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New lump, lump-kink, breather waves and other interaction solutions to the (3+1)-dimensional soliton equation

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Abstract

This study investigates the (3+1)-dimensional soliton equation via the Hirota bilinear approach and symbolic computations. We successfully construct some new lump, lump-kink, breather wave, lump periodic, and some other new interaction solutions. All the reported solutions are verified by inserting them into the original equation with the help of the Wolfram Mathematica package. The solution's visual characteristics are graphically represented in order to shed more light on the results obtained. The findings obtained are useful in understanding the basic nonlinear fluid dynamic scenarios as well as the dynamics of computational physics and engineering sciences in the related nonlinear higher dimensional wave fields.

Keywords: (3+1)-dimensional soliton equation, Hirota method, lump solution, breather waves

(Some figures may appear in colour only in the online journal)

1. Introduction

The lump soliton solutions have been commonly used in many natural sciences such as chemistry, biology, etc. In particular, in almost all branches of physics, engineering such as fluid dynamics, plasma physics, optics, etc [1–3] the lump soliton solutions play an important role. While some researchers used numerical simulation or analytical methods to investigate the performance of such structures, further study of the theoretical analysis of such systems is required [4–6]. Rogue waves (RW) are expansive and instinctive ocean waves that have drawn growing focus on both theoretical and experimental observations [7]. The RW for nonlinear Schrödinger equation in its simplest form has been proposed in [8]. It can be seen that there are huge wave phenomena in different fields such as plasmas, nonlinear optics, Bose–Einstein condensates, biophysics and even finance. [9–11]. In terms of a new combination of variable functions using the Hirota bilinear model, some researchers are working out some new solutions from the lump solution family and some groups of interaction solutions. We are reviewing some literature on the phenomena of lump solutions and their interaction. To this aim, there have been series of presentation of lump solutions from different perspectives, for instant, Zakharov [12], pump wave solution [13], lump solution through Hirota bilinear method [3, 14–16]. Through important properties of lump solutions it can be understood that amplitudes, shapes, speeds of solitons will be preserved after collision with another soliton and this is the elastic property of collision. Moreover, interaction between rouge wave and kink solitary wave solution have been established in [17]. Several other types of solution can also be found in [18–22].

In this study, we utilise the Hirota bilinear approach to construct some novel lump-type and interaction solutions for the (3+1)-dimensional soliton equation [18] given by

$$3\chi_{xz} - (2\chi_t + \chi_{xxx} - 2\chi\chi_x)_y + 2(\chi_x\partial_x^{-1}\chi_y)_x = 0. \quad (1)$$

The (3+1)-dimensional integrable equation (1) was first introduced in [23] in the study of the algebraic-geometrical solutions. The physical behaviour of the obtained solutions

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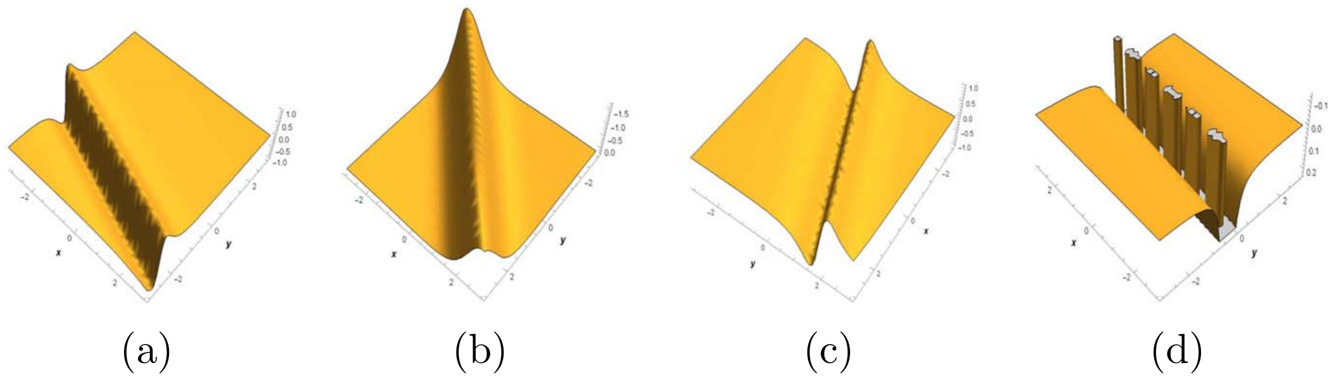


Figure 1. The 3D profiles of (a) imaginary part of equation (11) at $\sigma = 2.95$, $z = t = -10$ (b) real part of equation (11) at $\sigma = -0.95$, $z = t = -10$ (c) imaginary part of equation (14) at $\sigma = -2.45$, $z = t = 0.1$ (d) real part of equation (14) at $\sigma = -10$, $z = 0.1$, $t = -5.2$.

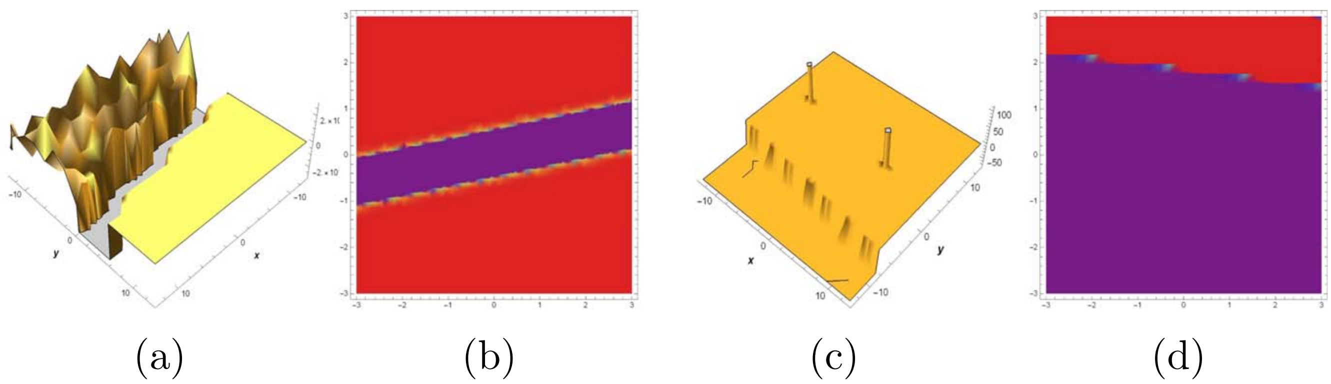


Figure 2. The 3D and density profiles of the real part of equation (24) (a), (b) at $\sigma = -4.88$, $z = t = 0$ (c), (d) at $\sigma = 8.04$, $z = -10$, $t = 1$.

are also depicted in figures 1–5 in order to shed more light on the presented solutions.

2. Lump and its interaction solutions

In part, we present the new lump and its interaction solutions to the (3+1)-dimensional soliton equation given in equation (1).

We first transform equation (1) into its bilinear form. Set

$$\chi = \Theta_x. \quad (2)$$

Substituting equation (2) into (1), yields

$$2\Theta_{xyt} + \Theta_{xxxxy} - 4\Theta_{xy}\Theta_{xx} - 3\Theta_{xxz} - 2\Theta_x\Theta_{xxy} - 2\Theta_y\Theta_{xxx} = 0. \quad (3)$$

Setting

$$\Theta(x, y, z, t) = \Theta(\xi), \quad \xi = x + \sigma y, \quad (4)$$

reduces equation (4) to the following (2+1)-dimensional soliton equation:

$$2\sigma\Theta_{\xi\xi t} + \sigma\Theta_{\xi\xi\xi\xi\xi} - 4\sigma\Theta_{\xi\xi}^2 - 3\Theta_{\xi\xi z} - 4\sigma\Theta_\xi\Theta_{\xi\xi\xi} = 0. \quad (5)$$

Consider the Cole–Hopf transformation

$$\Theta(\xi, z, t) = -3\frac{\partial(\ln f(\xi, z, t)}{\partial\xi}. \quad (6)$$

Substituting equation (6) into (5), gives the following bilinear form:

$$3\sigma f_{\xi\xi}^2 - 2\sigma f_t f_\xi + f(2\sigma f_{\xi t} - 3f_{\xi z}) + f_\xi(3f_z - 4\sigma f_{\xi\xi\xi}) + \sigma f f_{\xi\xi\xi\xi} = 0. \quad (7)$$

2.1. Lump solution

In this section, we report the lump solutions to equation (1).

Consider the positive quadratic solutions to the bilinear equation (7)

$$g = b_1\xi + b_2 z + b_3 t + b_4, \quad h = b_5\xi + b_6 z + b_7 t + b_8, \quad f = g^2 + h^2 + b_9. \quad (8)$$

Substituting equation (8) into (7) gives a polynomial in powers of ξ, z and t. Collecting the coefficients of the same power, and equating each summation to zero, produces an algebraic system of equations. We solve the system of equations to obtained the values of the parameters involved. Substituting the values of the parameters into equation (6) and then into equation (2), gives the following lump solutions to equation (1):

Case-1: When

$$b_1 = -\mathrm{i}b_5, \; b_2 = \frac{2b_3\sigma}{3}, \; b_6 = \frac{2b_7\sigma}{3},$$

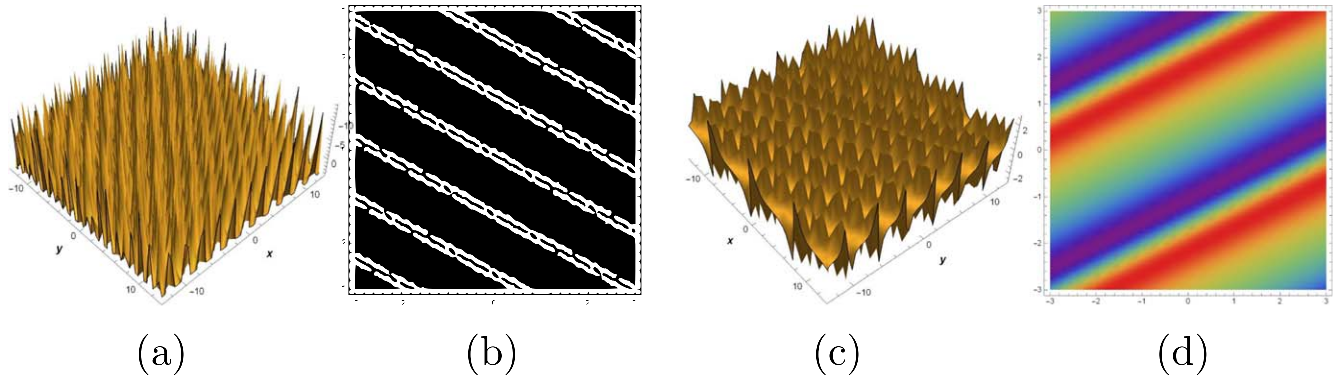


Figure 3. The 3D and density profiles of the (a), (b) real part of equation (31) at $\sigma = 1.82$, $z = 0.15$, $t = 1$ (c), (d) imaginary part of equation (31) at $\sigma = -2$, $z = -10$, $t = 2.52$.

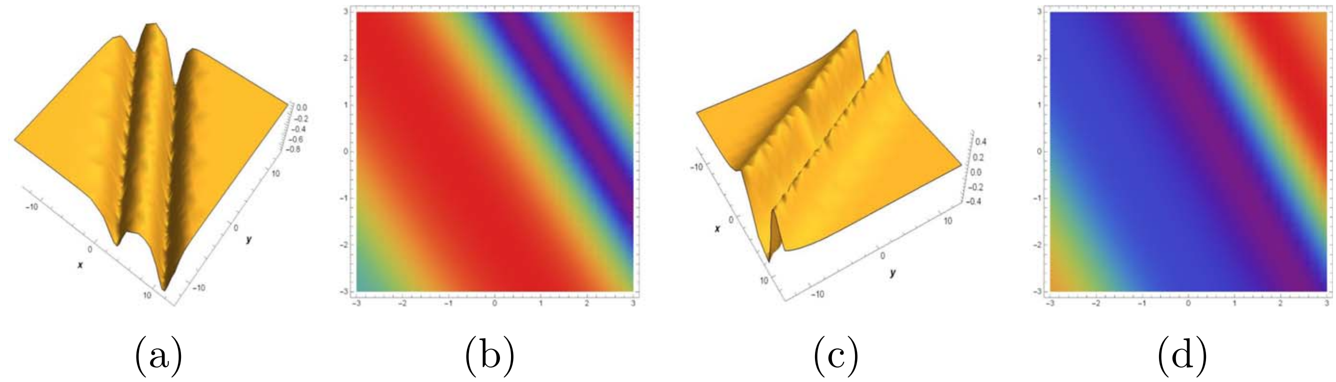


Figure 4. The (a) 3D and (b) density profiles of real part of equation (35) at $\sigma = 0.65$, $z = 0.4$, $t = 1.25$ (c) 3D and (d) density profiles of imaginary part of equation (35) at $\sigma = 0.5$, $z = 0.3$, $t = 0.5$.

we have

$$f = \left(-\mathrm{i}b_5\xi + b_3t + \frac{2}{3}b_3\sigma z + b_4\right)^2 + \left(b_5\xi + b_7t + \frac{2}{3}b_7\sigma z + b_8\right)^2 + b_9, \quad (9)$$

$$\Theta(\xi, z, t) = -\frac{3\left(2b_5\left(b_5\xi + b_7t + \frac{2}{3}b_7\sigma z + b_8\right) - 2\mathrm{i}b_5\left(-\mathrm{i}b_5\xi + b_3t + \frac{2}{3}b_3\sigma z + b_4\right)\right)}{\left(-\mathrm{i}b_5\xi + b_3t + \frac{2}{3}b_3\sigma z + b_4\right)^2 + \left(b_5\xi + b_7t + \frac{2}{3}b_7\sigma z + b_8\right)^2 + b_9}, \quad (10)$$

$$\chi(x, y, z, t) = \frac{3\left(\varphi_1 - 2\mathrm{i}b_5\left(b_3t - \mathrm{i}b_5(x + \sigma y) + \frac{2}{3}b_3\sigma z + b_4\right)\right)^2}{\left(\varphi_2 + \left(b_3t - \mathrm{i}b_5(x + \sigma y) + \frac{2}{3}b_3\sigma z + b_4\right)^2\right)^2}, \quad (11)$$

where

$$\varphi_1 = 2b_5\left(b_7t + b_5(x + \sigma y) + \tfrac{2}{3}b_7\sigma z + b_8\right),$$
$$\varphi_2 = \left(b_7t + b_5(x + \sigma y) + \tfrac{2}{3}b_7\sigma z + b_8\right)^2 + b_9.$$

Case-2: When

$$b_5 = -\mathrm{i}b_1,\ b_6 = -\mathrm{i}b_2,\ b_7 = \frac{\mathrm{i}(b_3\sigma - 3b_2)}{\sigma},\ b_9 = 0,$$

we have

$$f(\xi, z, t) = (b_1\xi + b_3t + b_2z + b_4)^2 + \left(-\mathrm{i}b_1\xi + \frac{\mathrm{i}t(b_3\sigma - 3b_2)}{\sigma} - \mathrm{i}b_2z + b_8\right)^2, \quad (12)$$

$$\Theta(\xi, z, t) = -\frac{3\left(2b_1(b_1\xi + b_3t + b_2z + b_4) - 2\mathrm{i}b_1\left(-\mathrm{i}b_1\xi + \frac{\mathrm{i}t(b_3\sigma - 3b_2)}{\sigma} - \mathrm{i}b_2z + b_8\right)\right)}{(b_1\xi + b_3t + b_2z + b_4)^2 + \left(-\mathrm{i}b_1\xi + \frac{\mathrm{i}t(b_3\sigma - 3b_2)}{\sigma} - \mathrm{i}b_2z + b_8\right)^2}, \quad (13)$$

$$\chi(x, y, z, t) = \frac{3(2b_1(b_3t + b_1(x + \sigma y) + b_2z + b_4) - \varphi_3)^2}{((b_3t + b_1(x + \sigma y) + b_2z + b_4)^2 + \varphi_4)^2}, \quad (14)$$

where $\varphi_3 = 2b_1\left(-\frac{t(b_3\sigma - 3b_2)}{\sigma} + b_1(x + \sigma y) + b_2z + \mathrm{i}b_8\right)$, $\varphi_4 = \left(\frac{\mathrm{i}t(b_3\sigma - 3b_2)}{\sigma} - \mathrm{i}b_1(x + \sigma y) - \mathrm{i}b_2z + b_8\right)^2$.

2.2. Lump-kink solutions

In this section, we reveal the lump-kink solution to equation (1).

Consider the exponential test function as a solution to the bilinear equation (7)

$$f(\xi, z, t) = (b_1\xi + b_3z + b_2t + b_4)^2 + (b_5\xi + b_6z + b_7t + b_8)^2 + \mathrm{e}^{b_9\xi + b_10z + b_11t + b_12} + a_{13}. \quad (15)$$

Substituting equation (15) into (7), gives a polynomial in the powers of ξ, z, t and an exponential function. Collecting the coefficients of the same power, and equating each summations to zero, yields an algebraic system of equations. We solve the system of equations to obtained the values of the parameters involved. Substituting the values of the parameters into equation (6) and then into equation (2), yields the following lump-kink solution to equation (1):

Case-1: When

$$b_1 = -\mathrm{i}b_5,\ \ b_2 = -\mathrm{i}b_6,\ \ b_3 = \frac{3\mathrm{i}(b_5b_9^2\sigma - b_6)}{2\sigma},$$
$$b_7 = -\frac{3(b_5b_9^2\sigma - b_6)}{2\sigma},\ \ b_8 = \mathrm{i}b_4,$$

$b_{10} = \frac{1}{3}(b_9^3 + 2b_{11})\sigma$, we have

$$f(\xi, z, t) = \mathrm{e}^{b_9\xi + b_{11}t + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}} + b_{13} + \varphi_5^2 + \varphi_6^2, \quad (16)$$

$$\Theta(\xi, z, t) = -\frac{3(b_9\mathrm{e}^{b_9\xi + b_{11}t + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}} + 2b_5\varphi_5 - 2\mathrm{i}b_5\varphi_6)}{\mathrm{e}^{b_9\xi + b_{11}t + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}} + b_{13} + \varphi_5^2 + \varphi_6^2}, \quad (17)$$

$$\chi(x, y, z, t) = \frac{3(b_9\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right)} + 2b_5\varphi_7 - 2\mathrm{i}b_5\varphi_8)^2}{(\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right)} + b_{13} + \varphi_7^2 + \varphi_8^2)^2} - \frac{3b_9^2\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right)}}{\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right) + b_{13} + \varphi_7^2 + \varphi_8^2}}, \quad (18)$$

where $\varphi_5 = b_5\xi - \frac{3t(b_5b_9^2\sigma - b_6)}{2\sigma} + b_6z + \mathrm{i}b_4$, $\varphi_6 = -\mathrm{i}b_5\xi + \frac{3\mathrm{i}t(b_5b_9^2\sigma - b_6)}{2\sigma} - b_6z\mathrm{i} + b_4$, $\varphi_7 = -\frac{3t(b_5b_9^2\sigma - b_6)}{2\sigma} + b_5(x + \sigma y) + b_6z + b_4\mathrm{i}$, $\varphi_8 = \frac{3\mathrm{i}t(b_5b_9^2\sigma - b_6)}{2\sigma} - b_5(x + \sigma y)\mathrm{i} - b_6\mathrm{i}z + b_4$.

Case-2: When

$$b_2 = \frac{1}{3}(-2)\mathrm{i}b_7\sigma,\ \ b_3 = -\mathrm{i}b_7,\ \ b_5 = \mathrm{i}b_1,$$
$$b_6 = \frac{2b_7\sigma}{3},\ \ b_8 = \mathrm{i}b_4,\ \ b_{10} = \frac{1}{3}(b_9^3 + 2b_{11})\sigma,$$

we have

$$f(\xi, z, t) = \mathrm{e}^{b_9\xi + b_{11}t + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}} + b_{13} + \varphi_9^2 + \varphi_{10}^2, \quad (19)$$

$$\Theta(\xi, z, t) = -\frac{3(b_9\mathrm{e}^{b_9\xi + b_{11}t + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}} + 2b_1\varphi_9 + 2\mathrm{i}b_1\varphi_{10})}{\mathrm{e}^{b_9\xi + b_{11}t + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}} + b_{13} + \varphi_9^2 + \varphi_{10}^2}, \quad (20)$$

$$\chi(x, y, z, t) = \frac{3(b_9\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right)} + 2b_1\varphi_{11} + 2\mathrm{i}b_1\varphi_{12})^2}{(\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right)} + b_{13} + \varphi_{11}^2 + \varphi_{12}^2)^2} - \frac{3b_9^2\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right)}}{\mathrm{e}^{\left(b_{11}t + b_9(x + \sigma y) + \frac{1}{3}(b_9^3 + 2b_{11})\sigma z + b_{12}\right) + b_{13} + \varphi_{11}^2 + \varphi_{12}^2}}, \quad (21)$$

where $\varphi_9 = b_1\xi - \mathrm{i}b_7t - \frac{2}{3}\mathrm{i}b_7\sigma z + b_4$, $\varphi_{10} = \mathrm{i}b_1\xi + b_7t + \frac{2}{3}b_7\sigma z + \mathrm{i}b_4$, $\varphi_{11} = -\mathrm{i}b_7t + b_1(x + \sigma y) - \frac{2}{3}\mathrm{i}b_7\sigma z + b_4$, $\varphi_{12} = b_7t + \mathrm{i}b_1(x + \sigma y) + \frac{2}{3}b_7\sigma z + \mathrm{i}b_4$.

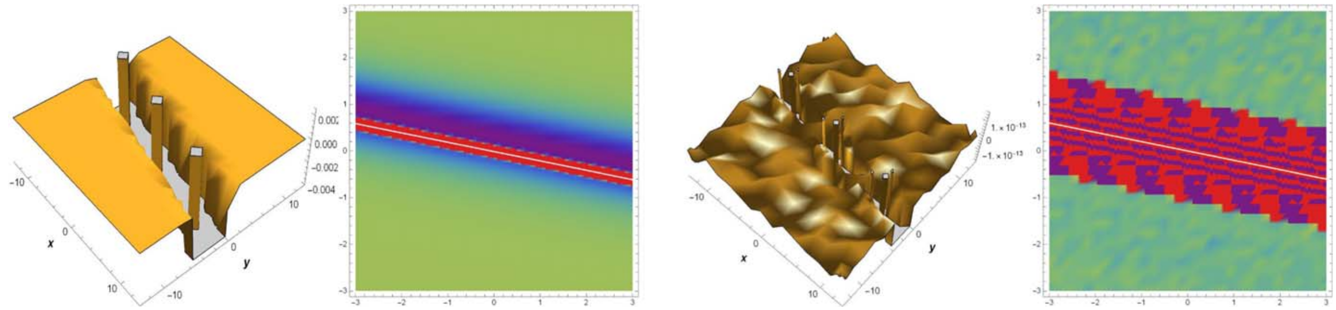
Case-3: When

$$b_2 = \frac{1}{3}(3b_1b_9^2 + 2b_3)\sigma,\ \ b_4 = \mathrm{i}b_8,\ \ b_5 = -\mathrm{i}b_1,$$
$$b_6 = -\frac{1}{3}\mathrm{i}(3b_1b_9^2 + 2b_3)\sigma,\ \ b_7 = -\mathrm{i}b_3,$$

$b_{11} = \frac{3b_{10} - b_9^3\sigma}{2\sigma}$, we have

$$f(\xi, z, t) = \mathrm{e}^{\left(b_9\xi + \frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_{10}z + b_{12}\right)} + b_{13} + \varphi_{13}^2 + \varphi_{14}^2, \quad (22)$$

$$\Theta(\xi, z, t) = -\frac{3(b_9\mathrm{e}^{\left(b_9\xi + \frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_{10}z + b_{12}\right)} - 2\mathrm{i}b_1\varphi_{13} + 2b_1\varphi_{14})}{\mathrm{e}^{\left(b_9\xi + \frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_{10}z + b_{12}\right)} + b_{13} + \varphi_{13}^2 + \varphi_{14}^2}, \quad (23)$$



(a) (b) (c) (d)

Figure 5. The 3D and density profiles of equations (42) and (45) at $\sigma = 5$, $z = t = 0$.

$$\chi(x, y, z, t) = \frac{3\left(b_9 e^{\left(\frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_9(x+\sigma y) + b_{10}z + b_{12}\right)} - 2ib_1\varphi_{15} + 2b_1\varphi_{16}\right)^2}{\left(e^{\left(\frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_9(x+\sigma y) + b_{10}z + b_{12}\right)} + b_{13} + \varphi_{15}^2 + \varphi_{16}^2\right)^2} - \frac{3b_9^2 e^{\left(\frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_9(x+\sigma y) + b_{10}z + b_{12}\right)}}{e^{\left(\frac{t(3b_{10} - b_9^3\sigma)}{2\sigma} + b_9(x+\sigma y) + b_{10}z + b_{12}\right)} + b_{13} + \varphi_{15}^2 + \varphi_{16}^2}, \quad (24)$$

where $\varphi_{13} = -ib_1\xi - ib_3t - \frac{1}{3}i(3b_1b_9^2 + 2b_3)\sigma z + b_8$, $\varphi_{14} = b_1\xi + b_3t + \frac{1}{3}(3b_1b_9^2 + 2b_3)\sigma z + ib_8$, $\varphi_{15} = -ib_3t - ib_1(x + \sigma y) - \frac{1}{3}i(3b_1b_9^2 + 2b_3)\sigma z + b_8$, $\varphi_{16} = b_3t + b_1(x + \sigma y) + \frac{1}{3}(3b_1b_9^2 + 2b_3)\sigma z + ib_8$.

2.3. Breather wave solutions

In this section, we construct the breather wave solutions to equation (1).

Consider the following test function as a solution to the bilinear equation (7):

$$f(\xi, z, t) = e^{-q_1(a_0z + b_0t + \xi)} + n_1\cos(q_0(c_0z + d_0t + \xi)) + n_2e^{q_1(a_0z + b_0t + \xi)}. \quad (25)$$

Substituting equation (25) into (7), yields a polynomial in the powers of trigonometric and exponential functions. Collecting the coefficients of the same power, and equating each summations to zero, produces an algebraic system of equations. We solve the system of equations to obtained the values of the parameters involved. Substituting the values of the parameters into equation (6) and then into equation (2), yields the following breather wave solutions to equation (1):

Case-1: When

$$n_2 = -\frac{n_1^2q_0^2}{4q_1^2}, \quad a_0 = \frac{1}{3}\sigma(-2d_0 + q_0^2 - 3q_1^2),$$
$$b_0 = -d_0 + 2q_0^2 - 2q_1^2,$$

$c_0 = \frac{1}{3}(2d_0\sigma + q_0^2(-\sigma) + 3q_1^2\sigma)$, we have

$$f(\xi, z, t) = -\frac{n_1^2q_0^2e^{\varphi_{17}}}{4q_1^2} + n_1\cos(\varphi_{18}) + e^{-\varphi_{17}}, \quad (26)$$

$$\Theta(\xi, z, t) = -\frac{3\left(-\frac{n_1^2q_0^2e^{\varphi_{17}}}{4q_1} - n_1q_0\sin(\varphi_{18}) - q_1e^{-\varphi_{17}}\right)}{-\frac{n_1^2q_0^2e^{\varphi_{17}}}{4q_1^2} + n_1\cos(\varphi_{18}) + e^{-\varphi_{17}}}, \quad (27)$$

$$\chi(x, y, z, t) = \frac{3\left(-\frac{n_1^2q_0^2e^{\varphi_{20}}}{4q_1} - n_1q_0\sin(\varphi_{19}) - q_1e^{-\varphi_{20}}\right)^2}{\left(-\frac{n_1^2q_0^2e^{\varphi_{20}}}{4q_1^2} + n_1\cos(\varphi_{19}) + e^{-\varphi_{20}}\right)^2} - \frac{3\left(-n_1q_0^2\cos(\varphi_{19}) - \frac{1}{4}n_1^2q_0^2e^{\varphi_{20}} + q_1^2e^{-\varphi_{20}}\right)}{-\frac{n_1^2q_0^2e^{\varphi_{20}}}{4q_1^2} + n_1\cos(\varphi_{19}) + e^{-\varphi_{20}}}, \quad (28)$$

where $\varphi_{17} = q_1\left(t(-d_0 + 2q_0^2 - 2q_1^2) + \frac{1}{3}\sigma z(-2d_0 + q_0^2 - 3q_1^2) + \xi\right)$, $\varphi_{18} = q_0\left(\frac{1}{3}z(2d_0\sigma + q_0^2(-\sigma) + 3q_1^2\sigma) + d_0t + \xi\right)$, $\varphi_{19} = q_0\left(\frac{1}{3}z(2d_0\sigma + q_0^2(-\sigma) + 3q_1^2\sigma) + d_0t + x + \sigma y\right)$, $\varphi_{20} = q_1\left(t(-d_0 + 2q_0^2 - 2q_1^2) + \frac{1}{3}\sigma z(-2d_0 + q_0^2 - 3q_1^2) + x + \sigma y\right)$.

Case-2: When

$$a_0 = \frac{2}{3}(2q_0^2\sigma - d_0\sigma), \quad b_0 = 4q_0^2 - d_0,$$
$$c_0 = \frac{1}{3}(-2)\sigma(2q_0^2 - d_0), \quad q_1 = -iq_0,$$

we have

$$f(\xi, z, t) = n_2e^{-\varphi_{21}} + n_1\cos(\varphi_{22}) + e^{\varphi_{21}}, \quad (29)$$

$$\Theta(\xi, z, t) = -\frac{3(-\mathrm{i}n_2 q_0 \mathrm{e}^{-\varphi_{21}} - n_1 q_0 \sin(\varphi_{22}) + \mathrm{i}q_0 \mathrm{e}^{\varphi_{21}})}{n_2 \mathrm{e}^{-\varphi_{21}} + n_1 \cos(\varphi_{22}) + \mathrm{e}^{\varphi_{21}}}, \tag{30}$$

$$\chi(x, y, z, t) = \frac{3(-\mathrm{i}n_2 q_0 \mathrm{e}^{-\varphi_{23}} - n_1 q_0 \sin(\varphi_{24}) + \mathrm{i}q_0 \mathrm{e}^{\varphi_{23}})^2}{(n_2 \mathrm{e}^{-\varphi_{23}} + n_1 \cos(\varphi_{24}) + \mathrm{e}^{\varphi_{23}})^2} - \frac{3(-n_2 q_0^2 \mathrm{e}^{-\varphi_{23}} - n_1 q_0^2 \cos(\varphi_{24}) + q_0^2(-\mathrm{e}^{\varphi_{23}}))}{n_2 \mathrm{e}^{-\varphi_{23}} + n_1 \cos(\varphi_{24}) + \mathrm{e}^{\varphi_{23}}}, \tag{31}$$

where $\varphi_{21} = \mathrm{i}q_0\Big(t(4q_0^2 - d_0) + \frac{2}{3}z(2q_0^2\sigma - d_0\sigma) + \xi\Big)$, $\varphi_{22} = q_0\Big(-\frac{2}{3}\sigma z(2q_0^2 - d_0) + d_0 t + \xi\Big)$, $\varphi_{23} = \mathrm{i}q_0(t(4q_0^2 - d_0) + \frac{2}{3}z(2q_0^2\sigma - d_0\sigma) + x + \sigma y)$, $\varphi_{24} = q_0(-\frac{2}{3}\sigma z(2q_0^2 - d_0) + d_0 t + x + \sigma y)$.

2.4. Lump-periodic solutions

In this section, we report the lump-periodic solutions to equation (1).

Consider the test function as a solution to the bilinear equation (7)

$$f(\xi, z, t) = q_1 \cosh(b_1\xi + b_3 t + b_2 z) + q_2 \cos(b_4\xi + b_6 t + b_5 z) + q_3 \cosh(b_7\xi + b_9 t + b_8 z). \tag{32}$$

Substituting equation (32) into (7), yields a polynomial in the powers of hyperbolic and trigonometric functions. Collecting the coefficients of the same power, and equating each summations to zero, produces an algebraic system of equations. We solve the system of equations to obtained the values of the parameters involved. Substituting the values of the parameters into equation (6) and then into equation (2), yields the following lump-periodic solution to equation (1):

Case-1: When

$$b_2 = \frac{1}{3}(b_1^3\sigma - 3b_4^2 b_1\sigma + 2b_3\sigma),$$
$$b_5 = \frac{1}{3}(-b_4^3 + 3b_1^2 b_4 + 2b_6)\sigma, \quad q_1 = \frac{\mathrm{i}b_4 q_2}{b_1}, \quad q_3 = 0,$$

we have

$$f(\xi, z, t) = q_2 \cos(\varphi_{25}) + \frac{\mathrm{i}b_4 q_2 \cosh(\varphi_{26})}{b_1}, \tag{33}$$

$$\Theta(\xi, z, t) = -\frac{3(-b_4 q_2 \sin(\varphi_{25}) + \mathrm{i}b_4 q_2 \sinh(\varphi_{26}))}{q_2 \cos(\varphi_{25}) + \frac{\mathrm{i}b_4 q_2 \cosh(\varphi_{26})}{b_1}}, \tag{34}$$

$$\chi(x, y, z, t) = \frac{3(-b_4 q_2 \sin(\varphi_{27}) + \mathrm{i}b_4 q_2 \sinh(\varphi_{28}))^2}{\left(q_2 \cos(\varphi_{27}) + \frac{\mathrm{i}b_4 q_2 \cosh(\varphi_{28})}{b_1}\right)^2} - \frac{3(-b_4^2 q_2 \cos(\varphi_{27}) + \mathrm{i}b_1 b_4 q_2 \cosh(\varphi_{28}))}{q_2 \cos(\varphi_{27}) + \frac{\mathrm{i}b_4 q_2 \cosh(\varphi_{28})}{b_1}}, \tag{35}$$

where $\varphi_{25} = b_4\xi + b_6 t + \frac{1}{3}(-b_4^3 + 3b_1^2 b_4 + 2b_6)\sigma z$, $\varphi_{26} = b_1\xi + b_3 t + \frac{1}{3}z(b_1^3\sigma - 3b_4^2 b_1\sigma + 2b_3\sigma)$, $\varphi_{27} = b_6 t + b_4(x + \sigma y) + \frac{1}{3}(-b_4^3 + 3b_1^2 b_4 + 2b_6)\sigma z$, $\varphi_{28} = b_3 t + b_1(x + \sigma y) + \frac{1}{3}z(b_1^3\sigma - 3b_4^2 b_1\sigma + 2b_3\sigma)$.

Case-2: When

$$b_5 = \frac{1}{3}(-b_4^3 + 3b_7^2 b_4 + 2b_6)\sigma,$$
$$b_8 = \frac{1}{3}(b_7^3 - 3b_4^2 b_7 + 2b_9)\sigma, \quad q_1 = 0, \quad q_2 = -\frac{\mathrm{i}b_7 q_3}{b_4},$$

we have

$$f(\xi, z, t) = q_3 \cosh(\varphi_{29}) - \frac{\mathrm{i}b_7 q_3 \cos(\varphi_{30})}{b_4}, \tag{36}$$

$$\Theta(\xi, z, t) = -\frac{3(b_7 q_3 \sinh(\varphi_{29}) + \mathrm{i}b_7 q_3 \sin(\varphi_{30}))}{q_3 \cosh(\varphi_{29}) - \frac{ib_7 q_3 \cos(\varphi_{30})}{b_4}}, \tag{37}$$

$$\chi(\xi, z, t) = \frac{3(b_7 q_3 \sinh(\varphi_{32}) + \mathrm{i}b_7 q_3 \sin(\varphi_{31}))^2}{\left(q_3 \cosh(\varphi_{32}) - \frac{\mathrm{i}b_7 q_3 \cos(\varphi_{31})}{b_4}\right)^2} - \frac{3(b_7^2 q_3 \cosh(\varphi_{32}) + \mathrm{i}b_4 b_7 q_3 \cos(\varphi_{31}))}{q_3 \cosh(\varphi_{32}) - \frac{\mathrm{i}b_7 q_3 \cos(\varphi_{31})}{b_4}}, \tag{38}$$

where $\varphi_{29} = b_7\xi + b_9 t + \frac{1}{3}(b_7^3 - 3b_4^2 b_7 + 2b_9)\sigma z$, $\varphi_{30} = b_4\xi + b_6 t + \frac{1}{3}(-b_4^3 + 3b_7^2 b_4 + 2b_6)\sigma z$, $\varphi_{31} = b_6 t + b_4(x + \sigma y) + \frac{1}{3}(-b_4^3 + 3b_7^2 b_4 + 2b_6)\sigma z$, $\varphi_{32} = b_9 t + b_7(x + \sigma y) + \frac{1}{3}(b_7^3 - 3b_4^2 b_7 + 2b_9)\sigma z$.

2.5. Some new interaction solutions

In this section, some new interaction solutions to equation (1) are reported.

Consider the following test function as a solution to the bilinear equation (7):

$$f(\xi, z, t) = c_1 \mathrm{e}^{(b_1\xi + b_2 z + b_3 t)} + c_2 \mathrm{e}^{-(b_1\xi + b_2 z + b_3 t)} + c_3 \sin(b_4\xi + b_5 z + b_6 t) + c_4 \sinh(b_7\xi + b_8 z + b_9 t). \tag{39}$$

Substituting equation (39) into equation (7), yields a polynomial in the powers of trigonometric, hyperbolic and exponential functions. Collecting the coefficients of the same power, and equating each summations to zero, produces an algebraic system of equations. We solve the system of equations to obtained the values of the parameters involved. Substituting the values of the parameters into equation (6) and then into equation (2), yields the following interaction solutions to equation (1):

Case-1: When

$$b_2 = \frac{1}{3}(b_1^3 + 3b_7^2 b_1 + 2b_3)\sigma,$$
$$b_8 = \frac{1}{3}(b_7^3 + 3b_1^2 b_7 + 2b_9)\sigma,\ \ c_2 = -\frac{b_7^2 c_4^2}{4b_1^2 c_1},\ \ c_3 = 0,$$

we have

$$f(\xi, z, t) = -\frac{b_7^2 c_4^2 e^{\varphi_{34}}}{4b_1^2 c_1} + c_1 e^{\varphi_{33}} + c_4 \sinh(\varphi_{35}), \tag{40}$$

$$\Theta(\xi, z, t) = -\frac{3\left(\frac{b_7^2 c_4^2 e^{\varphi_{34}}}{4b_1 c_1} + b_1 c_1 e^{\varphi_{33}} + b_7 c_4 \cosh(\varphi_{35})\right)}{-\frac{b_7^2 c_4^2 e^{\varphi_{34}}}{4b_1^2 c_1} + c_1 e^{\varphi_{33}} + c_4 \sinh(\varphi_{35})}, \tag{41}$$

$$\chi(x, y, z, t) = -\frac{3\left(b_1^2 c_1 e^{\varphi_{36}} - \frac{b_7^2 c_4^2 e^{\varphi_{37}}}{4c_1} + b_7^2 c_4 \sinh(\varphi_{38})\right)}{-\frac{b_7^2 c_4^2 e^{\varphi_{37}}}{4b_1^2 c_1} + c_1 e^{\varphi_{36}} + c_4 \sinh(\varphi_{38})} + \frac{3\left(\frac{b_7^2 c_4^2 e^{\varphi_{37}}}{4b_1 c_1} + b_1 c_1 e^{\varphi_{36}} + b_7 c_4 \cosh(\varphi_{38})\right)^2}{\left(-\frac{b_7^2 c_4^2 e^{\varphi_{37}}}{4b_1^2 c_1} + c_1 e^{\varphi_{36}} + c_4 \sinh(\varphi_{38})\right)^2}, \tag{42}$$

where $\varphi_{33} = b_1\xi + b_3 t + \frac{1}{3}(b_1^3 + 3b_7^2 b_1 + 2b_3)\sigma z$, $\varphi_{34} = b_1(-\xi) - b_3 t - \frac{1}{3}(b_1^3 + 3b_7^2 b_1 + 2b_3)\sigma z$, $\varphi_{35} = b_7\xi + b_9 t + \frac{1}{3}(b_7^3 + 3b_1^2 b_7 + 2b_9)\sigma z$, $\varphi_{36} = b_3 t + b_1(x + \sigma y) + \frac{1}{3}(b_1^3 + 3b_7^2 b_1 + 2b_3)\sigma z$, $\varphi_{37} = -b_3 t + b_1(-x - \sigma y) - \frac{1}{3}(b_1^3 + 3b_7^2 b_1 + 2b_3)\sigma z$, $\varphi_{38} = b_9 t + b_7(x + \sigma y) + \frac{1}{3}(b_7^3 + 3b_1^2 b_7 + 2b_9)\sigma z$.

Case-2: When

$$b_5 = \frac{1}{3}(-b_4^3 + 3b_7^2 b_4 + 2b_6)\sigma,$$
$$b_8 = \frac{1}{3}(b_7^3 - 3b_4^2 b_7 + 2b_9)\sigma,\ \ c_1 = 0,\ \ c_2 = 0,\ \ c_3 = \frac{b_7 c_4}{b_4},$$

we have

$$f(\xi, z, t) = \frac{b_7 c_4 \sin(\varphi_{40})}{b_4} + c_4 \sinh(\varphi_{39}), \tag{43}$$

$$\Theta(\xi, z, t) = -\frac{3(b_7 c_4 \cos(\varphi_{40}) + b_7 c_4 \cosh(\varphi_{39}))}{\frac{b_7 c_4 \sin(\varphi_{40})}{b_4} + c_4 \sinh(\varphi_{39})}, \tag{44}$$

$$\chi(x, y, z, t) = \frac{3(b_7 c_4 \cos(\varphi_{41}) + b_7 c_4 \cosh(\varphi_{42}))^2}{\left(\frac{b_7 c_4 \sin(\varphi_{41})}{b_4} + c_4 \sinh(\varphi_{42})\right)^2} - \frac{3(b_7^2 c_4 \sinh(\varphi_{42}) - b_4 b_7 c_4 \sin(\varphi_{41}))}{\frac{b_7 c_4 \sin(\varphi_{41})}{b_4} + c_4 \sinh(\varphi_{42})}, \tag{45}$$

where $\varphi_{39} = b_7\xi + b_9 t + \frac{1}{3}(b_7^3 - 3b_4^2 b_7 + 2b_9)\sigma z$, $\varphi_{40} = b_4\xi + b_6 t + \frac{1}{3}(-b_4^3 + 3b_7^2 b_4 + 2b_6)\sigma z$, $\varphi_{41} = b_6 t + b_4(x + \sigma y) + \frac{1}{3}(-b_4^3 + 3b_7^2 b_4 + 2b_6)\sigma z$, $\varphi_{42} = b_9 t + b_7(x + \sigma y) + \frac{1}{3}(b_7^3 - 3b_4^2 b_7 + 2b_9)\sigma z$.

The symbol $i = \sqrt{-1}$.

3. Conclusion

Via the Hirota bilinear approach and symbolic computation, the (3+1)-dimensional soliton equation is investigated in this study. Various new lump, lump-kink, breather wave, lump periodic, and some other new interaction solutions are successfully constructed. Using the Mathematica 12 package, all the acquired solutions are verified by inserting them into the original equation. The physical characteristics of the solution were graphically depicted to shed more light on the obtained results under the choice of suitable values of the parameters. The results obtained may be useful in understanding the basic nonlinear scenarios in fluid dynamics as well as the dynamics of computational physics and engineering sciences in nonlinear fields of higher dimensional motion.

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