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Entropy of higher-dimensional charged Gauss–Bonnet black hole in de Sitter space

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Abstract

The fundamental equation of the thermodynamic system gives the relation between the internal energy, entropy and volume of two adjacent equilibrium states. Taking a higher-dimensional charged Gauss–Bonnet black hole in de Sitter space as a thermodynamic system, the state parameters have to meet the fundamental equation of thermodynamics. We introduce the effective thermodynamic quantities to describe the black hole in de Sitter space. Considering that in the lukewarm case the temperature of the black hole horizon is equal to that of the cosmological horizon, we conjecture that the effective temperature has the same value. In this way, we can obtain the entropy formula of spacetime by solving the differential equation. We find that the total entropy contains an extra term besides the sum of the entropies of the two horizons. The corrected term of the entropy is a function of the ratio of the black hole horizon radius to the cosmological horizon radius, and is independent of the charge of the spacetime.

Keywords: entropy, Gauss–Bonnet gravity, black hole, de Sitter space

(Some figures may appear in colour only in the online journal)

1. Introduction

The Gauss–Bonnet (GB) black hole has been extensively studied along with the development of string theory. It was found that the GB black hole has interesting thermodynamic properties in anti-de Sitter space, especially, fruitful phase structures in higher-dimensional spacetime [1–6]. So what thermodynamic properties does the higher-dimensional GB black hole have in de Sitter space? There are black hole horizons and cosmological horizons for higher-dimensional charged GB black holes in de Sitter space. The thermodynamic quantities on the black hole horizon and the cosmological horizon all satisfy the first law of thermodynamics, moreover the corresponding entropies both fulfill the area formulae [7–10]. In recent years, the investigation of thermodynamic properties of black holes in de Sitter space has received a lot of attention [11–34]. In the early period of inflation, our universe was in a quasi-de Sitter space. On the other hand, with the inclusion of mysterious components with negative pressure, a large number of dark energy models have been proposed to explain the cosmic acceleration. The simplest candidate for dark energy is the cosmological constant (or vacuum energy density), with which our universe will naturally evolve into a new de Sitter phase. Finally, there has also been flourishing interest in the duality relation of de Sitter space, promoted by the recent success of AdS/CFT correspondence in theoretical physics. Therefore, from an observational and theoretical point of view, it is rewarding to have a better understanding of the classical and quantum properties of de Sitter space [35–41]. However, in general the radiation temperatures corresponding to the two horizons are not equal. For this reason, when taking the higher-dimensional charged GB black hole in de Sitter space (HGBDS) as a thermodynamic system, the system is usually unstable. The thermodynamic quantities on the black hole horizon and the cosmological one in de Sitter space are functions of mass M, charge Q and cosmological constant Λ respectively. These quantities, which correspond to the different horizons, are not independent of each other. Considering the relation between the thermodynamic quantities on the two horizons is very important for studying the thermodynamic properties of de Sitter space.

Considering the relation between the black hole horizon and the cosmological one of the HGBDS as a thermodynamic system, we obtain the effective thermodynamic quantities of the HGBDS. In the lukewarm case, the temperatures of the black hole horizon and that of the cosmological horizon are the same. We conjecture that the effective temperature should also take the same value in the special case. In [28, 29], the authors calculated the corrected entropy for hairy and Kerr black holes in de Sitter space, respectively. In this way, we provide the differential equation which the entropy of both horizons satisfies. We assume that the total entropy includes the sum of both horizons' entropy and the interaction term. The entropy corresponding to the two horizons is a function of horizon radius and the effective GB coefficient $\tilde{\alpha}$. So the interaction term of the corrected entropy is a function of horizon radius and the effective GB coefficient $\tilde{\alpha}$ and is independent of the charge of the spacetime. The result we obtained is self-consistent. In this work, we construct a self-consistent formula for the thermodynamic quantities of de Sitter spacetime and study the stability and the phase transition of de Sitter space. This work can provide further information on the gravity theory of the de Sitter space. The issue can help us get a clearer understanding of the classical and quantum properties of de Sitter space.

This paper is organized as follows. In section 2 we simply review the thermodynamic quantities of the black hole horizon and cosmological horizon of the HGBDS, and obtain the condition that the effective temperature of the horizon approaches to zero. By means of the first law of thermodynamics, we obtain the entropy and the effect temperature of the HGBDS. We study the condition that the HGBDS satisfies the stable equilibrium of the thermodynamics in section 3. Section 4 is devoted to conclusions. (We use the units $G_{n+1} = \hbar = k_B = c = 1$.)

2. Charged GB black hole in de Sitter space

The action of d-dimensional Einstein–GB–Maxwell theory with a bare cosmological constant Λ reads [3]

$$I = \frac{1}{16\pi}\int \mathrm{d}^d x\sqrt{-g}\,[R - 2\Lambda + \alpha(R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2) - 4\pi F_{\mu\nu}F^{\mu\nu}], \tag{2.1}$$

where the GB coupling α has dimension [length]^2 and can be identified with the inverse string tension with positive value if the theory is incorporated in string theory [42, 43], thus we shall consider only the case $\alpha > 0$. $F_{\mu\nu}$ is the Maxwell field strength defined as $F_{\mu\nu} = \partial_\mu A_\mu - \partial_\nu A_\nu$ with vector potential A_μ. In addition, let us mention here that the GB term is a topological term in $d = 4$ dimensions and has no dynamics in this case. Therefore we will consider $d \geqslant 5$ in what follows,

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2 h_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{2.2}$$

where $h_{ij}\mathrm{d}x^i\mathrm{d}x^j$ represents the line element of a $(d-2)$-dimensional maximal symmetric Einstein space with constant curvature $(d-2)(d-3)k$ and volume Σ_k. Without loss of generality, one may take $k = 1$, 0 and -1, corresponding to the spherical, Ricci flat and hyperbolic topology of the black hole horizon, respectively. The metric function $f(r)$ is given by [1–5]

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}}\left[1 - \sqrt{1 + \frac{64\pi\tilde{\alpha}M}{(d-2)\Sigma_k r^{d-1}} - \frac{2\tilde{\alpha}Q^2}{(d-2)(d-3)r^{2d-4}} + \frac{8\tilde{\alpha}\Lambda}{(d-1)(d-2)}}\right], \tag{2.3}$$

where $\tilde{\alpha} = (d-3)(d-4)\alpha$, M and Q are the mass and charge of the black hole respectively, and pressure P

$$P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2}. \tag{2.4}$$

In order to have a well-defined vacuum solution with $M = Q = 0$, the effective GB coefficient $\tilde{\alpha}$ and pressure P have to satisfy the following constraint

$$0 < \frac{64\pi\tilde{\alpha}P}{(d-1)(d-2)} \leqslant 1. \tag{2.5}$$

The black hole horizon r_+ and cosmological horizon r_c satisfy the equation $f(r_{+,c}) = 0$. The equations $f(r_+) = 0$ and $f(r_c) = 0$ are rearranged to

$$M = \frac{(d-2)\Sigma_k r_+^{d-3}}{16\pi}\left(k + \frac{k^2\tilde{\alpha}}{r_+^2}\right) - \frac{\Sigma_k r_+^{d-1}\Lambda}{8\pi(d-1)} + \frac{\Sigma_k Q^2}{32\pi(d-3)r_+^{d-3}}, \tag{2.6}$$

$$M = \frac{(d-2)\Sigma_k r_c^{d-3}}{16\pi}\left(k + \frac{k^2\tilde{\alpha}}{r_c^2}\right) - \frac{\Sigma_k r_c^{d-1}\Lambda}{8\pi(d-1)} + \frac{\Sigma_k Q^2}{32\pi(d-3)r_c^{d-3}}. \tag{2.7}$$

From equations (2.6) and (2.7), we can obtain

$$\begin{aligned} M &= \frac{(d-2)\Sigma_k r_c^{d-3}x^{d-3}}{16\pi}\frac{(1-x^2)}{(1-x^{d-1})}\left(k + \frac{k^2\tilde{\alpha}_c}{x^2}(1+x^2)\right) \\ &\quad + \frac{\Sigma_k Q^2}{32\pi(d-3)r_c^{d-3}(1-x^{d-1})x^{d-3}}(1-x^{2d-4}) \\ &= \frac{r_c^2\Sigma_k}{16\pi}f_{\mathrm{H1}}(x, \tilde{\alpha}_c) + \frac{Q^2}{r_c^2}\frac{\Sigma_k}{16\pi}f_{\mathrm{H2}}(x), \end{aligned} \tag{2.8}$$

here

$$\begin{aligned} f_{\mathrm{H1}} &= (d-2)x^{d-3}\frac{(1-x^2)}{(1-x^{d-1})}\left(k + \frac{k^2\tilde{\alpha}_c}{x^2}(1+x^2)\right), \\ f_{\mathrm{H2}} &= \frac{1}{2(d-3)(1-x^{d-1})x^{d-3}}(1-x^{2d-4}), \end{aligned} \tag{2.9}$$

$$\Lambda = \frac{(d-1)(d-2)}{2r_c^2(1-x^{d-1})}\left(k(1-x^{d-3}) + \frac{\tilde{\alpha}k^2}{r_c^2}(1-x^{d-5}\right) - \frac{(d-1)Q^2(1-x^{d-3})}{4(d-3)r_c^{2d-4}x^{d-3}(1-x^{d-1})}, \tag{2.10}$$

here $x = r_+/r_c$, $\tilde{\alpha}_c = \tilde{\alpha}/r_c^2$.

From equations (2.3), (2.8) and (2.10), we can obtain

$$f'(r_+) = -\frac{2k}{r_+} + \frac{(d-1)kr_+r_c^{d-3}(r_c^2-r_+^2)}{(r_+^2+2\alpha k)(r_c^{d-1}-r_+^{d-1})} \times \left(1+\frac{\alpha k(r_c^2+r_+^2)}{r_+^2r_c^2}\right) - \frac{Q^2}{2(r_+^2+2\alpha k)(d-2)r_+^{2d-7}} \times \left[1 - \frac{(d-1)r_+^{d-1}(r_c^{d-3}-r_+^{d-3})}{(d-3)r_c^{d-3}(r_c^{d-1}-r_+^{d-1})}\right] = A - Q^2B, \tag{2.11}$$

$$f'(r_c) = -\frac{2k}{r_c} + \frac{(d-1)kr_cr_+^{d-3}(r_c^2-r_+^2)}{(r_c^2+2\alpha k)(r_c^{d-1}-r_+^{d-1})} \times \left(1+\frac{\alpha k(r_c^2+r_+^2)}{r_+^2r_c^2}\right) - \frac{Q^2}{2(r_c^2+2\alpha k)(d-2)r_c^{2d-7}} \times \left[1 - \frac{(d-1)r_c^{d-1}(r_c^{d-3}-r_+^{d-3})}{(d-3)r_+^{d-3}(r_c^{d-1}-r_+^{d-1})}\right] = C - Q^2D, \tag{2.12}$$

here

$$A = \frac{1}{r_c}f_A(x, \tilde{\alpha}_c) = -\frac{2k}{r_cx} + \frac{(d-1)kx(1-x^2)}{r_c(x^2+2\tilde{\alpha}_ck)(1-x^{d-1})}\left[1+\frac{\tilde{\alpha}_ck(1+x^2)}{x^2}\right],$$

$$B = \frac{1}{r_c^{2d-5}}f_B(x, \tilde{\alpha}_c) = \frac{1}{2(d-2)(x^2+2\tilde{\alpha}_ck)r_c^{2d-5}x^{2d-7}} \times \left[1 - \frac{(d-1)x^{d-1}(1-x^{d-3})}{(d-3)(1-x^{d-1})}\right],$$

$$C = \frac{1}{r_c}f_C(x, \tilde{\alpha}_c) = -\frac{2k}{r_c} + \frac{(d-1)kx^{d-3}(1-x^2)}{r_c(1+2\tilde{\alpha}_ck)(1-x^{d-1})}\left[1+\frac{\tilde{\alpha}_ck(1+x^2)}{x^2}\right]$$

$$D = \frac{1}{r_c^{2d-5}}f_D(x, \tilde{\alpha}_c) = \frac{1}{2(d-2)(1+2\tilde{\alpha}_ck)r_c^{2d-5}} \times \left[1 - \frac{(d-1)(1-x^{d-3})}{(d-3)x^{d-3}(1-x^{d-1})}\right]. \tag{2.13}$$

Some thermodynamic quantities associated with the cosmological horizon are

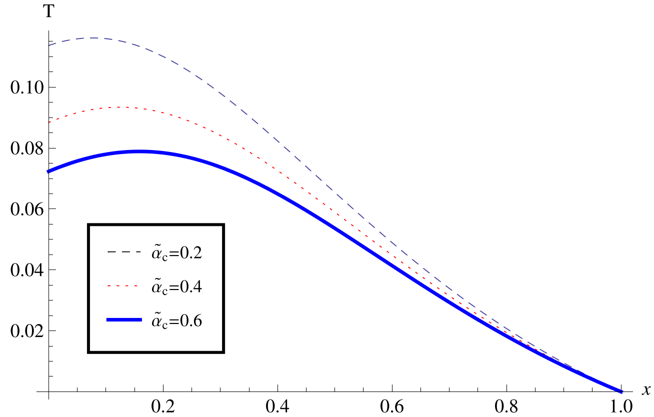

Figure 1. T with respect to x. We set $r_c = 1$.

$$T_c = -\frac{f'(r_c)}{4\pi} = -\frac{C-Q^2D}{4\pi},$$
$$S_c = \frac{\Sigma_kr_c^{d-2}}{4}\left[1+\frac{2\tilde{\alpha}_ck(d-2)}{(d-4)}\right],\ V_c = \frac{\Sigma_kr_c^{d-1}}{d-1}. \tag{2.14}$$

T_c, S_c and V_c denote the Hawking temperature, the entropy and the volume.

For the black hole horizon, associated thermodynamic quantities are

$$T_+ = \frac{f'(r_+)}{4\pi} = \frac{A-Q^2B}{4\pi r},$$
$$S_+ = \frac{\Sigma_kr_c^{d-2}x^{d-2}}{4}\left[1+\frac{2\tilde{\alpha}_ck(d-2)}{(d-4)x^2}\right],$$
$$V_+ = \frac{\Sigma_kr_c^{d-1}x^{d-1}}{d-1}. \tag{2.15}$$

From equations (2.10) and (2.11), when the charge Q of the spacetime satisfies

$$Q^2 = \frac{A+C}{B+D} = r_c^{2d-6}\frac{f_A(x, \tilde{\alpha}_c)+f_C(x, \tilde{\alpha}_c)}{f_B(x, \tilde{\alpha}_c)+f_D(x, \tilde{\alpha}_c)}, \tag{2.16}$$

the temperature of the black hole horizon and the ones of the cosmological horizon are equal,

$$T = T_+ = T_c = \frac{1}{4\pi}\frac{f_A(x, \tilde{\alpha}_c)f_D(x, \tilde{\alpha}_c) - f_C(x, \tilde{\alpha}_c)f_B(x, \tilde{\alpha}_c)}{r_c[f_B(x, \tilde{\alpha}_c)+f_D(x, \tilde{\alpha}_c)]}. \tag{2.17}$$

From equation (2.17), we plot the $T-x$ plane with different $\tilde{\alpha}_c$, $d=5$ and $k=1$ in figure 1.

3. The effective thermodynamics of HGBDS

Considering the connection between the black hole horizon and the cosmological horizon, we can derive the effective thermodynamic quantities and the corresponding first law of black hole thermodynamics

$$dM = T_{\text{eff}}dS - P_{\text{eff}}dV + \varphi_{\text{eff}}dQ, \tag{3.1}$$

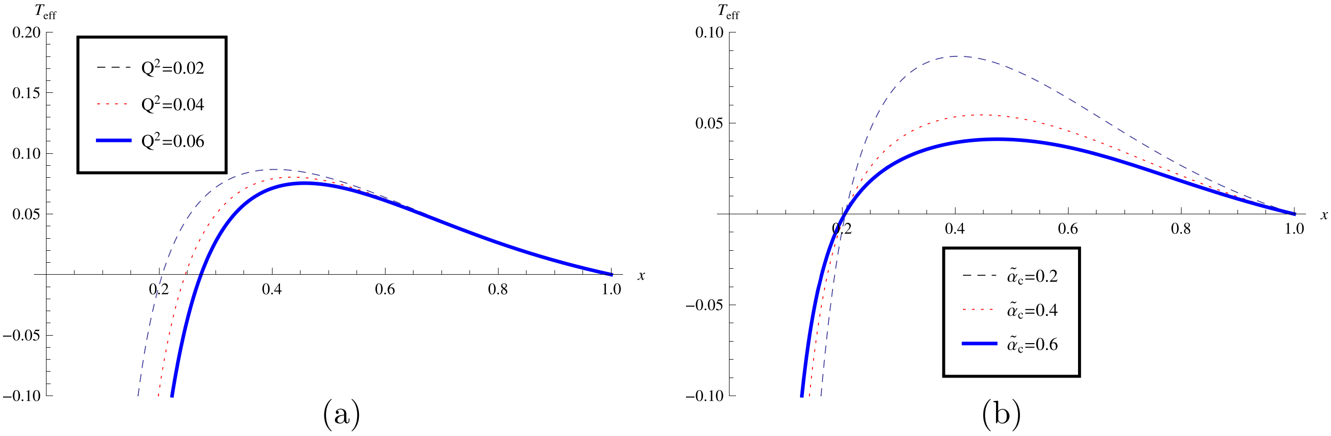

Figure 2. The effective temperature $T_{\rm eff}$ versus x. In (a), $T_{\rm eff}-x$ with different Q and fixed $\tilde{\alpha}_c=0.2$. In (b), $T_{\rm eff}-x$ with different $\tilde{\alpha}_c$ and fixed $Q^2=0.02$. We set $r_c=1$.

here the thermodynamic volume is that between the black hole horizon and the cosmological horizon, namely [6, 11]

$$\begin{aligned} V=V_c-V_+&=\frac{\Sigma_k}{d-1}(r_c^{d-1}-r_+^{d-1})\\ &=\frac{\Sigma_k}{d-1}r_c^{d-1}(1-x^{d-1}). \end{aligned} \tag{3.2}$$

Considering that $\tilde{\alpha}_c k$ is small, we can formally expand the total entropy in series of $\tilde{\alpha}_c k$,

$$S=\frac{\Sigma_k r_c^{d-2}}{4}\left[f(x)+\tilde{\alpha}_c k f_1(x)+\tilde{\alpha}_c^2 k^2 f_2(x)+\sum_{l=3}\tilde{\alpha}_c^l k^l f_l(x)\right], \tag{3.3}$$

here the undefined function $f(x)$, $f_1(x)$, $f_2(x)$ and $f_l(x)$ represents the extra contribution from the correlations of the two horizons. When taking Q, $\tilde{\alpha}_c$ as constant and substituting equations (2.8), (3.2) and (3.3) into equation (3.1), one obtains the effective temperature $T_{\rm eff}$

$$T_{\rm eff}=\frac{T_1(x)}{4\pi r_c T_2(x)}, \tag{3.4}$$

here

$$\begin{aligned} T_1(x)&=(1-x^{d-1})\left[f_{\rm H1}{}'(x,\tilde{\alpha}_c)+\frac{Q^2}{r_c^{2d-6}}f_{\rm H2}{}'(x)\right]\\ &\quad+x^{d-2}(d-3)\left[f_{\rm H1}(x,\tilde{\alpha}_c)-\frac{Q^2}{r_c^{2d-6}}f_{\rm H2}(x)\right],\\ T_2(x)&=(1-x^4)[f'(x)+\tilde{\alpha}_c k f_1{}'(x)+\tilde{\alpha}_c^2 k^2 f_2{}'(x)]\\ &\quad+3x^3[f(x)+\tilde{\alpha}_c k f_1(x)+\tilde{\alpha}_c^2 k^2 f_2(x)\\ &\quad+\textstyle\sum_{l=3}\tilde{\alpha}_c^l k^l f_l(x)\Big], \end{aligned} \tag{3.5}$$

here $f_l(x)=0$ with $l\geqslant 3$. When Q satisfies equation (2.16), the temperature of both horizons is equal. In this case we think that the effective temperature of spacetime is the

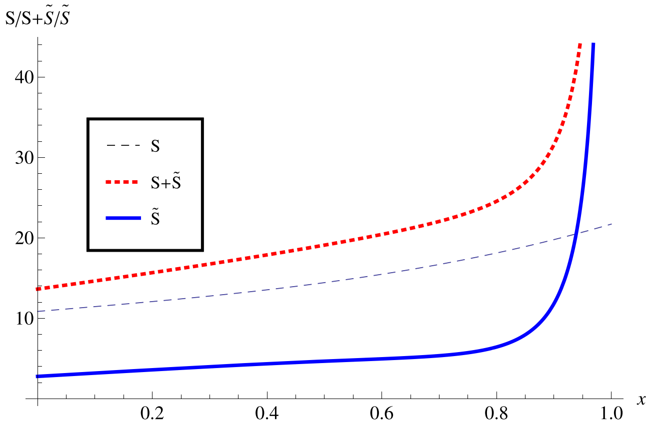

Figure 3. The uncorrected entropy S, the corrected terms $\tilde{S}$ for entropy and the total entropy $S+\tilde{S}$ with respect to x. We set $r_c=1$.

radiation temperature

$$\begin{aligned} T_{\rm eff}&=\frac{\tilde{T}_1(x)}{4\pi r_c T_2(x)}\\ &=\frac{1}{4\pi}\frac{f_A(x,\tilde{\alpha}_c)f_D(x,\tilde{\alpha}_c)-f_C(x,\tilde{\alpha}_c)f_B(x,\tilde{\alpha}_c)}{r_c[f_B(x,\tilde{\alpha}_c)+f_D(x,\tilde{\alpha}_c)]}, \end{aligned} \tag{3.6}$$

here

$$\begin{aligned} \tilde{T}_1(x)&=(1-x^{d-1})[f_{\rm H1}{}'(x,\tilde{\alpha}_c)\\ &\quad+f_{\rm H2}{}'(x)\frac{f_A(x,\tilde{\alpha}_c)+f_C(x,\tilde{\alpha}_c)}{f_B(x,\tilde{\alpha}_c)+f_D(x,\tilde{\alpha}_c)}\Bigg]+(d-3)x^{d-2}\\ &\quad\times\left[f_{\rm H1}(x,\tilde{\alpha}_c)-f_{\rm H2}(x)\frac{f_A(x,\tilde{\alpha}_c)+f_C(x,\tilde{\alpha}_c)}{f_B(x,\tilde{\alpha}_c)+f_D(x,\tilde{\alpha}_c)}\right]. \end{aligned} \tag{3.7}$$

From equation (3.6), one gets

$$T_2(x)=\frac{\tilde{T}_1(x)[f_B(x,\tilde{\alpha}_c)+f_D(x,\tilde{\alpha}_c)]}{f_A(x,\tilde{\alpha}_c)f_D(x,\tilde{\alpha}_c)-f_C(x,\tilde{\alpha}_c)f_B(x,\tilde{\alpha}_c)}. \tag{3.8}$$

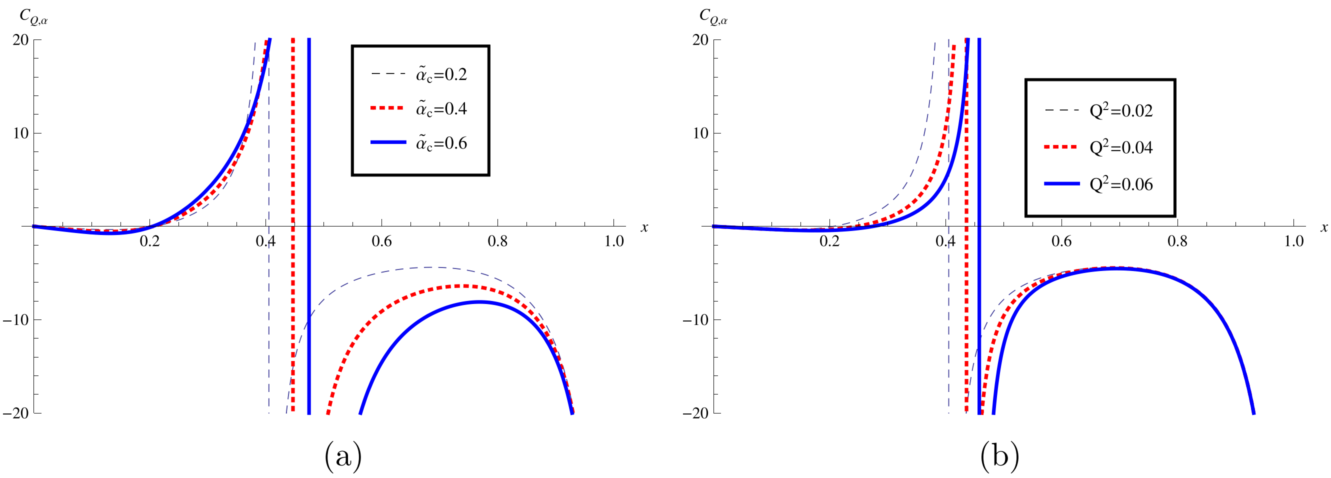

Figure 4. $C_{Q,\tilde{\alpha}_c}$ with respect to x. In (a), $C_{Q,\tilde{\alpha}_c} - x$ with different $\tilde{\alpha}_c$ and fixed $Q^2 = 0.02$. In (b), $C_{Q,\tilde{\alpha}_c} - x$ with different Q^2 and fixed $\tilde{\alpha}_c = 0.2$. We set $r_c = 1$.

When $d = 5$, from equation (3.8) one obtains

$$T_2(x) = 3\frac{x^2(1+x^5) + 2\tilde{\alpha}_c k(1+x^7)}{(1-x^4)} + 3kx^3\tilde{\alpha}_c\frac{[1+x+3x^2+x^3+x^4+2\tilde{\alpha}_c k(1-x+x^2)]}{(1+x)(1-x^2)}. \tag{3.9}$$

Substituting equation (3.5) into equation (3.9), compared with the power of $\tilde{\alpha}_c$ on the two sides of this equation, one can obtain that $f(x)$, $f_1(x)$ and $f_2(x)$ satisfy

$$\begin{aligned}(1-x^4)f'(x) + 3x^3 f(x) &= A_0,\\ (1-x^4)f_1{}'(x) + 3x^3 f_1(x) &= A_1,\\ (1-x^4)f_2{}'(x) + 3x^3 f_2(x) &= A_2,\end{aligned} \tag{3.10}$$

respectively, here

$$A_0 = \frac{3x^2(1+x^5)}{(1-x^4)},$$

$$A_1 = \frac{3(2+2x+x^3+x^4+4x^5+2x^6+6x^7+3x^8+x^9)}{(1+x)(1-x^4)},$$

$$A_2 = \frac{6x^3(1-x+x^2)}{(1+x)(1-x^2)} = \frac{6x^3(1+x^3)}{(1+x)^2(1-x^2)}. \tag{3.11}$$

When $x \to 0$, there is only a cosmological horizon in de Sitter space. The formula of entropy from equation (2.14) is $S_c = \frac{\Sigma_k r_c^{d-2}}{4}\left(1 + \frac{2\tilde{\alpha}_c k(d-2)}{(d-4)}\right)$. So we take the initial values $f(0) = 1, f_1(0) = 6, f_2(0) = 0$,

$$\begin{aligned}f(x) &= \frac{11}{7}(1-x^4)^{3/4} - \frac{4(1+x^4)(1-x+x^2) - 11x^3}{7(1-x)(1+x^2)}\\ &= \frac{11}{7}(1-x^4)^{3/4} - \frac{11 - x(11+3x^3)(1-x+x^2)}{7(1-x)(1+x^2)}\\ &\quad + 1 + x^3 = \tilde{f}(x) + 1 + x^3,\end{aligned} \tag{3.12}$$

$$\begin{aligned}f_1(x) &= \frac{1975}{308}(1-x^4)^{3/4} + \frac{1}{77(1+x)^2(1-x)(1+x^2)}\\ &\quad\times(-127 + 76x + 211x^2 + 8x^3\\ &\quad + 280x^4 + 140x^5 + 2x^6 + 76x^7 + 60x^8\\ &\quad + 259x(1-x^4)^{3/4}(1+x)(1-x^4)\,{}_2\mathrm{F}_1\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, x^4\right]\\ &\quad - 8x^2(1-x^4)^{3/4}(1+x)(1-x^4)\,{}_2\mathrm{F}_1\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, x^4\right]\Big)\\ &= \frac{1975}{308}(1-x^4)^{3/4} + \frac{1}{77(1+x)^2(1-x)(1+x^2)}\\ &\quad\times(-204 - 68x + 134x^2 + 8x^3\\ &\quad + 357x^4 + 284x^5 + 79x^6 + 76x^7 + 60x^8 + 1 + x\\ &\quad + 259x(1-x^4)^{3/4}(1+x)(1-x^4)\,{}_2\mathrm{F}_1\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, x^4\right]\\ &\quad - 8x^2(1-x^4)^{3/4}(1+x)(1-x^4)\,{}_2\mathrm{F}_1\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, x^4\right]\Big)\\ &= \tilde{f}_1(x) + 6(1+x),\end{aligned} \tag{3.13}$$

where the function ${}_2\mathrm{F}_1$ denotes the hypergeometric function. We can obtain $f_2(x)$ with the numerical calculation. From equations (2.14) and (2.15), we obtain the entropy of the black hole horizon and the cosmological horizon as

$$S = \frac{\Sigma_k}{4}r_c^{d-2}\left[1 + x^{d-2} + 2\frac{(d-2)\tilde{\alpha}_c k}{(d-4)}(1 + x^{d-4})\right]. \tag{3.14}$$

Comparing equations (3.12)–(3.14), we can obtain the corrected terms of the system entropy from the interaction of both horizons with $d = 5$

$$\tilde{S} = \frac{\Sigma_k}{4}r_c^3(\tilde{f}(x) + \tilde{f}_1(x) + f_2(x)). \tag{3.15}$$

Substituting equation (3.9) into equation (3.4), we can plot the $T_{\mathrm{eff}}(x) - x$ plane with different Q, $\tilde{\alpha}_c$ and $r_c = 1$, $k = 1$ in figure 2. From equations (3.3), (3.14) and (3.15), we can plot the $S-x$, $\tilde{S}-x$ and $(\tilde{S}+S)-x$ with $k=1$ in figure 3. The specific heat of the HGBDS can be defined as

$$C_{Q,\tilde{\alpha}_c} = T_{\text{eff}}\left(\frac{\partial S}{\partial T_{\text{eff}}}\right)_{Q,\tilde{\alpha}_c}. \tag{3.16}$$

From figure 4, when $x_0 < x < x_c$, the specific heat of the system is positive, while when $x_c < x < 1$ or $0 < x < x_0$, it is negative. This means that the HGBDS with $x_0 < x < x_c$ is thermodynamically stable. From figure 4, we can find that the region of stable state of the HGBDS is related to the charge Q and the effective GB coefficient $\tilde{\alpha}$, i.e., the region of stable state becomes bigger as the effective GB coefficient $\tilde{\alpha}$ or the charge Q increases.

In this letter, we have presented the entropy of the HGBDS. It is not only the sum of the entropies of the black hole horizon and the cosmological horizon, but also includes extra terms from the correlation between the two horizons. This idea has twofold advantages. First, without the extra terms in the total entropy, the effective temperature is different from that of the black hole horizon in the lukewarm case. This is not satisfactory. Second, the physical explanation of effective thermodynamic quantities is still unclear, however, taking advantage of the method, we obtain the corrected entropy of the HGBDS, which may make the method more acceptable.

4. Discussions and conclusions

The thermodynamic quantities of the black hole horizon and the cosmological one in de Sitter space are functions of mass M, charge Q and cosmological constant Λ respectively. These quantities, which correspond to the different horizons, are not independent of each other. It is not possible to fully realize the thermodynamic properties of de Sitter space by studying the thermodynamic system of the two horizons in de Sitter space separately. So, we can take the state parameters in de Sitter space as the state parameters in the whole system. We know that the state parameters in the whole system satisfy the first thermodynamics law. So, we can obtain the effective temperature T_{eff} and the total entropy $S+\tilde{S}$ by the above discussion and calculation.

Because the radiation temperatures of the two horizons are different, the spacetime did not meet the requirements of the stability of the thermodynamic equilibrium, so the system is unstable. Considering the correlation between the black hole horizon and the cosmological horizon, we can obtain the only effective temperature T_{eff} of the higher-dimensional charged GB black hole in de Sitter space from equation (3.4). From $C_{Q,\tilde{\alpha}_c}-x$ (figure 4), when $x > x_c$ or $x < x_0$, we know that the higher-dimensional charged GB black hole in de Sitter space is unstable. The system cannot be thermodynamically stable for the thermodynamic system does not meet the thermodynamic equilibrium conditions. In the Universe, there is only possible a higher-dimensional charged GB black hole in de Sitter space that meets the conditions of $x_0 < x < x_c$ because the cosmological constant is associated with a vacuum that describes the most fundamental theories of nature, such as quantum gravity, and de Sitter space is closely connected with the evolution of our Universe. According to these effective thermodynamic quantities, one can even discuss the entropic force between the two horizons, which can be used to explain the expansion of our universe [41, 44]. A deeper understanding of the quantum nature of de Sitter space is undoubtedly helpful to establish self-consistent quantum gravity theory.

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