

High-resolution 3D imaging through dense camouflage nets using single-photon LiDAR

Peng-Yu Jiang^{a,b,†}, Zheng-Ping Li^{a,b,c,†}, Wen-Long Ye^{a,b}, Ziheng Qiu^{a,b}, Da-Jian Cui^{c,d}, Feihu Xu^{a,b,c,*}

^aHefei National Research Center for Physical Sciences at the Microscale and School of Physical Sciences, University of Science and Technology of China, Hefei 230026, China

^bShanghai Research Center for Quantum Science and CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Shanghai 201315, China

^cHefei National Laboratory, University of Science and Technology of China, Hefei 230088, China

^dQuantum Information Chip & Device Chongqing Key Laboratory, Chongqing 400060, China

Abstract. The single-photon sensitivity and picosecond time resolution of single-photon light detection and ranging (LiDAR) can provide full-waveform profile for retrieving 3D profile of the target separated from foreground clutter. This capability has made single-photon LiDAR as a solution for imaging through obscurant, camouflage nets and semitransparent materials. However, the obstructive presence of the clutter and limited pixel number of single-photon detector arrays still pose challenges in achieving high-quality imaging. Here, we demonstrate a single-photon array LiDAR system combined with tailored computational algorithms for high-resolution 3D imaging through camouflage nets. For static targets, we develop a 3D sub-voxel scanning approach along with a photon-efficient deconvolution algorithm. Using this approach, we demonstrate 3D imaging through camouflage nets with a $3\times$ improvement in spatial resolution and a $7.5\times$ improvement in depth resolution compared with the inherent system resolution. For moving targets, we propose a motion compensation algorithm to mitigate the net's obstructive effects, achieving video-rate imaging of camouflaged scenes at 20 frames per second. More importantly, we demonstrate 3D imaging for complex scenes in various outdoor scenarios and evaluate the advanced features of single-photon LiDAR over both visible-light camera and mid-wave infrared (MWIR) camera. The results point a way forward for high-resolution real-time 3D imaging of multi-depth scenarios.

Keywords: single-photon LiDAR, single-photon imaging, Imaging through semitransparent materials, Time-of-flight, 3D imaging.

†Equal contribution. *Feihu Xu, feihuxu@ustc.edu.cn

1 Introduction

The ability to produce multi-layered 3D imagery is of utmost importance in various applications in remote sensing^{1,2} and rescue operations in surveillance and reconnaissance.^{3,4} The application scenarios include the viewing of partially obscured targets like camouflage nets, through semitransparent materials such as windows, and through distributed reflective media like foliage. However, conventional 2D optical imaging approaches exhibit poor performance in multi-layered scenarios with foreground clutter. This is primarily due to the lack of depth information, which prevents ef-

fective separation of the target from the foreground clutter. In recent years, active imaging methods have been proposed to tackle this challenge, including millimeter wave radar^{5,6} and Wi-Fi imaging.^{7,8} These methods leverage the inherent ability of mm-wave and Wi-Fi signals to penetrate the clutter in front of the target. Nevertheless, it is important to note that the resolution of these methods is relatively inferior compared to optical approaches. As a result, a combination of high-resolution optical imaging and active imaging methods has positioned LiDAR^{4,9} as a compelling technology for imaging partially concealed scenarios.

Single-photon LiDAR has witnessed rapid development because it can offer high temporal resolution and high sensitivity by using time-correlated single-photon counting (TCSPC) techniques.¹⁰⁻¹³ The high temporal resolution permits excellent surface-to-surface resolution for 3D imaging of multi-depth scenarios.¹⁴ The computational imaging algorithms have witnessed remarkable progress to process the single photon data of complex scenes efficiently.¹⁵⁻¹⁷ Moreover, the photon-efficient imaging algorithms¹⁸⁻²² have shown good performance dealing with low return signals and high background noise, which has been successfully demonstrated in several challenging scenarios including imaging through clutter,^{14,23} long-range depth imaging,²⁴⁻²⁹ non-line-of-sight imaging,³⁰⁻³² and imaging through high levels of scattering media.³³⁻³⁹

Researchers have foreseen the potential of single-photon LiDAR in imaging of camouflaged scenarios and carried out experiments. For instance, Jigsaw airborne system demonstrated the capability of detecting hidden objects using the foliage-penetrating 3D imaging.⁴ Wallace et al. reconstructed the depth profile of an object behind a wooden trellis fence using a scanning sensor.⁴⁰ Tobin et al. presented a scanning transceiver system for imaging targets through camouflaged nets.²³ These results employed a scanning-based single-photon LiDAR system, which requires a relatively long acquisition time. The emergence of single-photon avalanche diode (SPAD) detec-

tor arrays significantly decreases the acquisition time by collecting returned photons in a parallel way.⁴¹⁻⁴⁴ Tachella et al. utilized a SPAD array to show the remarkable real-time imaging through camouflage nets using the plug-and-play point cloud denoisers.¹⁵ However, due to technological constraints, SPAD array-based systems normally exhibit fewer pixels and lower time resolution compared to single-point scanning sensors, leading to reduced imaging resolution.

Here we present a single-photon LiDAR system based on an InGaAs/InP SPAD detector array to capture high-resolution 3D profiles of static and moving targets concealed by double-layer camouflage nets. For static objects, we reported a sub-voxel scanning approach^{45,46} combined with a 3D deconvolution algorithm to realize high-resolution imaging. Using this approach, we experimentally demonstrated 3D imaging through camouflage netting with a $3\times$ improvement in spatial resolution and a $7.5\times$ improvement in depth resolution. The average signal photons per pixel (PPP) is as few as 1.73 PPP and the acquisition time of each sub-voxel is 10 ms. For moving targets, we proposed a motion detection and compensation algorithm to mitigate the net's obstructive effects, achieving real-time imaging of camouflaged scenes at 20 frames per second. Different from ref.,¹⁵ our approach exploit the correlation between different frames which can avoid the loss of photons in some pixels due to the occlusion of the camouflage nets. To verify our approach, we perform a series of experiments in daylight and night for outdoor complex scene, and test the results through glass door and camouflage nets using single-photon LiDAR, visible-light camera, and mid-wave infrared (MWIR) camera.

2 Static target: 3D sub-voxel scanning approach

Constrained by factors such as material uniformity, circuit fabrication technology, data transmission, and cost, the format and time resolution of SPAD detector arrays are currently limited, pre-

venting them from achieving high-resolution imaging in multi-depth scenarios. To improve the depth resolution, Rapp et al. introduced subtractive dither to the temporal quantization of TC-SPC,⁴⁷ while Raghuram et al. achieved super-resolving transients by oversampled measurements.⁴⁸ Inspired by these works, we proposed the 3D sub-voxel scanning approach and a tailored spatial-temporal photon-efficient deconvolution algorithm (see Fig. 1), which can alleviate the block effect of the foreground clutter and achieve super-resolution image reconstruction with about 1 PPP. The sub-pixel scanning method, initially employed in conventional cameras, captures a series of low-resolution images during the sub-pixel displacement process, which can be reconstructed to obtain a high-resolution image.⁴⁹ This method has also been applied in single-photon imaging in recent years.⁵⁰ For instance, Li et al. utilized the sub-pixel scanning approach to achieve super-resolution imaging beyond the diffraction limit over long distances.^{45,46}

To achieve three-dimensional sub-voxel scanning, we employed a piezo tip/tilt platform with a mirror and an arbitrary function generator (AFG) for sub-pixel scanning in the spatial domain and sub-bin scanning in the temporal domain, respectively. To realize sub-pixel scanning, we set the inter-pixel scanning space smaller than the size of the field of view (FoV) of one pixel. As shown in Fig. 1(a), we illustrate an inter-pixel spacing of 1/2 FoV as an example. The inter-pixel shift was performed in both the x and y directions (Fig. 1(a)). After scanning all the pixels, a high-resolution image (320×320) was computed by combining frames of low-resolution images (64×64). In the temporal domain, considering that the time resolution of the time-to-digital converter (TDC) was 1 ns, the histogram bin width during the imaging process was set to 1 ns, corresponding to a depth resolution of 15 cm. To improve the depth resolution, we applied the AFG to perform sub-bin scanning with a step size of 1/10 of the bin width between the laser and the SPAD array (as shown in Fig. 1(b), an example of 1/3 sub-bin scanning). This enabled us to obtain 10 frames with

low temporal resolution, which could then be processed to derive an image with high temporal resolution.

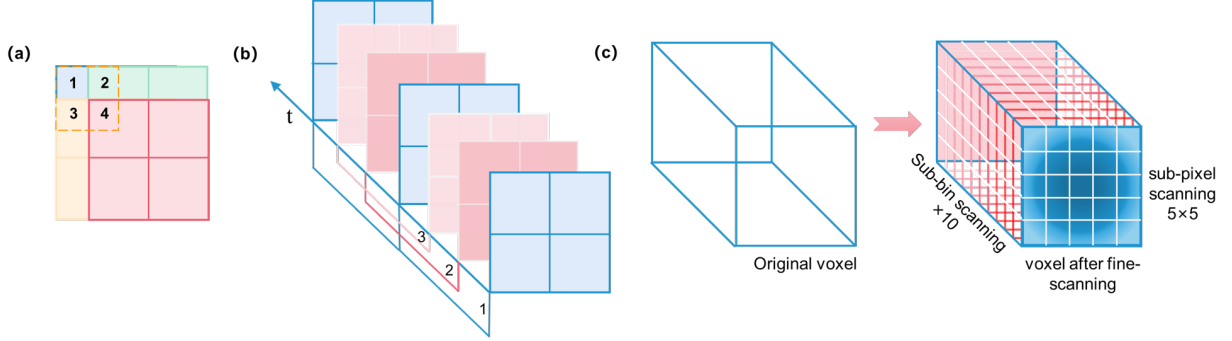


Fig 1 Schematic of the 3D sub-voxel scanning method. (a). An illustration of sub-pixel scanning using the SPAD array with 2×2 pixels and an inter-pixel spacing of $1/2$ FoV. (b). A scheme of sub-bin scanning in time domain with 3 steps. (c). One original voxel in the measurement matrix can be expanded to $5 \times 5 \times 10$ sub-voxels after fine scanning.

2.1 Spatial-temporal photon-efficient deconvolution algorithm

To cooperate with our 3D sub-voxel scanning approach, we develop a spatial-temporal photon-efficient deconvolutional algorithm to compute the high-resolution image from the sub-voxel scanning data with low signal levels at ~ 1 PPP. The algorithm takes the sub-voxel fine scanning process into account in the forward model and combines a 3D deconvolution method to retrieve the sub-voxel information from the acquired data.

In a single-photon LiDAR system, the laser emits pulses to periodically illuminate the target scene in either raster-scanned manner or flood-illuminated way. Using the technique of TCSPC, we measure the time-of-flight of each received photon and form a histogram which can be seen as the measured matrix. Taking the sub-voxel scanning process into consideration, the measured histogram in our experiment is spliced with all the histograms with low resolution in three dimensions according to their relative displacement. The integrated histogram is denoted as $Y_{x,y,t}$, where x, y, t represent the coordinates of the high-resolution matrix. We use $RD(x, y, \frac{2z}{c})$ to denote the

reflectivity and depth of the target scene whose (x, y) th element is a vector with only one nonzero entry to describe the (reflectivity, depth) pair of the scene. Combining the inhomogeneous Poisson photon-detection processing and our sub-voxel scanning process, the histogram matrix $Y_{x,y,t}$ can be written as:

$$Y_{x,y,t} \sim \text{Poisson}(IRF_{3D}(x, y, t) * RD(x, y, \frac{2z}{c}) + B) \quad (1)$$

$$IRF_{3D}(x, y, t) = PSF_{2D}(x, y) \otimes (Box(t) * IRF(t)) \quad (2)$$

where B is the background noise, IRF_{3D} represents the 3D instrument response function of our system combining the fine scanning process. As described in Eq. (2), the term PSF_{2D} is the point spread function of our system, which is discretized with the interval of sub-pixel after fine scanning. $Box(t)$ represents the box function with a width T equal to the temporal resolution of the TDC. $IRF(t)$ represents the instrument response function of our system in the temporal dimension, which includes the timing jitter of the laser, detector, and electronics circuit. And the symbol $*$, \otimes denotes the convolution and Cartesian product operation respectively. The IRF_{3D} can be calibrated by using a point object as the target. And the size of IRF_{3D} is dependent on the inter-voxel spacing of the scanning process.

To obtain the estimation of $RD(x, y, \frac{2z}{c})$ from the measured matrix $Y_{x,y,t}$, we treat the inverse problem as an optimization problem and adopted a deconvolutional convex optimization algorithm based on the forward model. Let $L_{RD}(RD; Y_{x,y,t}, IRF_{3D}, B)$ denotes the negative log-likelihood function of RD derived from Eq. (1). The inverse regularized convex problem can be described as:

$$\begin{aligned}
& \underset{\mathbf{RD}}{\text{minimize}} && L_{RD}(RD; Y_{x,y,t}, IRF_{3D}, B) + \beta \cdot \text{penalty}(RD) \\
& \text{subject to} && RD_{i,j,k} \geq 0, \forall i, j, k
\end{aligned} \tag{3}$$

Here, the constraint $RD_{i,j,k} \geq 0$ comes from the non-negativity of the reflectivity of the target scene. It is worth mentioning that our reconstruction framework is not tied to a particular choice of regularizers, and in our experiment, we exploited total variation (TV) constrains. We use the modified SPIRAL-TAP solver⁵¹ tailored for 3D spatial-temporal domain to solve this convex problem iteratively. To cooperate with the sub-voxel scanning scheme, the spacing size of the kernel IRF_{3D} needs to be adjusted according to the size of the inter-voxel spacing.

2.2 Numerical simulation

The numerical simulations are provided here to evaluate our sub-voxel scanning scheme. As shown in Fig. 2, we chose a typical scene from the Middlebury dataset of size 370×463 pixels as our target scene and simulated the practical low-light conditions by setting the signal-to-background ratio (SBR) ratio to 0.2, and the inter-pixel spacing to 1/5 FoV, which means a 5×5 PSF_{2D} . Here the PSF_{2D} was set to be a standard 2D Gaussian distribution while in the experiment we calibrated the PSF_{2D} with an approximate point light source. As for the IRF in temporal domain, we set it to be the convolution of a standard 1D Gaussian distribution representing the system’s jitter and a box function representing the 1/10 sub-bin scanning.

We conducted simulations using different numbers of detected signal photons per pixel, specifically 10, 5, and 1 PPP. We compared the reconstruction results with four different methods: no scanning, only two-dimensional spatial scanning, three-dimensional sub-voxel scanning using the maximum likelihood (ML) algorithm, and our proposed method. Quantitative results in terms of

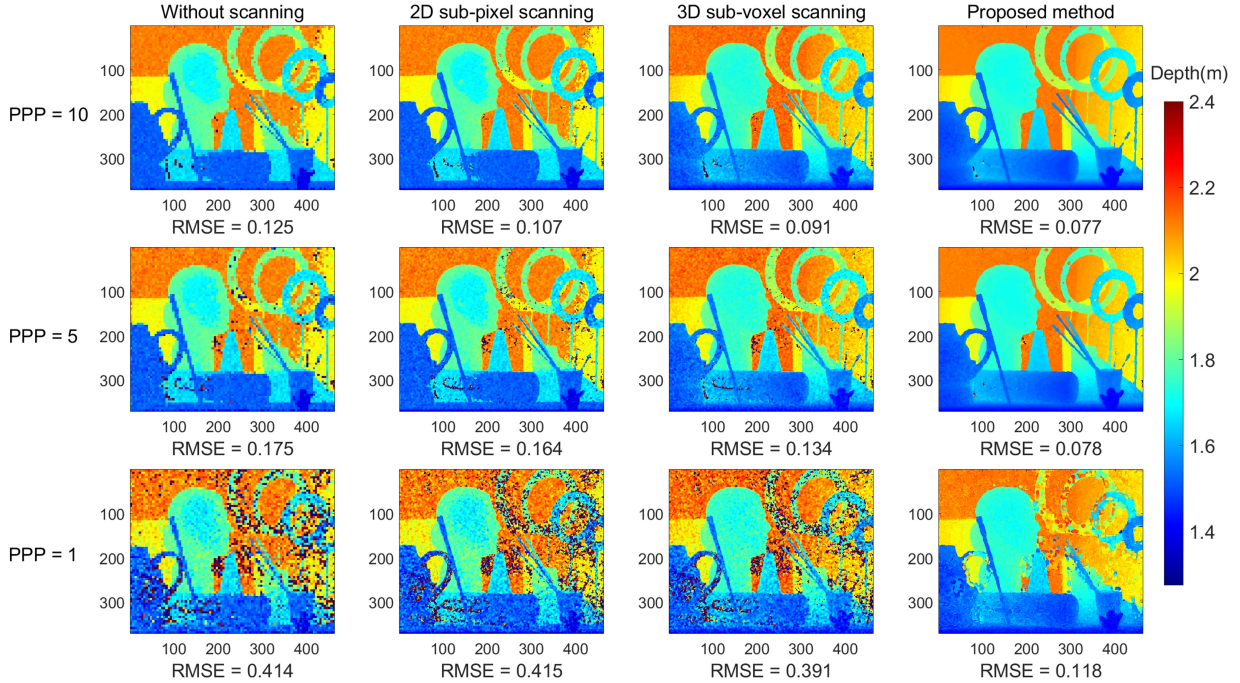


Fig 2 Numerical simulation of the proposed algorithm. The ground truth is a typical scene from the Middlebury dataset. In our simulation, the SBR is set to 0.2 and the average number of detected signal photons is set to 10, 5, and 1 PPP respectively. The first column shows the results without fine scanning. The second column shows the results with only 2D sub-pixel scanning and conventional pixel-wise ML processing. The third column shows the results with 3D sub-voxel scanning and pixel-wise ML processing. The last column shows the results with 3D sub-voxel scanning and our photon-efficient 3D deconvolutional algorithm. Quantitative results in terms of root mean square error (RMSE) are shown at the bottom of each figure. Clearly, our 3D sub-voxel method combined with the proposed algorithm has a smaller RMSE and superior performance to exhibit the details of the images.

root mean square error (RMSE) are presented at the bottom of each figure. As Fig. 2 shows, our algorithm outperforms the other methods in terms of imaging quality at 10, 5, and 1 average photon counts and exhibits the smallest RMSE.

3 Experiment setup

The experimental setup, depicted in Fig. 3, is compactly constructed on an aluminum breadboard and organized in a bistatic optical configuration. On the transmitting end, a pulsed fiber laser operating at 1550 nm is employed as the light source. The laser beam is transmitted through a fiber to provide flood illumination on the target scene. The laser operates at a repetition rate of

25 kHz, delivering an average power of 50 mW and a pulse width of 500 ps. The system exhibits a total jitter of approximately 1 ns. It is important to clarify that our method would have better outcomes when the system jitter is lower than the resolution of TDC. Limited by the jitter of our laser, the jitter of our system is slightly wider than the TDC resolution. However, compared with the approach of histogram interpolation, our method would still surpass it in results since that the interpolated histogram will suffer from severe distortion when the number of sample points is too small here. The utilization of a 1550 nm laser offers the advantage of being eye-safe, ensuring a safe distance for all ranges.

At the receiving end, a 64×64 pixel InGaAs/InP single-photon detector array is utilized for receiving returned photons parallel. The pixel pitch is $50 \mu\text{m}$, and each pixel is equipped with a time-to-digital converter providing a time resolution of 1 ns, corresponding to a distance resolution of 15 cm. The TDC modules integrated into the array measure the time difference between the start signal and photon events at each pixel. These measurements are then transferred to the computer via a CameraLink cable. The dark count rate of the detector is 2 kcps, and the width of the detection gate can be adjusted from 0 to 4000 ns. For the receiving optics, a commercially available camera lens (Thorlabs, MVL7000) is employed in the experiment. The chosen focal length is 25 mm, combined with the single-photon detector arrays, resulting in a field of view of 2 mrad per pixel. This field of view corresponds to a resolution of 6 cm at a distance of 30 m and the overall field of view of the system is 3.8 m. To eliminate solar noise and ensure continuous operation throughout the day, two filters are placed in front of the lens. One filter is a 1300 nm long-pass filter, and the other is a band-pass filter with a center wavelength of 1550.6 nm and a bandwidth of 1.8 nm.

An aluminum-coated mirror stucked on the piezo tip/tilt platform is adopted to achieve two-dimensional sub-pixel scanning of the target. The tip/tilt platform has a scanning range of 10

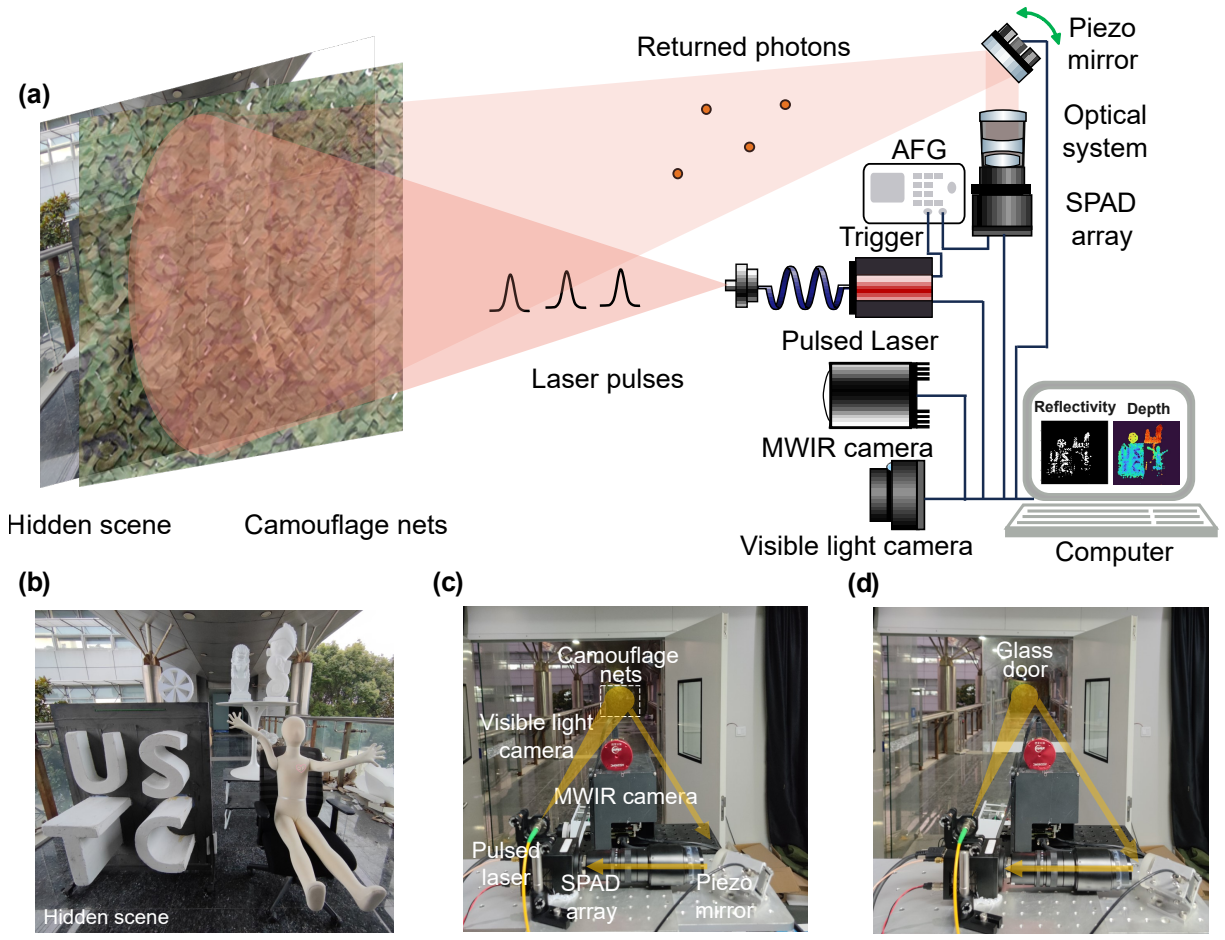


Fig 3 Schematic diagram of the experimental setup. (a). The schematic diagram of our experiment setup. The complex scenario is hidden behind a double-layer camouflage net, which is imaged by a visible-light camera, a MWIR camera, and our single-photon LiDAR system. (b). The photograph of the hidden scene. (c). The photograph of our experiment setup. (d). A photograph during the experiment with the glass door closed as an obstruction in the imaging path.

mrad and achieves a closed-loop accuracy as high as $1 \mu\text{rad}$, meeting the requirements for our fine scanning accuracy. Both the laser and SPAD detector arrays operate in external trigger mode and an arbitrary function generator (AFG) is employed in the system to provide a 25 kHz trigger signal for time synchronization. Sub-bin scanning in the temporal domain is achieved by scanning the delay between these two trigger signals.

In the experiment, a visible-light camera and an infrared camera are employed for imaging as a comparison among the three modalities. The visible-light camera (ASI294MC) has a resolution

of 4144×2822 pixels, with a pixel pitch of $4.63 \mu\text{m}$, and is equipped with a lens (Thorlabs, MVL25M23) which has a focal length of 25 mm. The infrared camera utilized is a cooled mid-wave infrared camera which employs HgCdTe as the detector material. It possesses a resolution of 640×512 pixels, with a pixel size of $15 \mu\text{m}$. During the experiment, the focal length of the infrared camera is set to 80 mm.

The experiment is conducted in an outdoor corridor where two layers of camouflage nets are placed 30 meters away from the imaging system to achieve dense occlusion of objects behind the nets. As shown in Fig. 3, the complex scene behind the net consists of letters, mannequins, sculptures, and other elements. The farthest point in the scene was approximately 2.3 meters away from the net. Additionally, to assess the penetration capability of different cameras through complex occlusions, a glass door was also used as an obstruction in the imaging path (shown in Fig. 3(d)).

4 Results for the static target

According to our forward model mentioned in section 2.1, we first calibrated the three-dimensional IRF_{3D} of the system to obtain a precise imaging model including the sub-voxel scanning process. We placed a $2 \text{ mm} \times 2 \text{ mm}$ square reflective sticker on a black plate to simulate an ideal point source. The target was finely scanned in the spatial domain with $1/5$ sub-pixel spacing (step size of $400 \mu\text{rad}$) and in the temporal domain with $1/10$ sub-bin spacing (step size of 100 ps). After this scanning process, we acquired the IRF_{3D} of the system corresponding to the process of $5 \times 5 \times 10$ sub-voxel scanning. The fitted spatial PSF PSF_{2D} and temporal response $Box * IRF$ are shown in the figure below:

After calibrating the system's IRF, we validated the three-dimensional super-resolution imag-

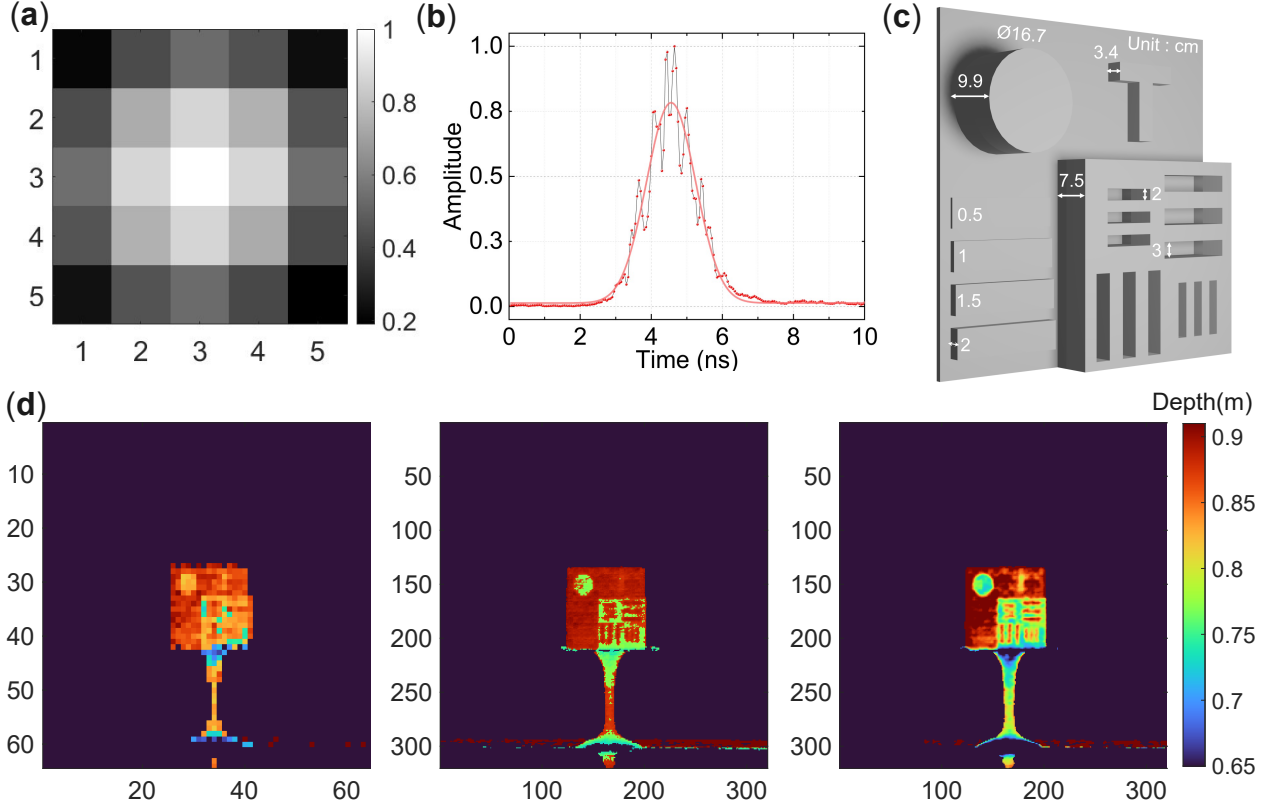


Fig 4 Calibration results of our imaging system. (a). The calibration result of the PSF_{2D} including 5×5 pixels. **(b).** The calibration result of $Box(t) * IRF(t)$. The red line is the Gaussian fit of the raw data. **(c).** The 3D model of the resolution chart. **(d).** The results were captured by single-photon LiDAR without fine scanning, with 2D sub-pixel scanning, and with 3D sub-voxel scanning, separately.

ing capability of our system by conducting measurements on a self-made 3D resolution chart. The photograph and dimensions of the resolution chart are illustrated in the figure below. We performed a $5 \times 5 \times 10$ steps sub-voxel scanning on the scene and applied the photon-efficient 3D deconvolution algorithm, incorporating the calibrated IRF. To evaluate the effectiveness of our method, we compared the results with those obtained without fine scanning and with only two-dimensional sub-pixel scanning. The figure below presents the comparative outcomes. From the results, it is evident that our method significantly improves both spatial and temporal resolution, achieving a spatial resolution of 2 cm and a depth resolution of 2 cm. In comparison to the system's inherent resolution, our method provides a spatial resolution improvement of three times and a depth resolution improvement of 7.5 times.

Subsequently, we employed our system to perform measurements of the complex scene behind the camouflage nets under different conditions and compared the imaging results with those captured by visible-light camera and MWIR camera. During all the experiments, the laser power was set to 50 mW, and the acquisition time for each frame of fine scanning was 10 ms, resulting in a total imaging time of 2.5 s. We first did the experiment in daylight of imaging through the double-layer camouflage nets and the results of different modalities are illustrated in Fig. 5. The four images stand for the results of visible-light camera, MWIR camera, single-photon LiDAR without fine scanning and our proposed method. In Fig. 5(a) and (b), most of the details of the scenario behind the nets are missing since these two passive modalities are lack of depth information to separate the nets from the camouflaged scenarios. In Fig. 5(d), it can be seen that our method exhibits superior capabilities to achieve high-quality imaging through the camouflage nets. The average photon count per pixel was 2.36 PPP, which demonstrated the photon-efficient capability of our algorithm.

We further carried out experiment in daylight with a glass door closed as another obstruction. The results are shown in Fig. 6. Compared with Fig. 5, it shows that the visible-light camera and single-photon LiDAR are almost unaffected while the MWIR camera failed to penetrate the glass, which means that the application of MWIR camera would be restricted when there are glasses involved in the path. With the same PPP as low as 1.73, the reconstructed result of our method in Fig. 6(d) is much better than the results without fine scanning shown in Fig. 6(c), which demonstrates that our method is efficient for dealing with the obstruction of the camouflage nets.

In Fig. 7, we show the results of the experiment at night to demonstrates the system's ability to operate effectively throughout the whole day. It can be seen that the imaging quality of the visible-light camera and the MWIR camera significantly degrades at night, while the reconstructed

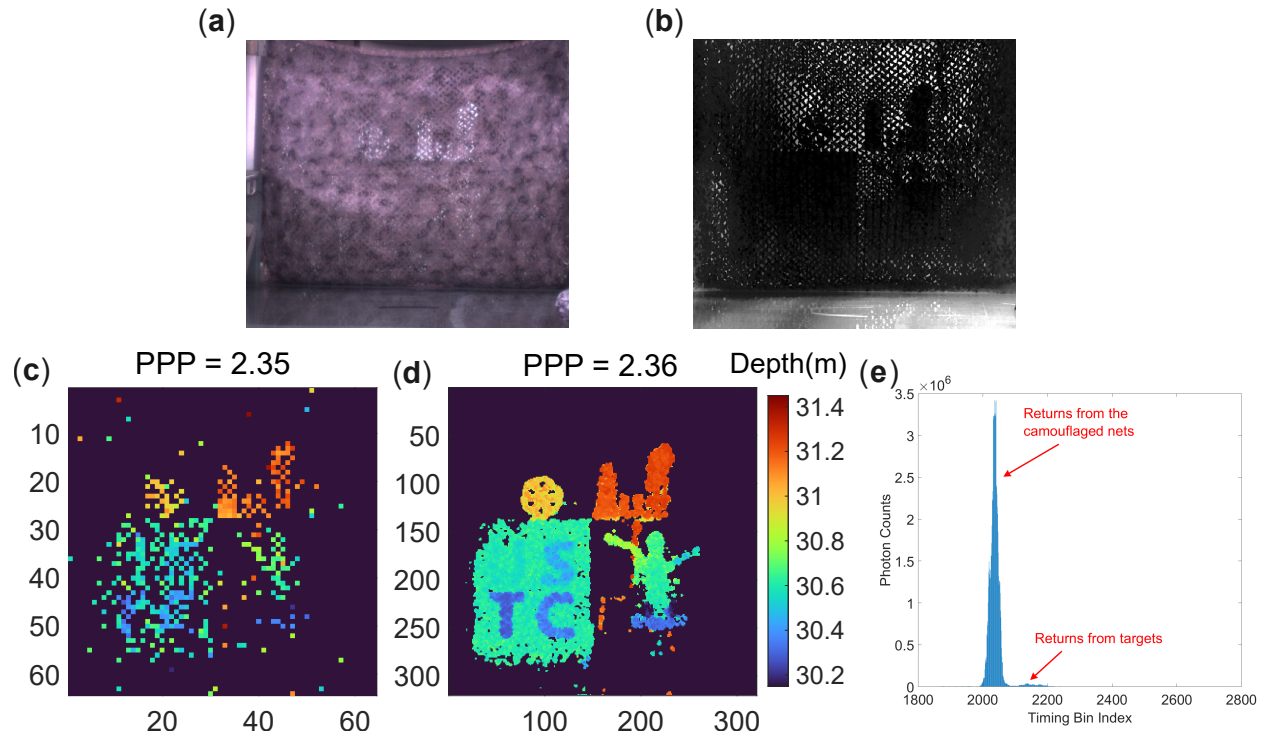


Fig 5 Experiment results of the static scenario behind the camouflage nets in daylight. The results of (a) visible-light camera, (b) MWIR camera, (c) single-photon LiDAR without fine scanning, (d) Single-photon LiDAR with 3D sub-voxel scanning, and (e) The timing histogram of the data in (d)

result of our method is not affected. By comparing the three experiments, we demonstrate 3D imaging for complex scene in various outdoor scenarios throughout the whole day and evaluate the advanced features of single-photon LiDAR over both visible-light camera and MWIR camera.

5 Moving target: Motion compensation method

For moving targets, sub-voxel scanning can result in image blurring and distortion due to the object's movement. If the SPAD detector array with 64×64 pixels is directly used for imaging of dynamic targets through camouflage nets, each frame will have a portion of the object blocked by the grid, and the limited number of pixels can lead to poor results. However, when the target is in motion, the blocked portion changes with each frame. If it is possible to detect the moving targets (rigid bodies) and their positions in each frame,⁵² the impact of grid obstruction can be

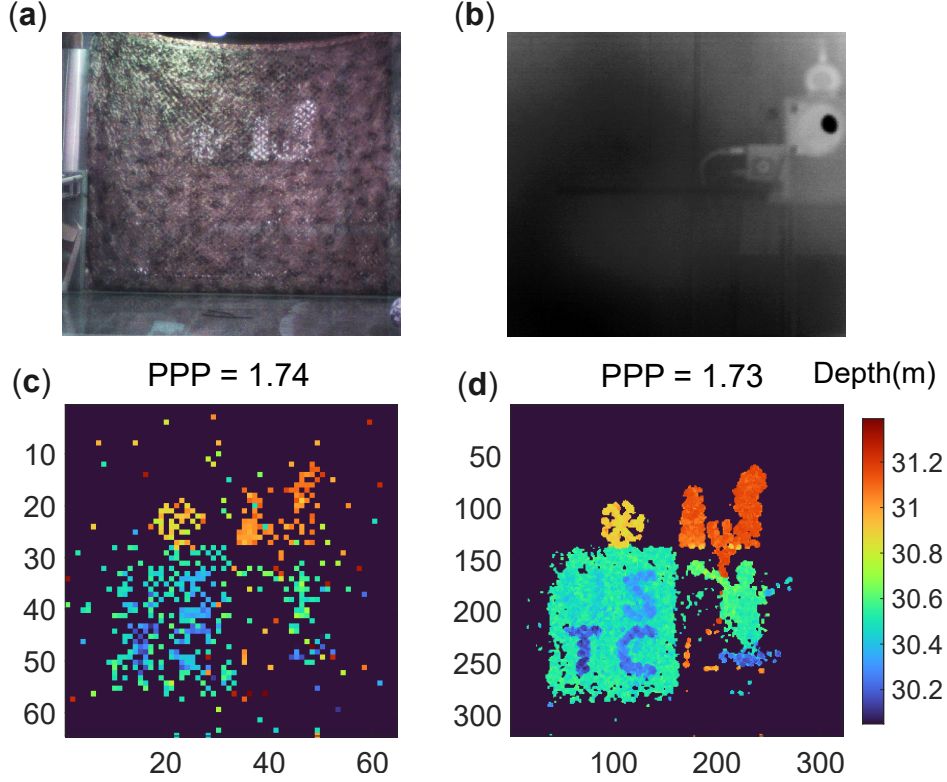


Fig 6 Experiment results of the static scenario behind the camouflage nets in daylight with a glass door. The results of (a) visible-light camera, (b) MWIR camera, (c) single-photon LiDAR without fine scanning, and (d) Single-photon LiDAR with 3D sub-voxel scanning. (a) and (b) are applied with contrast enhancement for better visual effects.

eliminated by using information collected from multiple frames. Moreover, due to the high frame rate requirements for imaging of moving target, the number of signal photons in a single frame is limited, which means that the results are easily affected by noise. The signal-to-noise ratio (SNR) of single-photon LiDAR can be obtained as:

$$SNR = \frac{n_s}{\sqrt{n_s + n_b}} \quad (4)$$

where n_s is the number of signal photons while n_b denotes the number of background noise photons. From Eq. (4), it can be seen that the SNR would be improved by \sqrt{n} by averaging over n frames as long as the inter-frame motion can be compensated effectively. Here we developed a motion compensation method for obstruction elimination and SNR improvement, by which the

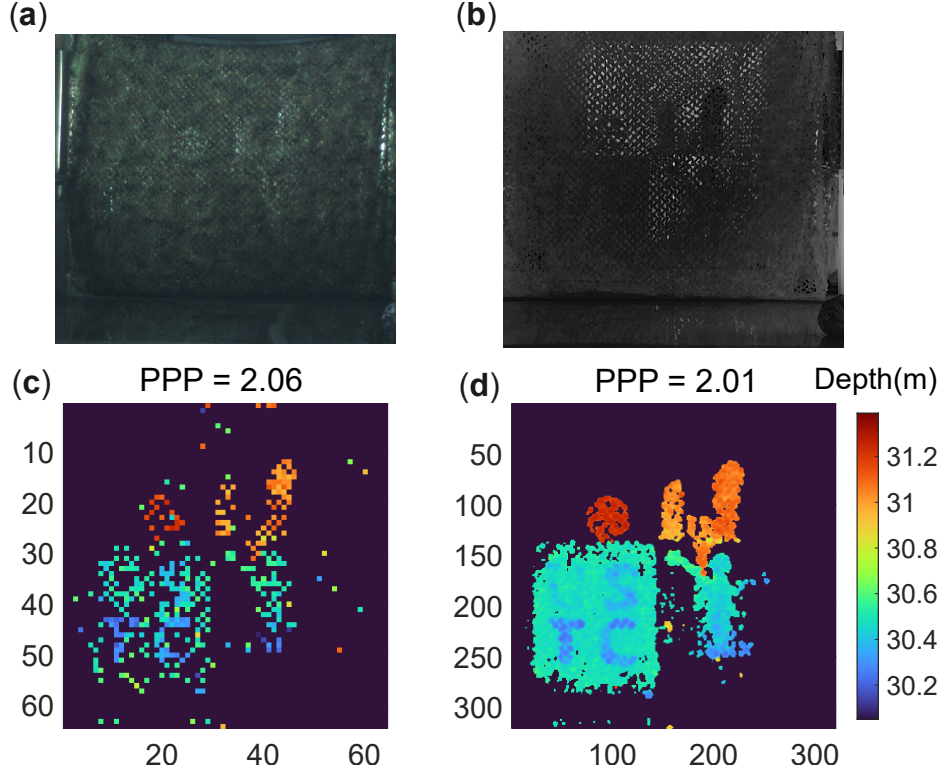


Fig 7 Experiment results of the static scenario behind the camouflage nets at night. The results of (a) visible-light camera, (b) MWIR camera, (c) single-photon LiDAR without fine scanning, and (d) Single-photon LiDAR with 3D sub-voxel scanning. (a) and (b) are applied with contrast enhancement for better visual effects.

motion states can be estimated and compensated before the low-quality sub-images are combined into a high-resolution image.

5.1 Motion compensation algorithm

Pre-processing: We use an approximate maximum likelihood estimator for each frame to perform the preliminary reconstruction. Firstly, for every pixel, the median of the bins is calculated as the ambient level which is denoted by b . Taking consideration of the Poisson noise on the bin counts, we set a threshold of h ,

$$h = b + 3\sqrt{b} \quad (5)$$

For each bin in the pixel, if the threshold h is exceeded, the bin count is reserved as signal photons while others are discarded as noise. After this denoising step, we estimate the depth and reflectivity of each pixel using the following approximate ML estimator:⁴⁴

$$d = \frac{\sum_{t=d_{max}-\Delta t}^{d_{max}+\Delta t} t \cdot \max(0, h_t - b)}{\sum_{t=d_{max}-\Delta t}^{d_{max}+\Delta t} \max(0, h_t - b)} \cdot \frac{c}{2} \quad (6)$$

$$r = \sum_{t=d_{max}-\Delta t}^{d_{max}+\Delta t} \max(0, h_t - b) \quad (7)$$

where h_t represents the histogram bin count at the given index of t , d_{max} denotes the index of the bin with the maximum count, and Δt is chosen corresponding to the width of the peak of the target. After calculating the depth and reflectivity of each pixel, we then obtained a depth map and a reflectivity map of every frame, which are used for object segmentation.

Object Segmentation: By utilizing the depth and reflectivity map of each frame acquired in the pre-processing step, we implement the object segmentation to the 3D imaging results with two steps: Firstly, using the depth map, we simply divide the pixels into groups with different depths, which can be seen as splitting the 3D scenario into depth slices. In details, we obtained the histogram of the depth map and found peaks in it. For each peak, we calculate the bottom width and gathered those pixels in the depth map which have values within that interval. In our experiment, we got two peaks and acquired two depth slice. One contains the mannequin while the other contains the man and the ball. Secondly, for each depth slice, we further use the reflectivity map and split pixels in each slice into groups according to the difference in reflectivity. The specific operations are the same as those in the first step. We used the differences in reflectivity to split the

ball and man, getting three separated objects. Up to here, we have got the boundary position of each object in vertical, horizontal, and depth axis, we can segment each object with a 3D box in the measurement histogram matrix.

Motion Estimation: For rigid objects, we utilize cross-correlation to obtain the motion of the target. We directly perform auto-correlation using the three-dimensional matrices obtained from the compensated frames and the reference frame. The three-dimensional matrices are first subjected to denoising processing used in the pre-processing step to remove most of the noise. The formula is as follows:

$$C = DFT^{-1}[DFT(H_{ref})^* \cdot DFT(H_c)] \quad (8)$$

where C is the correlation function, DFT denotes the 3D discrete Fourier transform, H_{ref} represents the measured histogram of the reference frame and H_c represents the measured histogram of current frame. Considering the limited motion velocity, we can only search the voxel near the position of the object in the previous frame, which would avoid ambiguity with similar objects and reduce the computation cost. At present, we didn't cope with the deformable objects. In the future, we intend to utilize state-of-art motion estimation algorithms such as block-matching algorithm and optical flow algorithm to deal with more complex motion.

Image Reconstruction With Motion Compensation: After object segmentation and motion estimation, we can extract the box containing objects from the reconstructed image, then we perform inverse motion transformation to keep the position of the object consistent with the reference state. After superimposing enough image subsets, we used the same method in pre-processing step to recover the depth and reflectivity map, for that the signals and SNR are high after compensation. Thus a high-resolution image, which we called the reference image, can be reconstructed.

Data Super-Resolution: The last optional step relates to data super-resolution to improve the spatial quality of the images. Since the acquired data from the SPAD detector array is at the relatively low spatial resolution of 64×64 , it is, therefore, necessary to simulate an improved spatial resolution using super-resolution approaches to improve visualization. Inspired by the depth maximum a-posteriori estimate, super-resolution can be performed using a combination of a weighted median operator with a point cloud filtering step. As for implementation, we used the approach described in,³⁴ which can be divided into three steps including interpolation, edge improvement, and filtered smoothing.

The framework of the motion compensation algorithm is described in Algorithm 1.

Algorithm 1 Motion compensation algorithm

- 1: Input:
 - 2: Histogram of current frame \mathbf{H}_k , number of frames for compensation n
 - 3: Pre-processing step:
 - 4: Denoising with the threshold \mathbf{h}_k defined in Eq. (5)
 - 5: Estimate $\mathbf{d}_k, \mathbf{r}_k$ from \mathbf{H}_k using Eq. (6) and Eq. (7)
 - 6: Object Segmentation:
 - 7: Segment each object in depth and reflectivity map then project to the histogram
 - 8: Motion Estimation:
 - 9: Doing cross-correlation between \mathbf{H}_k and \mathbf{H}_{k-n} (the histogram of frame k-n to frame k-1) using Eq. (8) and get the motion matrix \mathbf{T}_k for all segmented objects
 - 10: Image reconstruction:
 - 11: Get the histogram after motion compensation $\tilde{\mathbf{H}}_k = \mathbf{H}_k + \mathbf{T}_k \mathbf{H}_k$
 - 12: Compute $\tilde{\mathbf{d}}_k$ and $\tilde{\mathbf{r}}_k$ using Eq. (6) and Eq. (7)
 - 13: Super resolution step:
 - 14: Compute $\tilde{\mathbf{d}}_k^{HR}$ and $\tilde{\mathbf{r}}_k^{HR}$ with interpolation, self weight median and smoothing operations
 - 15: Output:
 - 16: $\tilde{\mathbf{d}}_k^{HR}, \tilde{\mathbf{r}}_k^{HR}$
-

5.2 Experiment and results

The experimental setup remains consistent, as illustrated in Fig. 3. For moving targets, we employed the motion compensation algorithm which was discussed in the previous section. The

scene behind the camouflage nets included a mannequin moving from right to left which was positioned approximately 1.5 meters away from the net, as well as a person bouncing a basketball at a distance of about 3 meters. During the experiment, we collected data over a period of 10 seconds, which amounted to 250,000 frames for the SPAD detector array. A total of 1,250 frames of data were processed to generate a single processed image, resulting in an imaging frame rate of 20 fps.

The existence of the double-layer camouflage nets severely limited the number of signal photons in each frame of the image, particularly in regions with low reflectivity where the signal was substantially attenuated. Directly applying the Poisson maximum likelihood estimation algorithm to each frame of image data would result in poor image reconstruction quality. Therefore, we employed the motion compensation algorithm described in the previous section to process the data. Specifically, we first performed pre-processing step on current frame and conducted target segmentation of the scene using the processed image and the signal histogram. This allowed us to isolate and extract targets such as the mannequin, body, arms, and ball. Then we calculated the displacement of each target relative to the previous n frames using Eq. (8), and subsequently applied motion compensation to each component accordingly. This compensation step accounted for the motion and helped align the targets in subsequent frames, improving the overall image quality. Finally, we upsampled the compensated frames from 64×64 pixels to 256×256 pixels by adopting the super-resolution algorithm for dynamic targets, which incorporates interpolation, self-guided weighted-median filtering, and smoothing filtering techniques to enhance the details and overall resolution of the reconstructed image. By implementing these steps, we aimed to overcome the limitations imposed by the double-layer camouflage nets and improve the quality and clarity of the final images.

We extracted four frames from the processed video (please refer to the supplementary video) for

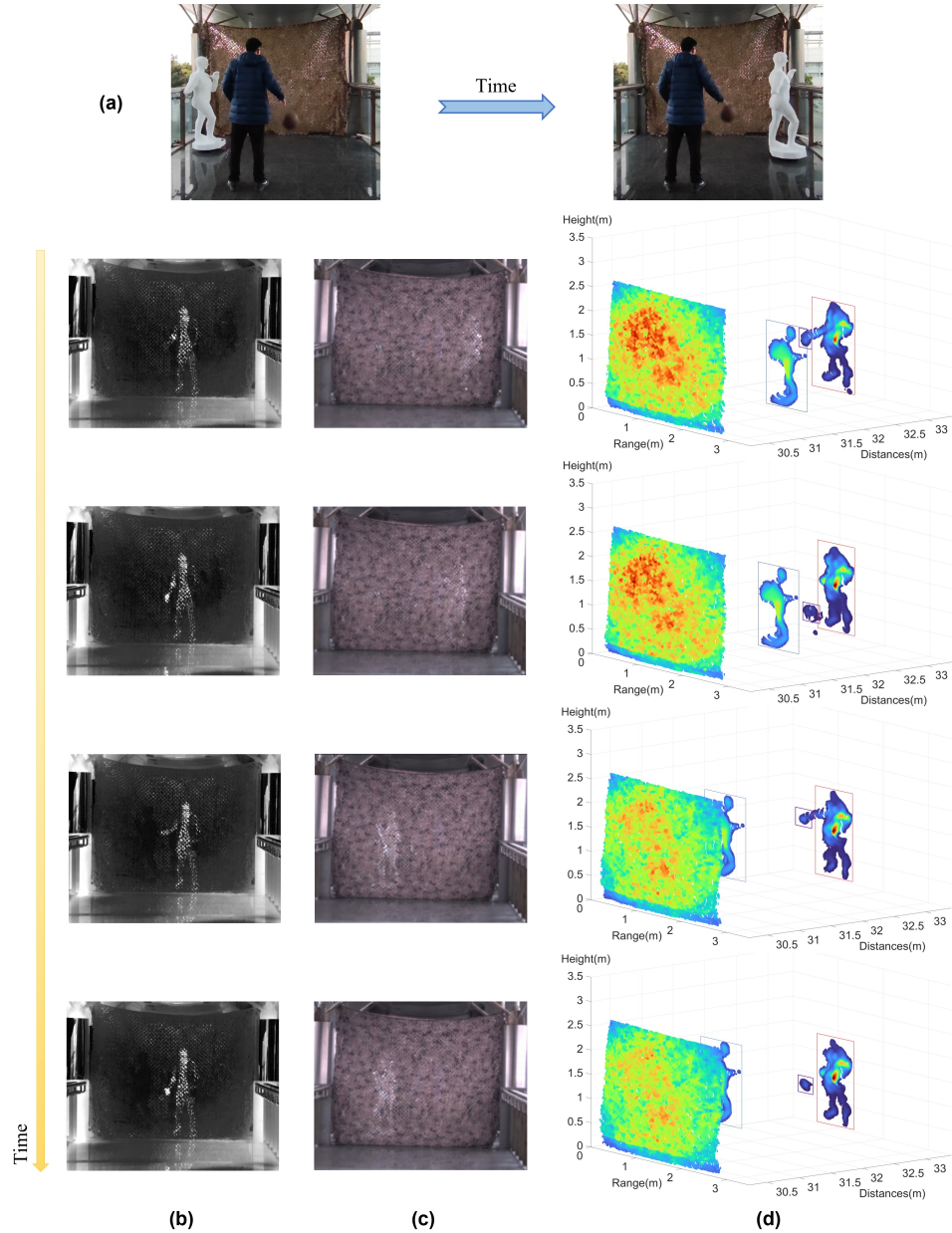


Fig 8 Experiment results of the moving scenario behind the camouflage nets. (a). The photographs of the scene behind the camouflage nets taken from backward. **(b).** The photographs were captured by MWIR camera. **(c).** The photographs were captured by a visible-light camera. **(d).** The reconstructed 3D profile of the multi-layer scenario. The movement of the mannequin and the basketball can be seen in the image sequences. The boxes of different colors indicate the segmented objects in our experiment.

presentation, as depicted in Fig. 8. The first column displays images extracted from the video captured by a visible-light camera, the second column shows images from the MWIR camera, and the third column presents the reconstructed 3D profile acquired from the SPAD array. From the results,

we can see that the visible-light camera exhibits poor performance in imaging the human body with low reflectivity, while the MWIR camera performs poorly in capturing the mannequin with low temperatures. However, our approach using single-photon LiDAR achieves high-resolution three-dimensional imaging of all targets behind the net, providing significant advantages compared to the previous two cameras, which verifies the effectiveness of our motion compensation algorithm for single-photon imaging through the camouflage nets.

As shown in Fig.9, we also compared our algorithm with cross-correlation method and the real-time plug-and-play denoiser.¹⁵ Due to the obstruction of the dense camouflaged nets, many of the pixels contains few photon signals especially in the region with low reflectivity (the legs of the man for wearing black pants), which leads to the loss of information in the reconstructed results shown in Fig 8(a) and (b). Our proposed motion compensation algorithm mitigates the net’s obstructive effect by using spatio-temporal correlation between each frame. The reconstruction result of our algorithm is more complete and continuous, which also captures additional details in the contours of the 3D target.

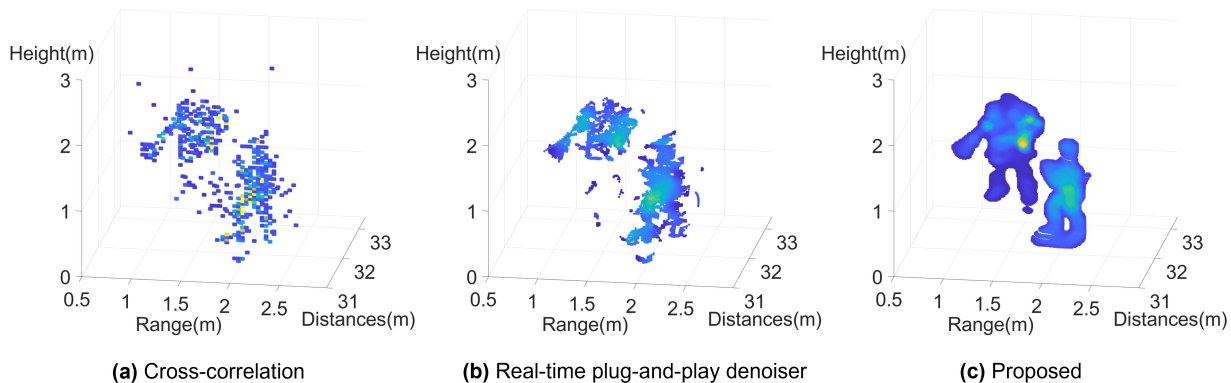


Fig 9 Comparison of 3D reconstruction methods for moving targets behind the camouflage nets. The data is the same as the first frame displayed in Fig. 8. Reconstruction results of (a) cross-correlation, (b) real-time plug-and-play denoiser,¹⁵ and (c) the proposed method.

6 Conclusion

This paper presents reconstructions of high-resolution 3D profiles for both static and moving targets that are obscured by camouflage nets. For static objects, we developed a sub-voxel scanning approach combined with a photon-efficient 3D deconvolution algorithm, which was demonstrated through numerical and experimental analysis. In our experiments, we achieved a $3\times$ improvement in spatial resolution and a $7.5\times$ improvement in depth resolution. The captured results, obtained in both daylight and night conditions, highlight the adaptability of our system for all-time applications. Moreover, our imaging results exhibit significant advantages compared to visible-light cameras and MWIR cameras. Regarding moving objects, we proposed a 3D motion compensation algorithm, enabling the capture of high-quality 3D video at 20 fps through camouflage nets. The method presented in this paper demonstrates the potential for implementing single-photon LiDAR in remote sensing, rescue operations, and defense, particularly when objects of interest are partially concealed or viewed through semitransparent surfaces, such as windows. Future work will focus on reducing the processing time for real-time applications.

Disclosures

The authors declare no conflicts of interest.

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References

- 1 J. J. Degnan, “Photon-counting multikilohertz microlaser altimeters for airborne and spaceborne topographic measurements,” *Journal of Geodynamics* **34**(3-4), 503–549 (2002).
- 2 A. M. Wallace, A. McCarthy, C. J. Nichol, *et al.*, “Design and evaluation of multispectral lidar for the recovery of arboreal parameters,” *IEEE Transactions on Geoscience and Remote Sensing* **52**(8), 4942–4954 (2013).
- 3 M. A. Albota, B. F. Aull, D. G. Fouche, *et al.*, “Three-dimensional imaging laser radars with geiger-mode avalanche photodiode arrays,” *Lincoln Laboratory Journal* **13**(2), 351–370 (2002).
- 4 R. M. Marino and W. R. Davis, “Jigsaw: a foliage-penetrating 3d imaging laser radar system,” *Lincoln Laboratory Journal* **15**(1), 23–36 (2005).
- 5 D. M. Sheen, D. L. McMakin, and T. E. Hall, “Three-dimensional millimeter-wave imaging for concealed weapon detection,” *IEEE Transactions on microwave theory and techniques* **49**(9), 1581–1592 (2001).
- 6 V. M. Patel, J. N. Mait, D. W. Prather, *et al.*, “Computational millimeter wave imaging: Problems, progress, and prospects,” *IEEE Signal Processing Magazine* **33**(5), 109–118 (2016).
- 7 F. Adib and D. Katabi, “See through walls with wifi!,” in *Proceedings of the ACM SIGCOMM 2013 conference on SIGCOMM*, 75–86 (2013).
- 8 C. R. Karanam and Y. Mostofi, “3d through-wall imaging with unmanned aerial vehicles

- using wifi,” in *Proceedings of the 16th ACM/IEEE International Conference on Information Processing in Sensor Networks*, 131–142 (2017).
- 9 V. Molebny, P. McManamon, O. Steinvall, *et al.*, “Laser radar: historical prospective—from the east to the west,” *Optical Engineering* **56**(3), 031220–031220 (2017).
 - 10 A. McCarthy, R. J. Collins, N. J. Krichel, *et al.*, “Long-range time-of-flight scanning sensor based on high-speed time-correlated single-photon counting,” *Applied optics* **48**(32), 6241–6251 (2009).
 - 11 J. Rapp, J. Tachella, Y. Altmann, *et al.*, “Advances in single-photon lidar for autonomous vehicles: Working principles, challenges, and recent advances,” *IEEE Signal Processing Magazine* **37**(4), 62–71 (2020).
 - 12 C. Zhang, Y. Wang, Y. Yin, *et al.*, “High precision 3d imaging with timing corrected single photon lidar,” *Optics Express* **31**(15), 24481–24491 (2023).
 - 13 R. H. Hadfield, J. Leach, F. Fleming, *et al.*, “Single-photon detection for long-range imaging and sensing,” *Optica* **10**(9), 1124–1141 (2023).
 - 14 D. Shin, F. Xu, F. N. Wong, *et al.*, “Computational multi-depth single-photon imaging,” *Optics express* **24**(3), 1873–1888 (2016).
 - 15 J. Tachella, Y. Altmann, N. Mellado, *et al.*, “Real-time 3d reconstruction from single-photon lidar data using plug-and-play point cloud denoisers,” *Nature communications* **10**(1), 1–6 (2019).
 - 16 A. Halimi, R. Tobin, A. McCarthy, *et al.*, “Robust restoration of sparse multidimensional single-photon lidar images,” *IEEE Transactions on Computational Imaging* **6**, 138–152 (2019).

- 17 J. Tachella, Y. Altmann, X. Ren, *et al.*, “Bayesian 3d reconstruction of complex scenes from single-photon lidar data,” *SIAM Journal on Imaging Sciences* **12**(1), 521–550 (2019).
- 18 A. Kirmani, D. Venkatraman, D. Shin, *et al.*, “First-photon imaging,” *Science* **343**(6166), 58–61 (2014).
- 19 D. Shin, F. Xu, D. Venkatraman, *et al.*, “Photon-efficient imaging with a single-photon camera,” *Nature communications* **7**(1), 1–8 (2016).
- 20 J. Rapp and V. K. Goyal, “A few photons among many: Unmixing signal and noise for photon-efficient active imaging,” *IEEE Transactions on Computational Imaging* **3**(3), 445–459 (2017).
- 21 D. B. Lindell, M. O’Toole, and G. Wetzstein, “Single-photon 3d imaging with deep sensor fusion,” *ACM Trans. Graph.* **37**(4), 113–1 (2018).
- 22 J. Peng, Z. Xiong, H. Tan, *et al.*, “Boosting photon-efficient image reconstruction with a unified deep neural network,” *IEEE Transactions on Pattern Analysis and Machine Intelligence* (2022).
- 23 R. Tobin, A. Halimi, A. McCarthy, *et al.*, “Long-range depth profiling of camouflaged targets using single-photon detection,” *Optical Engineering* **57**(3), 031303–031303 (2018).
- 24 M. Laurenzis, F. Christnacher, and D. Monnin, “Long-range three-dimensional active imaging with superresolution depth mapping,” *Optics letters* **32**(21), 3146–3148 (2007).
- 25 A. M. Pawlikowska, A. Halimi, R. A. Lamb, *et al.*, “Single-photon three-dimensional imaging at up to 10 kilometers range,” *Optics express* **25**(10), 11919–11931 (2017).
- 26 Z. Li, E. Wu, C. Pang, *et al.*, “Multi-beam single-photon-counting three-dimensional imaging lidar,” *Optics express* **25**(9), 10189–10195 (2017).

- 27 Z.-P. Li, X. Huang, Y. Cao, *et al.*, “Single-photon computational 3d imaging at 45 km,” *Photonics Research* **8**(9), 1532–1540 (2020).
- 28 Z.-P. Li, J.-T. Ye, X. Huang, *et al.*, “Single-photon imaging over 200 km,” *Optica* **8**(3), 344–349 (2021).
- 29 P.-Y. Jiang, Z.-P. Li, and F. Xu, “Compact long-range single-photon imager with dynamic imaging capability,” *Optics Letters* **46**(5), 1181–1184 (2021).
- 30 M. O’Toole, D. B. Lindell, and G. Wetzstein, “Confocal non-line-of-sight imaging based on the light-cone transform,” *Nature* **555**(7696), 338–341 (2018).
- 31 X. Liu, I. Guillén, M. La Manna, *et al.*, “Non-line-of-sight imaging using phasor-field virtual wave optics,” *Nature* **572**(7771), 620–623 (2019).
- 32 C. Wu, J. Liu, X. Huang, *et al.*, “Non-line-of-sight imaging over 1.43 km,” *Proceedings of the National Academy of Sciences* **118**(10), e2024468118 (2021).
- 33 G. Satat, M. Tancik, and R. Raskar, “Towards photography through realistic fog,” in *2018 IEEE International Conference on Computational Photography (ICCP)*, 1–10, IEEE (2018).
- 34 R. Tobin, A. Halimi, A. McCarthy, *et al.*, “Robust real-time 3d imaging of moving scenes through atmospheric obscurant using single-photon lidar,” *Scientific reports* **11**(1), 1–13 (2021).
- 35 Y. Zhang, S. Li, J. Sun, *et al.*, “Three-dimensional single-photon imaging through realistic fog in an outdoor environment during the day,” *Optics Express* **30**(19), 34497–34509 (2022).
- 36 H. Shi, G. Shen, H. Qi, *et al.*, “Noise-tolerant bessel-beam single-photon imaging in fog,” *Optics Express* **30**(7), 12061–12068 (2022).

- 37 P.-Y. Jiang, Z.-P. Li, W.-L. Ye, *et al.*, “Long range 3d imaging through atmospheric obscurants using array-based single-photon lidar,” *Optics Express* **31**(10), 16054–16066 (2023).
- 38 A. Maccarone, F. M. Della Rocca, A. McCarthy, *et al.*, “Three-dimensional imaging of stationary and moving targets in turbid underwater environments using a single-photon detector array,” *Optics express* **27**(20), 28437–28456 (2019).
- 39 A. Maccarone, K. Drummond, A. McCarthy, *et al.*, “Submerged single-photon lidar imaging sensor used for real-time 3d scene reconstruction in scattering underwater environments,” *Optics Express* **31**(10), 16690–16708 (2023).
- 40 A. M. Wallace EURASIP Member, J. Ye EURASIP Member, N. J. Krichel, *et al.*, “Full waveform analysis for long-range 3d imaging laser radar,” *EURASIP Journal on Advances in Signal Processing* **2010**, 1–12 (2010).
- 41 D. Bronzi, F. Villa, S. Tisa, *et al.*, “Spad figures of merit for photon-counting, photon-timing, and imaging applications: a review,” *IEEE Sensors Journal* **16**(1), 3–12 (2015).
- 42 R. K. Henderson, N. Johnston, F. M. Della Rocca, *et al.*, “A 192x128 time correlated spad image sensor in 40-nm cmos technology,” *IEEE Journal of Solid-State Circuits* **54**(7), 1907–1916 (2019).
- 43 K. Morimoto, A. Ardelean, M.-L. Wu, *et al.*, “Megapixel time-gated spad image sensor for 2d and 3d imaging applications,” *Optica* **7**(4), 346–354 (2020).
- 44 I. Gyongy, S. W. Hutchings, A. Halimi, *et al.*, “High-speed 3d sensing via hybrid-mode imaging and guided upsampling,” *Optica* **7**(10), 1253–1260 (2020).
- 45 Z.-P. Li, X. Huang, P.-Y. Jiang, *et al.*, “Super-resolution single-photon imaging at 8.2 kilometers,” *Optics express* **28**(3), 4076–4087 (2020).

- 46 E. Wade, A. McCarthy, R. Tobin, *et al.*, “Micro-scanning of a focal plane detector array in a single-photon lidar system for improved depth and intensity image reconstruction,” in *Emerging Imaging and Sensing Technologies for Security and Defence VII*, **12274**, 16–22, SPIE (2022).
- 47 J. Rapp, R. M. Dawson, and V. K. Goyal, “Improving lidar depth resolution with dither,” in *2018 25th IEEE International Conference on Image Processing (ICIP)*, 1553–1557, IEEE (2018).
- 48 A. Raghuram, A. Pediredla, S. G. Narasimhan, *et al.*, “Storm: Super-resolving transients by oversampled measurements,” in *2019 IEEE International Conference on Computational Photography (ICCP)*, 1–11, IEEE (2019).
- 49 S. C. Park, M. K. Park, and M. G. Kang, “Super-resolution image reconstruction: a technical overview,” *IEEE signal processing magazine* **20**(3), 21–36 (2003).
- 50 C. Callenberg, A. Lyons, D. d. Brok, *et al.*, “Super-resolution time-resolved imaging using computational sensor fusion,” *Scientific reports* **11**(1), 1689 (2021).
- 51 Z. T. Harmany, R. F. Marcia, and R. M. Willett, “This is spiral-tap: Sparse poisson intensity reconstruction algorithms—theory and practice,” *IEEE Transactions on Image Processing* **21**(3), 1084–1096 (2011).
- 52 G. M. Martín, A. Halimi, R. K. Henderson, *et al.*, “High-ambient, super-resolution depth imaging with a spad imager via frame re-alignment,” in *International Image Sensor Workshop 2021*, (2021).

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