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The influence of Landau quantization on the propagation of solitary structures in collisional plasmas

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Abstract

In this work, we study damped ion acoustic solitary wave structures in magnetized dense plasmas. The collisional effects of ions with electrons and neutrals are considered. The trapping effects of electrons and Landau quantization are included in the plasma model under consideration. We assume that magnetic field is quantized such that the condition $k_B T \ll \hbar\omega_{\rm ce}$ is satisfied. We have derived the damped Korteweg–de Vries (dKdV) equation by using small amplitude reductive perturbation technique. The time-dependent analytical and numerical solutions of the dKdV equation are presented. For numerical solutions we apply a two level finite difference scheme with the help of the Runge Kutta method. The effects of variations of different plasma parameters on the propagation characteristics of damped solitary structures in the presence of collisions are discussed.

Keywords: soliton, collisional plasmas, finite difference method, reductive perturbation method

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum plasma is gaining the attention of many researchers due to its importance in modern technology [1–13]. Different characteristics of plasma waves have been investigated in the quantum plasmas by considering different quantum mechanical effects. Haas *et al* [14] studied two-species quantum hydrodynamic model and observed periodic patterns in such type of plasmas. A quantum parameter was identified in their work which is proportional to diffraction effects of the waves in these plasmas. Three species quantum hydrodynamic model which contains positrons with ions and electrons was studied by Ali *et al* [15].

In the presence of magnetic field, plasma waves in quantum plasmas have also been investigated by many researchers. Mushtaq *et al* [16] studied the low frequency magnetosonic waves by incorporating the spin effects via spin force and macroscopic spin magnetization current. The authors discussed the slow and fast dynamics of nonlinear structures (solitons and shocks) and found that spin quantum plasmas differ significantly from the non-spin quantum plasmas. Low frequency waves propagating in dense astrophysical plasmas in the presence of exchange-correlation potential effects of degenerate electrons are studied by Hussain *et al* [17].

The electrons rotate in circular orbits in the presence of magnetic field $\boldsymbol{B}(0, 0, B_0)$. In the perpendicular direction the energy is quantized while along the direction of magnetic field the energy is expressed as $E_z = \frac{p_z^2}{2m_e}$. In the presence of spin the energy is expressed as $\varepsilon_e^{l,\delta} = (2l + 1 + \delta)\beta B_0 + \frac{p_z^2}{2m_e}$ where $\beta = \frac{|e|\hbar}{2m_e c}$ is Bohr magneton and δ describes the spin orientation $\vec{s} = \frac{1}{2}\vec{\delta}$ $(\delta = \pm 1)$. Here l resents the Landau level having values 0, 1, 2,$\cdots$. The lowest energy level is achieved for $l = 0$ and $s = -1$ and pair of degenerate level is obtained for $s = +1$ case. Thus each value with $l \neq 0$ occurs twice. In the non relativistic limit total energy is expressed as $\varepsilon_e^{l,\delta} = \varepsilon_e^l = 2l\beta H_0 + \frac{p_z^2}{2me}$. The quantized energy in the presence of trapping potential is given by

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expression $\varepsilon_e^l = l\hbar\omega_{ce} + \frac{p_z^2}{2me} - e\phi$, where $e\phi$ is the electrostatic potential energy of well in which electrons are trapped [18]. In the presence of trapping potential and quantization of magnetic field plasma waves have also been investigated. The effective influence of the quantization of orbital motion of electrons and the spin of electrons on the propagation of plasma waves in quantum plasmas has been reported [19]. The excitation of nonlinear structures in the presence of Landau quantization have been investigated by Shaukat [20] and it is shown that compressive solitary structures are formed under these effects.

Many researchers have studied plasma waves by taking into account the collisional phenomena between plasma species and neutrals as well [21–23]. For example, Haas and Bret [24] investigated Buneman instability in quantum collisional plasmas and showed that due to this instability, density oscillations are excited. Ion acoustic solitary structures in the presence of collisions in quantum plasma are governed by damped Korteweg–de Vries (dKdV) [25]. Using quantum hydrodynamic model in relativistic quantum plasma, ion acoustic solitons are studied by Sahu *et al* [26]. It was shown that both positive and negative type solitary structures exist in the presence of collisions.

The objective of this research is to investigate the ion acoustic solitary structures in dense collisional plasmas in the presence of trapping effects and Landau quantization. The set of equations, governing the plasma system are presented in section 2. Normalization of basic equation is written in section 3. We have separated the equations in component form in section 4. The equations governing the dynamics of nonlinear structure are described in section 5. The results are discussed in section 6 and work is summarized in section 7 of the manuscript.

2. Set of dynamic equations

We consider two component ion electron magnetized quantum plasma. The ions are taken classical, owing the fact that they have large inertia in comparison to electrons. The collisions, which are responsible for dissipation, of plasma species with neutrals are also considered. The continuity equation representing the ion dynamics is written as follows,

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla} \cdot (n_i \boldsymbol{v}_i) = 0, \tag{1}$$

here n_i and $\boldsymbol{v}_i$ are representing the density and velocity of ions respectively.

The momentum equation of ion is given by

$$\frac{\partial \boldsymbol{v}_i}{\partial t} + (\boldsymbol{v}_i.\boldsymbol{\nabla})\boldsymbol{v}_i = \frac{e}{m_i}\left(\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v}_i \times \boldsymbol{B})\right) + \nu_i \boldsymbol{v}_i, \tag{2}$$

here $\boldsymbol{E}$ and $\boldsymbol{B}$ are electric and magnetic fields, ν_i is the collision frequency of ions with electrons and neutrals such that $\nu_i = \nu_{ie} + \nu_{in}$ [27].

Poisson equation is written as,

$$\boldsymbol{\nabla}.\boldsymbol{E} = 4\pi e(n_i - n_e). \tag{3}$$

By introducing trapping and Landau quantization effects the electron density after integration over the polar coordinates and change of variables from momentum p to energy ε is given by [20],

$$n_e = \frac{p_{\mathrm{Fe}}^2 \eta}{2\pi^2\hbar^3}\sqrt{\frac{m_e}{2}}\sum_{l=0}^{\infty}\int_0^{\infty}\frac{\varepsilon^{-\frac{1}{2}}}{\exp\left\{\frac{\varepsilon - e\phi - \mu + l\hbar\omega_{ce}}{T}\right\} + 1}\mathrm{d}\varepsilon. \tag{4}$$

In the above expression p_{Fe} is the electron Fermi momentum, η is defined as $\hbar\omega_{ce}/\varepsilon_{\mathrm{Fe}}$, where ω_{ce} is electron cyclotron frequency, $\varepsilon_{\mathrm{Fe}}$ is Fermi energy and $\hbar$ is the Plank's constant divided by 2π. $e\phi$ is the electrostatic potential energy of the well in which electrons are trapped and μ is the chemical potential. T is system temperature and l is Landau levels having values $l = 0, 1, 2, \ldots$.

3. Scaling for normalization of model equation

$$\begin{aligned}
&v \to v_i/c_s && t \to t\omega_{ci}\\
&\nu_i \to \nu_i/\omega_{ci} && \eta \to \eta = \hbar\omega_{ce}/\varepsilon_{\mathrm{Fe}}\\
&x \to x/\rho_q && n_j \to n_j/n_{j0}\\
&\Phi \to e\Phi/\varepsilon_{\mathrm{Fe}} && T \to \pi T/2^{3/2}\varepsilon_{\mathrm{Fe}}.
\end{aligned}$$

4. Set of equations in component form

Now linearizing equations (1)–(3) and assuming that wave is propagating obliquely in xy plane i.e. $\nabla = (\partial_x, \partial_y, 0)$ and magnetic field is lying along x-axis. We obtain the basic equations in component form as follow.

The ion continuity equation is expressed as,

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_{ix})}{\partial x} + \frac{\partial(n_i v_{iy})}{\partial y} = 0. \tag{5}$$

x, y, z components of ion momentum equation are described in the following way,

$$\frac{\partial v_{ix}}{\partial t} + \left(v_{ix}\frac{\partial}{\partial x} + v_{iy}\frac{\partial}{\partial y}\right)v_{ix} = -\frac{\partial \Phi}{\partial x} + \nu_i v_{ix}, \tag{6}$$

$$\frac{\partial v_{iy}}{\partial t} + \left(v_{ix}\frac{\partial}{\partial x} + v_{iy}\frac{\partial}{\partial y}\right)v_{iy} = -\frac{\partial \Phi}{\partial y} + v_{iz} + \nu_i v_{iy}, \tag{7}$$

$$\frac{\partial v_{iz}}{\partial t} + \left(v_{ix}\frac{\partial}{\partial x} + v_{iy}\frac{\partial}{\partial y}\right)v_{iz} = -v_{iy} + \nu_i v_{iz}. \tag{8}$$

Poisson equation is written in the following form,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi = n_e - n_i. \tag{9}$$

The equilibrium density of plasma species is $N_0 = \frac{p_{\mathrm{Fe}}^3}{3\pi^2\hbar^3}$ with Fermi energy given as $\varepsilon_{\mathrm{Fe}} = \left(\frac{\hbar^2}{2m_e}\right)(3\pi^2 N_0)^{2/3}$ [28, 29]. The density of electrons by using the procedure given in [30] can be expressed in the following form,

$$n_e = \frac{3}{2}\eta(1+\Phi)^{1/2} + (1+\Phi-\eta)^{3/2} - \frac{T^2}{2}\eta(1+\Phi)^{-3/2} + T^2(1+\Phi-\eta)^{-1/2}. \quad (10)$$

5. dKdV equation

To investigate the damping effects due to collisions of ions with neutrals and electrons in the presence of trapping potential and Landau quantization we use the reductive perturbation scheme [31] as follows

$$\xi = \epsilon^{1/2}(l_x x + l_y y - \lambda t),\quad \tau = \epsilon^{3/2} t,$$

where λ is the normalized phase velocity of the wave propagating obliquely in xy plane and direction cosines along x and y axes are l_x and l_y respectively such that $l_x^2 + l_y^2 = 1$. The size of perturbed amplitude is determined by small expansion parameter ϵ such that $(0 < \epsilon \leqslant 1)$.

The variables n_i, v_{ix}, v_{iy}, v_{iz}, Φ and ν in terms of the smallness parameter ϵ, are expanded in the following form,

$$\begin{aligned} n_i &= 1 + \epsilon n_{i1} + \epsilon^2 n_{i2} + \cdots,\\ v_{ix} &= \epsilon v_{ix1} + \epsilon^2 v_{ix2} + \cdots,\\ v_{iy} &= \epsilon^2 v_{iy1} + \epsilon^3 v_{iy2} + \cdots,\\ v_{iz} &= \epsilon^{\frac{3}{2}} v_{iz1} + \epsilon^{\frac{5}{2}} v_{iz2} + \cdots,\\ \Phi &= \epsilon \Phi_1 + \epsilon^2 \Phi_2 + \cdots,\\ \nu &= \epsilon^{\frac{3}{2}} \nu_0. \end{aligned} \quad (11)$$

Now putting the above perturbation scheme in the equations (5)–(10) and obtaining the lowest order of ϵ as follows:

$\sim\epsilon^{3/2}$ of ion continuity equation gives,

$$n_{i1} = \frac{l_x v_{ix1}}{\lambda}. \quad (12)$$

$\sim\epsilon^{3/2}$ of x component of ion momentum equation yields the following relation,

$$\Phi_1 = \frac{\lambda v_{ix1}}{l_x}. \quad (13)$$

$\sim\epsilon^{3/2}$ of y component of ion momentum equation is expressed as follows,

$$l_y \frac{\partial \Phi_1}{\partial \xi} = v_{iz1}. \quad (14)$$

$\sim\epsilon^2$ of z component of ion momentum equation gives a relation between v_{iz1} and v_{iy1},

$$\lambda \frac{\partial v_{iz1}}{\partial \xi} = v_{iy1}. \quad (15)$$

$\sim\epsilon$ of Poisson equation is expressed as follows,

$$n_{i1} - \frac{3}{2}\left[\frac{\eta}{2}(1+T^2) + (1-\eta)^{1/2} - \frac{T^2}{3}(1-\eta)^{-3/2}\right]\Phi_1 = 0. \quad (16)$$

The phase velocity of the wave propagating obliquely is obtained in the following form,

$$\lambda = \frac{l_x}{\sqrt{\frac{3}{2}\left[\frac{\eta}{2}(1+T^2) + (1-\eta)^{1/2} - \frac{T^2}{3}(1-\eta)^{-3/2}\right]}}. \quad (17)$$

Next higher order i.e. $\sim\epsilon^{5/2}$ of continuity equation gives,

$$-\lambda \frac{\partial n_{i2}}{\partial \xi} + \frac{\partial n_{i1}}{\partial \tau} + l_x \frac{\partial v_{ix2}}{\partial \xi} + l_y \frac{\partial v_{iy1}}{\partial \xi} + l_x \frac{\partial (n_{i1} v_{ix1})}{\partial \xi} = 0. \quad (18)$$

Similarly next higher order i.e. $\sim\epsilon^{5/2}$ of x component of momentum equation gives,

$$\frac{\partial v_{ix1}}{\partial \tau} - \lambda \frac{\partial v_{ix2}}{\partial \xi} + v_{ix1} l_x \frac{\partial v_{ix1}}{\partial \xi} = -l_x \frac{\partial \Phi_2}{\partial \xi} + \nu_0 v_{ix1}. \quad (19)$$

$\sim\epsilon^{5/2}$ of y component of momentum equation is written in the following way,

$$-\lambda \frac{\partial v_{iy1}}{\partial \xi} = -l_y \frac{\partial \Phi_2}{\partial \xi} + v_{iz2}. \quad (20)$$

$\sim\epsilon^{5/2}$ of z component of momentum equation of ions is expressed as,

$$-\lambda \frac{\partial v_{iz2}}{\partial \xi} + \frac{\partial v_{iz1}}{\partial \tau} + v_{ix1} l_x \frac{\partial v_{iz1}}{\partial \xi} = -v_{iy2} + \nu_0 v_{iz1}. \quad (21)$$

$\sim\epsilon^2$ of Poisson's equation gives the following expression,

$$\begin{aligned} \frac{\partial^2 \Phi_1}{\partial \xi^2} = {} & \frac{3}{2}\left[\frac{\eta}{2}(1+T^2) + (1-\eta)^{1/2} - \frac{T^2}{3}(1-\eta)^{-3/2}\right]\Phi_2 \\ & - \frac{3}{8}\left[\frac{\eta}{2}(1+5T^2) - (1-\eta)^{-1/2} - T^2(1-\eta)^{-5/2}\right](\Phi_1)^2 - n_{i2}. \end{aligned} \quad (22)$$

Finally, eliminating n_{i2}, Φ_2, v_{ix2}, v_{iy2} and v_{iz2} from equations (18)–(22), we obtain the dKdV equation in the following form,

$$\frac{\partial \Phi_1}{\partial \tau} + A\Phi_1 \frac{\partial \Phi_1}{\partial \xi} + B \frac{\partial^3 \Phi_1}{\partial \xi^3} + C\Phi_1 = 0, \quad (23)$$

where the nonlinear, dispersive and dissipative coefficients are defined as

$$A = \frac{\lambda}{2 l_x^2}\left(\frac{3 l_x^4}{\lambda^2} - 2\gamma\lambda^2\right),$$

where γ is defined as

$$\gamma = \frac{3}{8}\left[(1-\eta)^{-1/2} - \frac{\eta}{2}(1+5T^2) + T^2(1-\eta)^{-5/2}\right],$$
$$B = \frac{\lambda^3}{2l_z^2}(l_x^2 + l_y^2),$$
$$C = \frac{\nu_0}{2}.$$

For limiting case i.e. $\nu_0 = 0$, which represents the absence of collisions we get well known KdV equation, which describes a completely integrable Hamiltonian system and has a solution [25] in the following form,

$$\Phi_1 = \Phi_0 \sec\mathrm{h}^2\left(\frac{\xi - V_0\tau}{\Delta}\right). \tag{24}$$

The maximum amplitude Φ_0 and width Δ of the solitons are expressed as follows:

$$\Phi_0 = \frac{3V_0}{A}, \qquad \Delta = \sqrt{\frac{4B}{V_0}}. \tag{25}$$

Now in the presence of collisions the energy of the system is not conserved and time-dependent solution of dKdV is presented in the following form [32, 33]:

$$\Phi_1(\xi, \tau) = \Phi_0(\tau) \times \sec\mathrm{h}^2\left(\sqrt{\frac{A\Phi_0(\tau)}{12B}}\left(\xi - \frac{A}{3}\int_0^t \Phi_0(\acute{\tau})\mathrm{d}\acute{\tau}\right)\right). \tag{26}$$

$V_0(\tau)$ is the velocity of the soliton, and $\Phi_m(\tau)$ and $\Delta(\tau)$ represent the time-dependent peak amplitude and width of single pulsed soliton respectively,

$$V_0(\tau) = \frac{A\Phi_0(0)}{3}\exp\left(\frac{-4C\tau}{3}\right), \tag{27}$$

$$\Phi_m(\tau) = \Phi_0(0)\exp\left(\frac{-4C\tau}{3}\right), \tag{28}$$

$$\Delta(\tau) = \sqrt{\frac{12B}{A\Phi_0(0)}}\exp\left(\frac{2C\tau}{3}\right). \tag{29}$$

The solution given in equation (26) is approximated soliton solution which exhibits fairly well for systems having dissipation. The above solution indicates that velocity, amplitude and width of the soliton depend on the dissipation, present in the plasma system. Velocity and amplitude of the structure decay exponentially with passage of time whilst width enhances exponentially.

6. Numerical plots and discussion

In this section we investigate the excitation of damped solitary structures in the presence of trapping potential and Landau quantization. We solved equation (26) numerically by using two level finite difference scheme with the help of Runge Kutta method. For this purpose we use the solution (24) of KdV equation as initial Guess. For time-dependent numerical solution, normalized values of different plasma parameters are taken and have observed the damped solitary

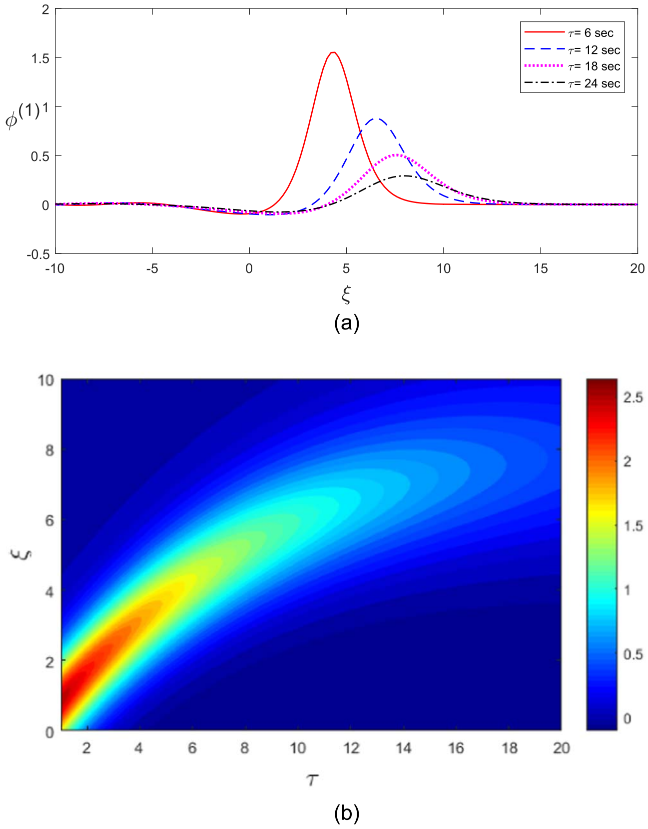

Figure 1. (a) Time evolution of solitary structures in the presence of collisions at different times (b) propagation of nonlinear structure in time-space plane for the same condition with plasma parameters $T = 0.4$, $\eta = 0.5$, $\nu_0 = 0.15$, $l_y = 0.3$ respectively.

structures at different times. It is seen in the presence of dissipation in the plasma system, with the passage of time as structure propagates, the amplitude of the soliton decreases and width increases which demonstrate the process of damping as shown in figures 1(a) and (b) respectively. Now we have studied the variations in the obliqueness angle and observed that by increasing the value of l_y the amplitude of the wave does not alter and width of the nonlinear structure decreases as depicted in figure 2. Now we have increased the value of quantizing parameter η which is defined as $\hbar\omega_{ce}/\varepsilon_{Fe}$, and it depends on Fermi energy and cyclotron frequency and have observed that width of solitary structures increases at certain fixed value of time as shown in figure 3. Physically this result indicates that for fixed values of plasma density the increment in magnetic field strength, the width of the structure flourishes. This was obvious, η is directly proportional to the ω_{ce} which depends on the magnetic field strength. In the presence of stronger magnetic field the solitary structure with larger width would be retrieved. The variations in the collisions frequency ν_0 of ions with electrons and neutrals have revealed the fact that at high values of collision frequency the nonlinear structures are damped and by lowering the value of collision frequency we get the nonlinear structure without damping as displayed in figure 4. It can be seen that with gradual decrease in dissipation the stable structure are appearing. The damping is decreased with decreasing values

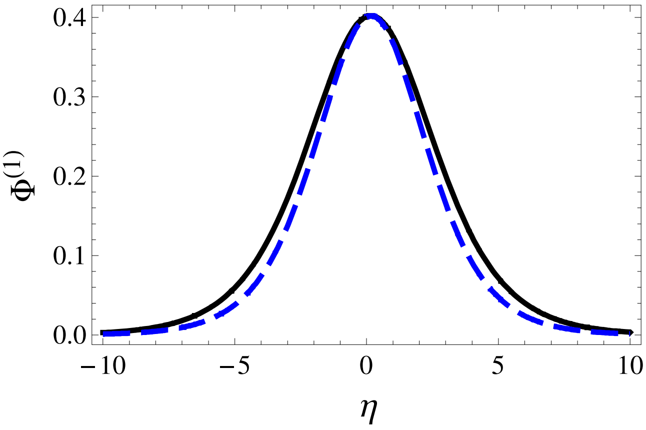

Figure 2. Variations in the width of the solitary structures are shown for different values of obliqueness parameter i.e. $l_y = 0.3$ (bold curve), 0.9 (dashed curve) with other plasma parameters $\eta = 0.3$, $\nu_0 = 0.3$, $T = 0.1$ respectively.

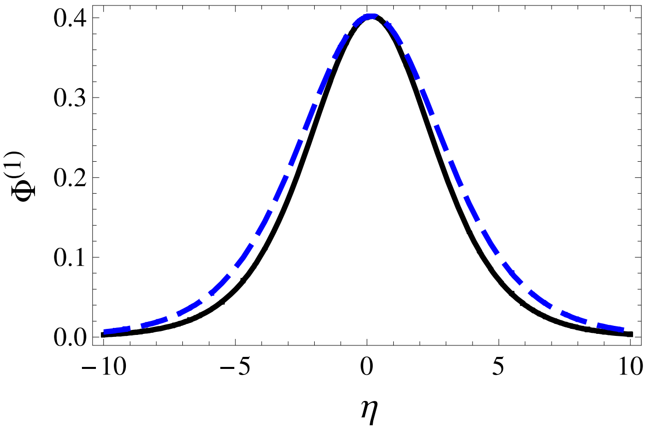

Figure 3. Increment in the width of the solitary structures are shown for different values of quantizing parameter i.e. $\eta = 0.3$ (bold curve), 0.9 (dashed curve) with other plasma parameters $l_y = 0.3$, $\nu_0 = 0.3$, $T = 0.1$ respectively.

of damping frequency. The figure 5 is showing same result in time-space plane indicating that in the presence of collisions the width is enhancing or the structure is damping as shown in figure 5(a). The figures 5(b) and (c) are showing that at lower values of collisions smooth curves in the space-time plane are obtained.

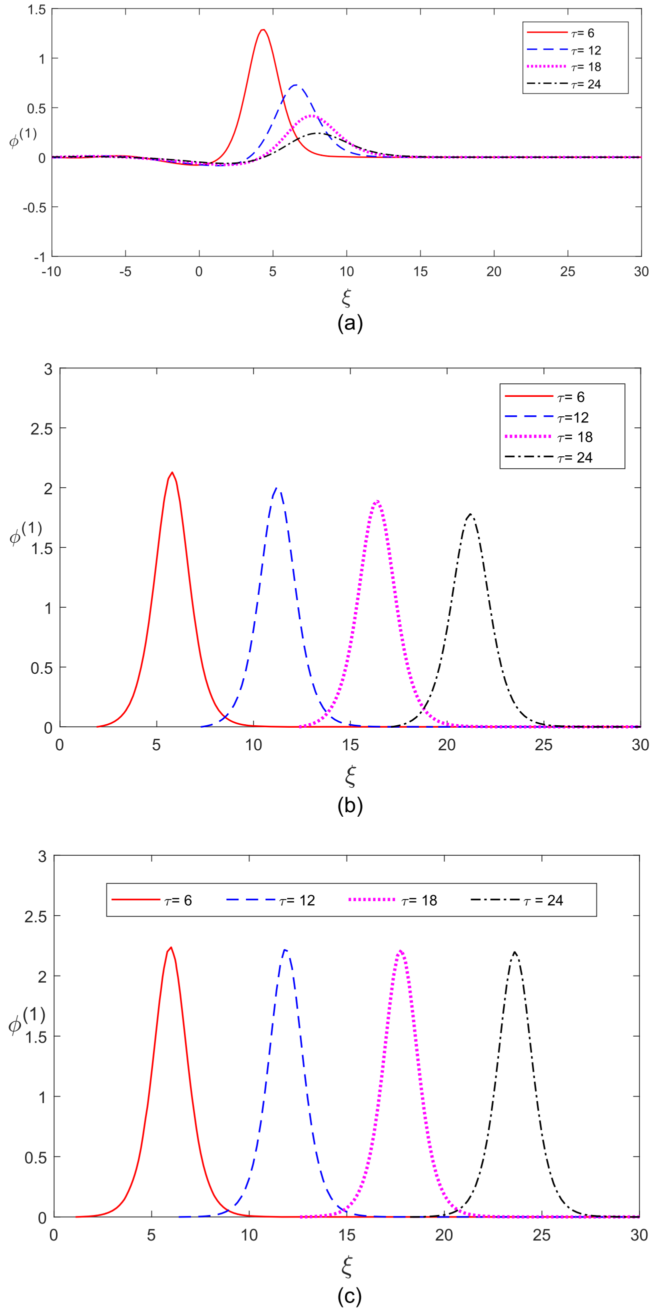

Figure 4. Time evolution of solitary structures with different values of collision frequency i.e. (a) $\nu_0 = 0.15$ (b) $\nu_0 = 0.015$ and (c) $\nu_0 = 0.0015$ with other plasma parameters $T = 0.4$, $\eta = 0.35$, $l_y = 0.3$ respectively.

7. Summary

In summary we have investigated damped ion acoustic solitary waves in degenerate collisional plasmas in the presence of trapping potential and Landau quantization. For this purpose, we have derived the dKdV equation by employing reductive perturbation technique. A time-dependent solution of dKdV is discussed by applying two level finite difference scheme with the help of the Runge Kutta method. We have observed the damped solitary wave structures at different times, which indicate that with passage of time the amplitude of nonlinear structures is diminished and width is enhanced. The comparison between the solitary structures for different values of collision is also discussed. Our findings are helpful in laser plasmas where Landau quantization plays an important role in finding the equation of state (EOS) and astrophysical plasmas, where strong magnetic field exists. For example strong magnetic fields upto $0.7GG$ have been reported in high power pulsed laser interaction with matter.

In such high magnetic fields Landau quantization is an important factor in determining the transport coefficients and EOS data, which describes the fundamental relation between

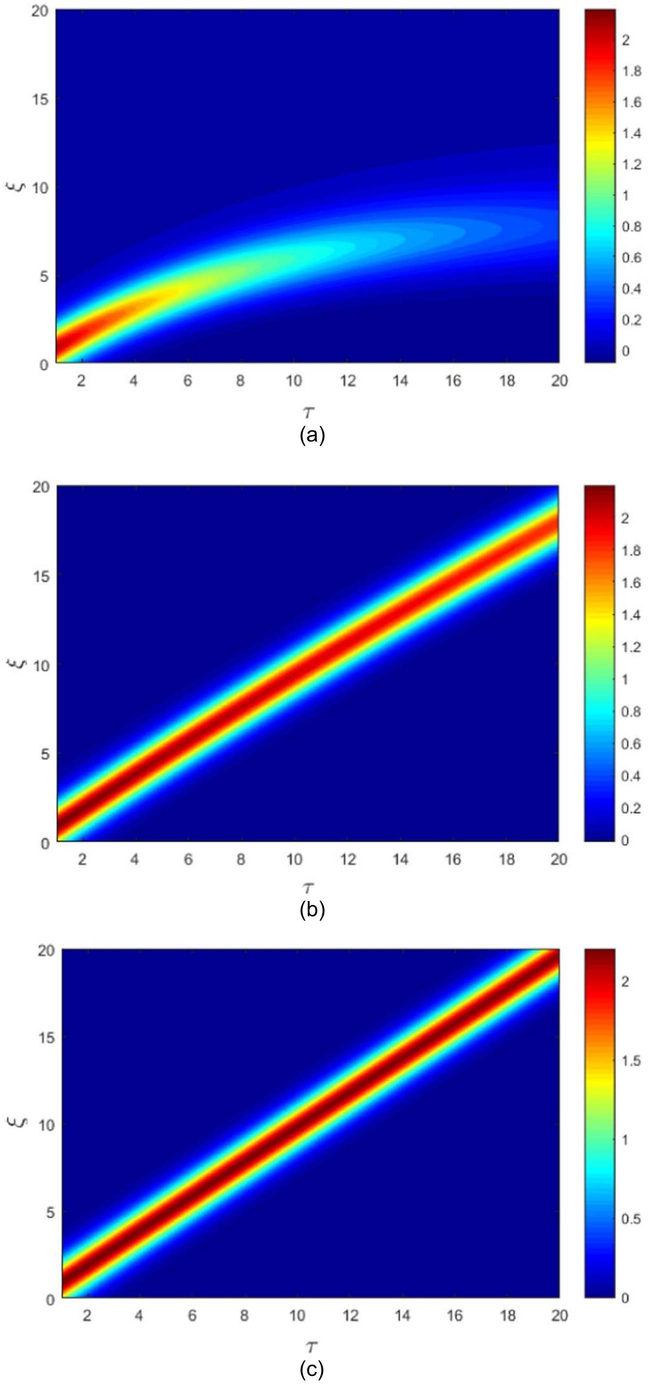

Figure 5. Propagation of nonlinear structure in time-space plane for the same conditions given in figure 4.

the macroscopically quantities of a physical system in equilibrium. On the other hand, a magnetic field in the *MG* range with high dense plasmas is responsible for weak Landau quantization. In the presence of 100–500*MG* magnetic field with high density exists in white dwarfs. In such situations electrons occupy several high energy Landau levels and quantities determined by thermal electrons near the Fermi level of plasmas are affected [34, 35].

References

[1] Calvayrac F, Reinhard P-G, Suraud E and Ullrich C A 2000 *Phys. Rep.* **337** 493
[2] Brorson S D, Fujimoto J G and Ippen E P 1987 *Phys. Rev. Lett.* **59** 1962
[3] Suaŕez C, Bron W E and Juhasz T 1995 *Phys. Rev. Lett.* **75** 4536
[4] Bigot J-Y, Halté V, Merle J-C and Daunois A 2000 *Chem. Phys.* **251** 181
[5] Domps A, Reinhard P-G and Suraud E 1997 *Ann. Phys.* **260** 171
[6] Kluksdahl N C, Kriman A M, Ferry D K and Ringhofer C 1989 *Phys. Rev.* B **39** 7720
[7] Luscombe J H, Bouchard A M and Luban M 1992 *Phys. Rev.* B **46** 10262
[8] Arnold A and Steinruc¨k H 1989 *Z. Angew. Math. Phys.* **40** 793
[9] Haas F 2005 *Phys. Plasmas* **12** 062117
[10] Melrose D B 2008 *Quantum Plasma Dynamics: Unmagnetized Plasmas* (New York: Springer)
[11] Shukla P K and Eliasson B 2010 *Phys.-Usp.* **53** 51
[12] Manfredi G and Hass F 2001 *Phys. Rev.* B **64** 075316
[13] Manfredi G 2005 *Fields Inst. Commun.* **46** 263
[14] Haas F, Garcia L G and Goedert J 2003 *Phys. Plasmas* **10** 3858
[15] Ali S, Moslem W M, Shukla P K and Schlickeiser R 2007 *Phys. Plasmas* **14** 082307
[16] Mushtaq A and Vladimirov S V 2010 *Phys. Plasmas* **17** 102310
[17] Hussain S and Mahmood S 2018 *Chaos Solitons Fractals* **106** 266
[18] Hussain S, Ur-Rehman H and Mahmood S 2019 *Phys. Plasmas* **26** 052105
[19] Tsintsadze L N 2010 *AIP Conf. Proc.* **1306** 89
[20] Shaukat M I 2017 *Phys. Plasmas* **24** 102301
[21] Hussain S and Rizvi H 2019 *Eur. Phys. J. Plus* **134** 25
[22] Hussain S and Akhtar N 2018 *Phys. Plasmas* **25** 062109
[23] Hussain S, Hasnain H and Haseeb M Q 2019 *Chin. Phys.* B **28** 015202
[24] Hass F and Bret A 2012 *Europhys. Lett.* **97** 26001
[25] Ghosh S 2012 *Europhys. Lett.* **99** 36002
[26] Sahu B, Sinha A and Roychoudhury R 2017 *Phys. Plasmas* **24** 112111
[27] Vranjes J, Petrovic D, Pandey B P and Poedts S 2008 *Phys. Plasmas* **15** 072104
[28] Bonitz M, Moldabekov Zh A and Ramazanov T S 2019 *Phys. Plasmas* **26** 090601
[29] Hussain S and Akhtar N 2016 *Phys. Plasmas* **23** 092110
[30] Landau L D and Lifshitz E M 1980 *Statistical Physics* I (I) (New York: Pergamon)
[31] Washimi H and Tanuiti T 1966 *Phys. Rev. Lett.* **17** 996
[32] Sultana S 2018 *Phys. Lett.* A **382** 1368
[33] Elkamash I S and Kourakis I 2016 *43rd EPS Conf. on Plasma Physics (Leuven, Belgium)* P5.097
[34] Heyl J S and Hernquist L 1998 *Phys. Rev.* D **58** 043005
[35] Eliezer S, Norreys P, Mendonça J T and Lancaster K 2005 *Phys. Plasmas* **12** 052115