, t). \quad (2)$$

where $\hat{S}_{\alpha} S(\mathbf{r}, t) = \langle \alpha | \hat{S}(\mathbf{r}, t) | \alpha \rangle$ and $\hat{S}_{\alpha} S(\mathbf{r}, t) = \langle \alpha | \hat{S}(\mathbf{r}, t) | \alpha \rangle$. The dynamics of the atoms is controlled by the Heisenberg equation of motion for the atomic operators $\hat{S}_{\alpha}$, i.e., $i\hbar \partial \hat{S}_{\alpha} / \partial t = \{\hat{H}, \hat{S}_{\alpha}\}$. Taking expectation values on both sides of this equation, we obtain the optical Bloch equation involving one- and two-body reduced density matrices, which can be cast into the form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \rho] - \Gamma [\rho], \quad (3)$$

where $\rho(\mathbf{r}, t)$ is reduced density matrix (DM) in the single-particle basis $\{|1\}, \{|2\}, \{|3\}\}$, with the matrix elements defined by $\rho_{\alpha \beta} (\mathbf{r}, t) \equiv \langle \alpha | \rho(\mathbf{r}, t) | \beta \rangle$; $\Gamma$ is a $3 \times 3$ relaxation matrix describing the spontaneous emission and dephasing. Due to the existence of the Rydberg-Rydberg interaction, two-body reduced DM [i.e., $\rho_{\text{twobody}}$ with DM elements $\rho_{\alpha \beta \gamma \delta}(\mathbf{r}, \mathbf{r'}, t)$] is involved in this equation, denoting the contribution from the Rydberg-Rydberg interaction. The explicit expression of Eq. (3) is presented in Supplemental Material [39].
The results shown in Fig. 1(c) is \( \chi_{(3)}^{(2)} \) as a function of \( x \) for \( y = 0 \). The real part \( \text{Re}\{\chi_{(3)}^{(2)}\} \) and imaginary parts \( \text{Im}\{\chi_{(3)}^{(2)}\} \) are plotted on the condition that \( \Delta_2 \gg \Gamma_2 \) (i.e. the system works in dispersive nonlinearity regime) by the solid black line and the dashed red line, respectively. We see that \( \text{Im}\{\chi_{(3)}^{(2)}\} \) is much smaller than \( \text{Re}\{\chi_{(3)}^{(2)}\} \), which means that the optical absorption of the probe field is negligible, originated by the EIT effect and the condition of large one-photon detuning; moreover, \( \text{Re}\{\chi_{(3)}^{(2)}\} \) is an attractive potential well and there is a saturation near \( x = 0 \), which is due to the Rydberg blockade effect (with blockade radius \( \sim 7 \mu m \)) that suppresses the excitation of atoms to the Rydberg state and hence causes the nonlinear kernel \( \chi_{(3)}^{(2)} \) to saturate to a finite value. By virtue of the strong Rydberg-Rydberg interaction, the nonlocal optical nonlinearities can reach to \( |\text{d}x^2|\chi_{(3)}^{(2)}(r_0) \sim 10^{-8} \text{m}^2 / \text{V}^2 \), which have many order of magnitudes larger than conventional EIT systems [6, 14].

(3+1)D NONLINEAR ENVELOPE EQUATION

Our main aim is to implement an active control of LBS by the medium. To make the related physical mechanism transparent, we first derive the equation describing the non-linear evolution of the probe-field envelope. For a modulated plane-wave of the probe field, we assume \( \Omega_\eta \sim F \exp(i(K_z - \omega t)) \) [41]. The equation for the envelope \( F \) in the presence of the magnetic field can be derived by means of the multiple-scale perturbation method, similar to that carried out in Ref. [42]. We obtain

\[
\frac{\partial u}{\partial s} = -\left( \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) u - g_d \frac{\partial^2 u}{\partial \tau^2} - V_m(\xi, \eta) u + \left| W_1 \right| u^2 \\
+ \int d\mathbf{r}_0 W_2(\mathbf{r}_0 - \mathbf{r}_1)(u(\mathbf{r}_1, s))^2 \big| u + i\delta_0 u. \tag{6}
\]

Here \( s = z/(2L_{\text{diff}}) \); \( \mathbf{r}_1 = (\xi, \eta) / R_0 \); \( \tau = (1 - z / V_s) / \tau_0 \); \( V_s = (\partial K / \partial \omega)^{-1} \) is group velocity, with \( K = K(\omega) \) the linear dispersion relation; \( u = (F / U_0) \exp(-i\omega_0 \tau) \), with \( \omega_0 \equiv \Omega(\mathbf{K}) \) a decay constant; \( g_d \equiv -L_{\text{diff}} K_2 / \tau_0^2 \); \( W_1 \equiv -b \Delta_2 R_0^2 / c \); \( W_2 \equiv -b \Delta_2 R_0^2 / 3 \); and \( \delta_0 \equiv -2L_{\text{diff}} / L_A \) are dimensionless coefficients of dispersion, local Kerr nonlinearity, nonlocal Kerr nonlinearity, and absorption, respectively. In these coefficients, \( b = \hbar / (3 \omega_0^2) \); \( K_2 \equiv \partial^2 K / \partial \omega^2 \) describes group-velocity dispersion, \( L_{\text{diff}} \equiv k / \partial \omega^2 \) and \( L_A \equiv 1 / \alpha_0 \) are respectively the typical diffraction and absorption lengths, \( U_0 \) and \( R_0 \) are respectively the typical half Rabi frequency and transverse size of the probe field [39]. Since we are interested only in the dispersive nonlinearity regime of the system, where the \( L_A \) is much larger than the other typical lengths and hence \( \delta_0 \) is very small and the imaginary parts of the coefficients in Eq. (6) are negligible.

In Eq. (6), \( V_m = -k_2 R_0^2 \mu_3^{(2)} \) is a dimensionless linear potential contributed by the gradient magnetic field. It has the form

\[
V_m(\xi, \eta) = V_1 \xi + V_2 \eta, \tag{7a}
\]

\[
V_1 = \frac{\kappa_{12} R_0 L_{\text{diff}} ((\omega + \Delta_3) \Delta_3 \mu_{21} + |\Omega_\eta|^2 \mu_{31})}{\left[ (\omega + \Delta_2) ((\omega + \Delta_3) - |\Omega_\eta|^2) \right]^2} B_1, \tag{7b}
\]

\[
V_2 = \frac{\kappa_{12} R_0 L_{\text{diff}} ((\omega + \Delta_3) \Delta_3 \mu_{21} + |\Omega_\eta|^2 \mu_{31})}{\left[ (\omega + \Delta_2) ((\omega + \Delta_3) - |\Omega_\eta|^2) \right]^2} B_2. \tag{8}
\]

We then consider the formation of LBS when the gradient magnetic field is absent (i.e., \( V_m = 0 \)). In this case, stable (3+1)D LBS and vortices can form, with the result by a numerical simulation shown in Fig. 1(d). From the figure, we see that the (3+1)D LB (upper part) and vortex (lower part) relax to self-cleaned forms quite close to the unperturbed ones [43], and their shapes undergo no apparent change during propagation. The physical parameters used in the simulation are chosen \( \Delta_2 = -15 \Gamma_2, \Delta_3 = -0.02 \Gamma_2, R_0 = 10 \mu m, \tau_0 = 9 \times 10^{-7} \text{s}, N_e = 3 \times 10^{10} \text{cm}^{-3}, U_0 = 0.3 \Gamma_2, \) and \( C_0 \approx 2 \pi \times 81.6 \text{MHz} \mu \text{m} \) (for the principal quantum number \( n = 60 \)). With these parameters, we obtain \( L_{\text{diff}} = 1.36 \text{mm}, L_A = 907 \text{mm}, R_0 = 6 \mu m, g_d = 0.134, \delta_0 = -0.03 \). Such a LB can form in a very short distance and extremely low light power (~ 1 nW), which is due to the giant Kerr nonlinearity (contributed by the Rydberg-Rydberg interaction) and the ultra-slow propagation velocity of the probe pulse (~ 2.3 \times 10^{-6} c, contributed from the EIT effect).

MANIPULATION OF LBS

We now turn to investigate what will happen for a nonlocal LB when an external gradient magnetic field is present. As a first step, we consider the absence of the Kerr nonlinearity (i.e., \( W_1 = W_2 = 0 \)). Using the transformation \( u = u' \exp[i(V_1 \xi + V_2 \eta + V_3 s^2/3 + V_4 s^2/3)s], \) with \( \xi' = \xi - V_1 s^2/2, \) and \( \eta' = \eta - V_2 s^2/2, \) Eq. (6) is converted into the form

\[
\frac{\partial u'}{\partial s} = -i/2(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \eta'^2} + g_0 \frac{\partial^2}{\partial \tau^2}) u'. \tag{6}
\]

It is easy to obtain the expression of the central position of the probe pulse in the \( \xi' - \eta' \) plane, which is given by \( (\xi, \eta) = (V_1 s^2/2, V_2 s^2/2) \). Returning to the original variables, the central position of the pulse reads

\[
(x, y) = \kappa_{12} \frac{\left[ (\omega + \Delta_3) \Delta_3 \mu_{21} + |\Omega_\eta|^2 \mu_{31} \right] R_0^2}{\left[ (\omega + \Delta_2) ((\omega + \Delta_3) - |\Omega_\eta|^2) \right]^2 L_{\text{diff}}} \big( B_1, B_2 \big). \tag{8}
\]

We see that, due to the presence of the magnetic field, the motion of the linear wave is changed and its trajectory in the \( x-y \) plane has a deflection with a quadratic dependence on the propagation coordinate \( z \); moreover, the trajectory can be controlled by tuning the gradient of the magnetic field, i.e. by manipulating the parameters \( B_1 \) and \( B_2 \).
In the presence of the Kerr nonlinearities, it is hard to get an exact expression for the motion trajectory of the probe pulse. In this situation, however, one can obtain the trajectory deflection by resorting to a numerical simulation for solving Eq. (6). Fig. 2(a) shows the result of the 3D motion trajectory of a LB as a function of \(x/R_0\), \(y/R_0\), and \(z/(2L_{\text{diff}})\) in the presence of the gradient magnetic field \((B_1, B_2) = (3.2, 0) \text{ mG cm}^{-1}\). The corresponding trajectory in the \(x-z\) plane is illustrated in panel (b). We see clearly that the LB experiences a deflection due to the existence of the magnetic field. Shown in Fig. 2(c) is the result of the 3D motion trajectory of the LB for an increased magnetic-field gradient in the \(x\)-direction, by taking \((B_1, B_2) = (6.4, 0) \text{ mG cm}^{-1}\) (b). One sees that the trajectory of the LB is changed significantly due to the increase of the magnetic field.

In addition, more rich motion trajectories of the LB can be obtained by using different magnetic fields. To prove this, we consider a time-varying gradient magnetic field of the form

\[
B(x, t) = \mathbf{\hat{x}} B_0 \cos(\omega_0 t) \mathbf{x}, \tag{9}
\]

where \(\omega_0\) characterizes the motion period of the magnetic field in time. Fig. 3(a) shows the motion trajectory of the LB under the action of such a magnetic field. We see that the trajectory of the LB follows the variation of the magnetic field. Illustrated in Fig. 3(b) is the corresponding sinusoidal trajectory of the LB in the \(x-z\) plane. Obviously, one can use various magnetic fields to manipulate the motion of LBs; conversely, the trajectory-deflections of the LBs may be exploited to measure external magnetic fields.

**CONCLUSION**

We have shown that nonlocal LBs created in a cold Rydberg atomic gas can be actively manipulated by using a weak gradient magnetic field. In particular, the LBs can experience significant Stern-Gerlach deflections when a weak external magnetic field is applied, and their motion paths may be controlled through the adjustment of the magnetic-field gradient. The results reported here are useful not only for understanding novel properties of the LBs in nonlocal optical media but also for finding new ways for precision measurements of magnetic fields.

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[41] The frequency and wave number of the probe field are given by $\omega + \omega_p + K(\omega)$, respectively. Thus $\omega = 0$ corresponds to the center frequency of the probe field.


[43] In the related numerical simulation, a 10% random disturbance has been added into the initial condition.