Directed transport of coupled Brownian motors in a two-dimensional traveling-wave potential*

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Considering an elastically coupled Brownian motors system in a two-dimensional traveling-wave potential, we investigate the effects of the angular frequency of the traveling wave, wavelength, coupling strength, free length of the spring, and the noise intensity on the current of the system. It is found that the traveling wave is the essential condition of the directed transport. The current is dominated by the traveling wave and varies nonmonotonically with both the angular frequency and the wavelength. At an optimal angular frequency or wavelength, the current can be optimized. The coupling strength and the free length of the spring can locally modulate the current, especially at small angular frequencies. Moreover, the current decreases rapidly with the increase of the noise intensity, indicating the interference effect of noise on the directed transport.

Keywords: directed transport, Brownian motors, coupled system, two-dimensional traveling-wave potential

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1. Introduction

In the last few decades, Brownian motors have attracted much interest due to the special energy conversion mechanism and huge potential applications in physics, biology, and other fields. Brownian motors can convert the nonequilibrium fluctuation into the directed motion in a periodic asymmetric potential in the absence of external bias.[1,2] A large variety of motor proteins (kinesin, myosin, dynein, etc.) in biological cells can all be regarded as Brownian motors, which can convert the chemical energy from ATP hydrolysis into mechanical work to achieve directed transport along tracks. Up to now the directed transport mechanism of Brownian motors has been a hot topic in physics and biology.[3–12] Inspired by the Smoluchowski–Feynman ratchet, many theoretical models for Brownian motors, called Brownian ratchets, have been proposed, such as flashing ratchets,[13,14] rocking ratchets,[15,16] and correlation ratchets.[17,18]

Most studies of Brownian ratchets have been concerned with the directed transport mechanism of a single Brownian motor in a one-dimensional asymmetric periodic potential.[19,20] However, experiments have shown that the collective directed transport widely exists in many biological active processes. Vershinin *et al.* found that multiple kinesin motors could work in groups to achieve a long distance transport and apply significantly larger forces without the need of additional factors.[21] Shtridelman *et al.* suggested that motors could work cooperatively to attain higher transport force and velocity through comparing the transport force and velocity of the collective transport with that of a single molecular motor.[22] Ali *et al.* observed that different kinds of molecular motors, such as kinesin and myosin, could also cooperate to transport cargos for a longer distance.[23] These studies indicate that the collective transport has more complex effects compared to the single-motor transport. Therefore, collective transport dynamics of Brownian motors have attracted the interest of researchers in recent years. Julicher and Prost investigated the dynamics of multiple rigid ratchets in an asymmetric potential and found the existence of phase transition.[24] Csahók *et al.* studied the transport of a chain of elastically coupled particles in a ratchet potential.[25] Li *et al.* explored the collective mechanism of coupled Brownian motors in a flashing ratchet in the presence of coupling symmetry breaking and space symmetry breaking, and reversed motion was found.[26] Besides, Zheng *et al.* proposed an

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interesting two-dimensional coupled transport mechanism in which with the aid of interactions between two motors, coordinated processive motion in the ratchet potential could be achieved when external forces were exerted to the non-ratchet potential.[27] Wang *et al.* investigated the transport properties of coupled Brownian particles in a two-dimensional rocking ratchet, and found that ratchet movement and collective effect could coexist.[28] Zhang *et al.* explored a flashing ratchet model of a two-headed molecular motor in a two-dimensional potential, and obtained the result which accorded with the experimental observations.[29]

In this paper, we consider a model of two elastically coupled Brownian motors in a two-dimensional traveling-wave potential. The effects of the traveling wave parameters (the angular frequency and the wavelength), the system parameters (the coupling strength and the free length of the spring), and the noise intensity on the current of the system are discussed. The results show that the current is dominated by the angular frequency and the wavelength of the traveling wave. With the increase of the angular frequency and the wavelength, the current varies nonmonotonically, and has its maximum value at an optimal angular frequency or wavelength. The coupling interaction between motors is not the necessary condition to achieve the directed transport, but the coupling strength and the free length of the spring can locally modulate the current. Moreover, with the increase of the noise intensity, the current decreases rapidly.

2. Model

We consider a coupled system consisting of two Brownian motors connected by an elastic spring in a two-dimensional potential. The overdamped dynamics of the system can be written as

$$\gamma\dot{x}_i(t) = -\frac{\partial V(x_i,y_i)}{\partial x_i} - \frac{\partial U_{\text{int}}}{\partial x_i} + \xi_i(t), \tag{1}$$

$$\gamma\dot{y}_i(t) = -\frac{\partial V(x_i,y_i)}{\partial y_i} - \frac{\partial U_{\text{int}}}{\partial y_i}, \tag{2}$$

where γ is the friction coefficient. $\boldsymbol{r}_i(t) = (x_i(t), y_i(t))$ is the position of the i-th Brownian motor at time t. The two-dimensional potential $V(x,y) = V_x(x,t) + V_y(y)$ is shown in Fig. 1. Here, we apply the traveling wave form for the potential in x direction

$$V_x(x,t) = V_0 \cos\left(\frac{2\pi}{\lambda}x - \omega t\right), \tag{3}$$

where V_0, λ, and ω are the wave amplitude, the wavelength, and the angular frequency of the traveling wave, respectively. Then, the wave velocity can be expressed as $u = \omega\lambda/2\pi$, and the energy flow density of the wave can be determined by $E = k'V_0^2\omega^2 u$, where k' is the proportionality factor. The larger the energy flow density is, the greater the energy of the wave is.

The potential in y direction is chosen as the parabolic form

$$V_y(y) = \frac{1}{2}\varepsilon y^2, \tag{4}$$

where ε is the parameter which determines the concavity of the potential in y direction

In Eqs. (1) and (2), U_{int} is the interaction energy between two Brownian motors which is given by the harmonic form $U_{\text{int}} = k\left[\|\boldsymbol{r}_2 - \boldsymbol{r}_1\| - a\right]^2/2$, where k is the coupling strength, and a is the free length of the spring. $\xi_i(t)$ is Gaussian white noise with $\langle\xi_i(t)\rangle = 0$ and $\langle\xi_i(t)\xi_j(t')\rangle = 2D\delta_{ij}\delta(t-t')$, where D is the noise intensity.

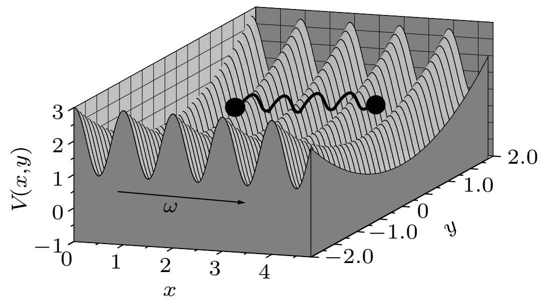

Fig. 1. Schematic diagram of coupled Brownian motors in the two-dimensional traveling-wave potential $V(x,y)$.

We simulate numerically Eqs. (1) and (2) by using the stochastic Runge–Kutta algorithm. Each trajectory evolves 5×10^6 steps with the time step $\Delta t = 10^{-3}$. Two motors are initially placed at (2.5, 0) and (7.5, 0), respectively. Due to the limitation of the potential barrier, it is difficult to achieve effective directed transport in y direction. So, we are only concerned with the current in x direction which is determined by

$$\upsilon = \lim_{T\to\infty}\frac{1}{2T}\int_0^T \sum_{i=1}^{2}\dot{x}_i\,\mathrm{d}t. \tag{5}$$

3. The results and discussion

3.1. Effects of the traveling wave parameters on the directed transport

In this model, the angular frequency and the wavelength are two important parameters for the traveling wave. In order to investigate the influence of the traveling wave on the directed transport, firstly, we calculate the current υ as a function of the angular frequency ω for different values of the coupling strength k and the noise intensity D in Figs. 2(a) and 2(b), respectively. It is found that when ω is zero, namely, there is no fluctuation, there is no directed transport. It can be explained that in this case no ordered energy generated from fluctuation is transmitted to the system to induce the directed transport. However, when ω is not zero, the current υ varies nonmonotonically with the angular frequency ω. For both very small and very large angular frequencies, the current υ is small, and has its maximum at an optimal angular frequency. It can be understood that when the angular frequency ω is very small, the wave travels very slowly, so the motors system can be tightly coupled to the potential and travels with the wave. If the angular frequency increases, namely, the wave velocity increases, the current also increases. But when the angular frequency ω is very large, the wave travels very fast, so the motors system is loosely coupled to the potential so that it cannot keep up with the traveling wave. Thus the current decreases. These results indicate that the wave velocity can obviously affect the directed transport of the system, and at an optimal wave velocity the current can be optimized, which is in agreement with the result given in Ref. [2]. Moreover, it is also shown that the current decreases with the increase of the coupling strength k in Fig. 2(a). This is because that the rigidity of the spring between motors increases so that the coupling of the system to the potential becomes looser. Figure 2(b) shows that with the increase of the noise intensity D, the current also decreases, indicating the interference effect of noise (i.e., the thermal fluctuation) on the directed transport.

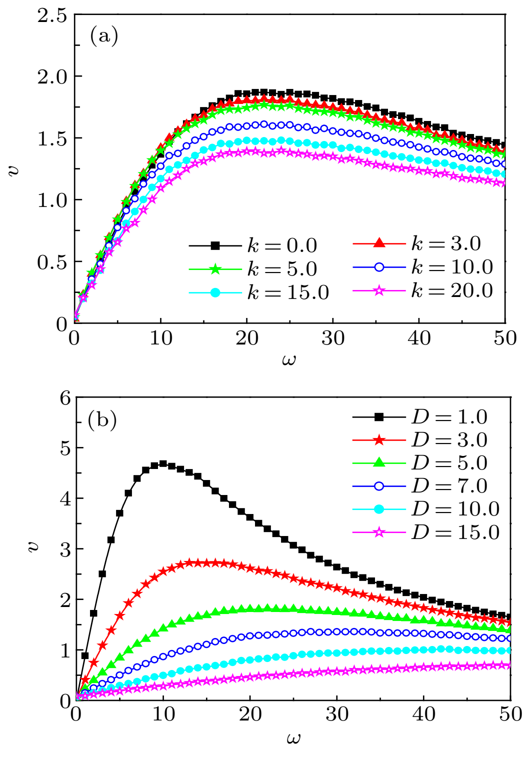

Fig. 2. The current υ versus the angular frequency ω for different values of (a) the coupling strength k at $D = 5.0$ and (b) the noise intensity D at $k = 3.0$, with other parameters being $\gamma = 1.0$, $a = 5.0$, $V_0 = 3.0$, $\lambda = 3.0$, and $\varepsilon = 0.2$.

In Fig. 3, we study the current υ versus the wavelength λ for different values of the angular frequency ω. It can be clearly seen that the variation of the current υ with the wavelength λ is very similar to that of υ with ω, that is, υ is also a nonmonotonous function of λ. At an optimal λ the current can also be optimized. The reason is similar to that in Fig. 2. When λ is small, the wave travels slowly so that the motors system is tightly coupled to the potential. In this case, the current increases with the increase of the wave velocity. But when λ is very large, the wave travels very fast so that the coupling is very loose. Thus, in this case the current is also very small.

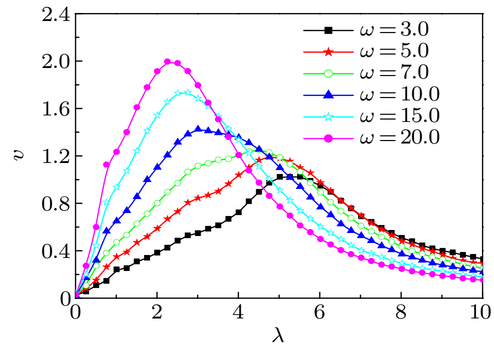

Fig. 3. The current υ versus the wavelength λ for different values of the angular frequency ω, with other parameters being $\gamma = 1.0$, $a = 5.0$, $V_0 = 3.0$, $k = 3.0$, $D = 5.0$, and $\varepsilon = 0.2$.

3.2. Effects of the system parameters on the directed transport

We investigate the effects of the traveling wave parameters on the directed transport in Subsection 3.1. In this section, we explore the influences of the system parameters, namely, the coupling strength and the free length of the spring on the directed transport. In Fig. 4(a), we plot the current υ versus the coupling strength k for different values of the angular frequency ω. It can be observed that when k is zero, there is still a large current, implying that the coupling interaction between motors is not the necessary condition to achieve directed transport. In the range of low-to-moderate coupling strength, the current varies obviously with k. Moreover, for small angular frequencies, the current can obtain its maximum at an optimal coupling strength. For this, it can be understood that when k is zero, because two motors are independent of each other, almost all of the energy to achieve the directed transport comes from the traveling wave. For the small coupling strength, besides the energy from the traveling wave, the elastic energy can also be transmitted to the system to promote the directed transport. But, with the increase of the coupling strength, the rigidity of the spring increases so that the coupling of the system to the potential becomes looser. So the current decreases. Therefore, at small angular frequencies, only for an optimal coupling strength, can the current obtain its maximum. Here, in fact, it also indicates the effect of the coupling on the coordination between motors. When the wave travels slowly, smaller coupling can enhance the coordination between motors so that the system can keep up with the traveling wave. Thus the current is larger than that of the single motor. But when the coupling strength is larger, the coordination between motors decreases so that the system cannot keep up with the wave. So the current decreases. In Fig. 4(a), it is also shown that at strong coupling, the current is hardly affected by the coupling strength. The reason is that in this case the two motors can be regarded as a whole due to the rigid connection.

In order to further explore the influence of the coupling strength on the directed transport in the traveling-wave potential, the current υ versus the coupling strength k and the angular frequency ω is presented in Fig. 4(b). We can find that when the coupling strength k is given, υ varies nonmonotonically with ω, which is consistent with that shown in Fig. 2(a). At a given angular frequency ω, the curve of υ versus k accords with that in Fig. 4(a). On the whole, the current is dominated by the angular frequency, and the coupling strength can locally modulate the current.

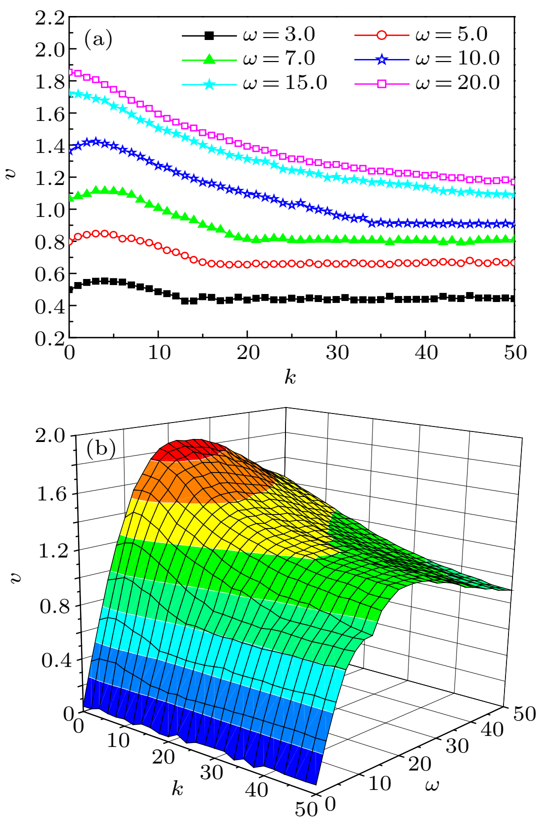

Fig. 4. The current υ versus (a) the coupling strength k for different values of the angular frequency ω, and (b) the coupling strength k and the angular frequency ω with other parameters being $\gamma = 1.0$, $a = 5.0$, $V_0 = 3.0$, $\lambda = 3.0$, $D = 5.0$, and $\varepsilon = 0.2$.

Next, we continue to investigate the other important parameter of the system, namely, the free length of the spring on the directed transport. Figure 5(a) shows the current υ as a function of the free length a for different values of the angular frequency ω. It can be clearly found that at small angular frequencies, υ oscillates periodically with a. When a/λ is approximately an integer, the two motors are trapped in the wells of the potential so that they can be tightly coupled to the potential. In this case, the current υ can reach a local maximum. While, when the angular frequency is very large, the coupling of the system to the potential becomes loose. Thus, the dependence of the current on the free length a decreases.

In order to study synthetically the influence of the free length of the spring on the directed transport in the traveling-wave potential, the current υ versus the free length a and the angular frequency ω is plotted in Fig. 5(b). Similar to Fig. 4(b), the current in the a–ω space is still determined by the angular frequency, and the free length only modulates the current locally.

According to the above results, it is fully shown that the traveling wave is the essential condition of the directed transport. The current is dominated by the angular frequency and the wavelength. The coupling strength and the free length of the spring can locally modulate the current of the system.

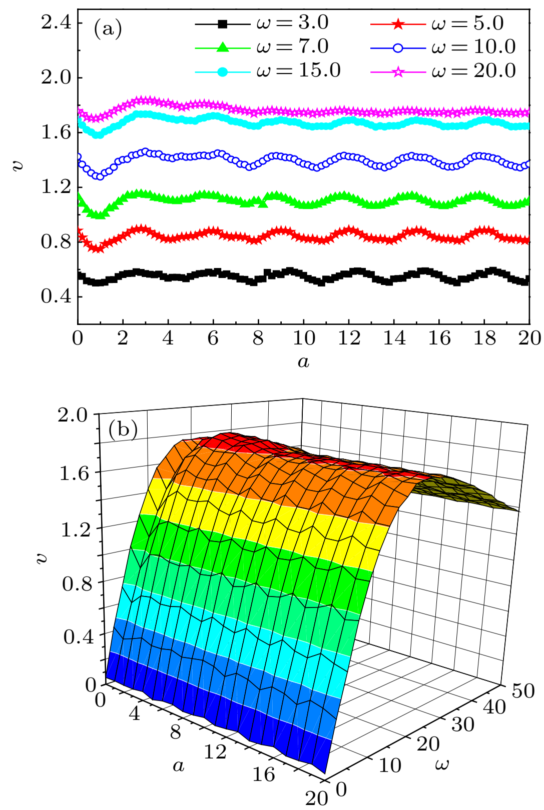

Fig. 5. The current υ versus (a) the free length of the spring a for different values of the angular frequency ω, and (b) the free length of the spring a and the angular frequency ω, with other parameters being $\gamma = 1.0$, $k = 3.0$, $V_0 = 3.0$, $\lambda = 3.0$, $D = 5.0$, and $\varepsilon = 0.2$.

3.3. Effect of noise on the directed transport

For Brownian motors, noise is an important factor for the directed transport. In many models of Brownian motors, the stochastic resonance effect can often be shown, namely, noise can enhance the directed transport.[27,30,31] In order to investigate whether the stochastic resonance effect appears in this model, we calculate the current υ as a function of the noise intensity D for different values of the angular frequency ω in Fig. 6(a). We can clearly see that with the increase of the noise intensity D, the current decreases monotonically. This illustrates that the thermal fluctuation cannot be effectively rectified to promote the directed transport. So the stochastic resonance effect is difficult to be found in this model.

Figure 6(b) shows the current υ as a function of the noise intensity D and the angular frequency ω. We can see that when there is no noise, the current can reach the maximum at an optimal angular frequency. In the case of noise, the current decreases rapidly with the increase of the noise intensity.

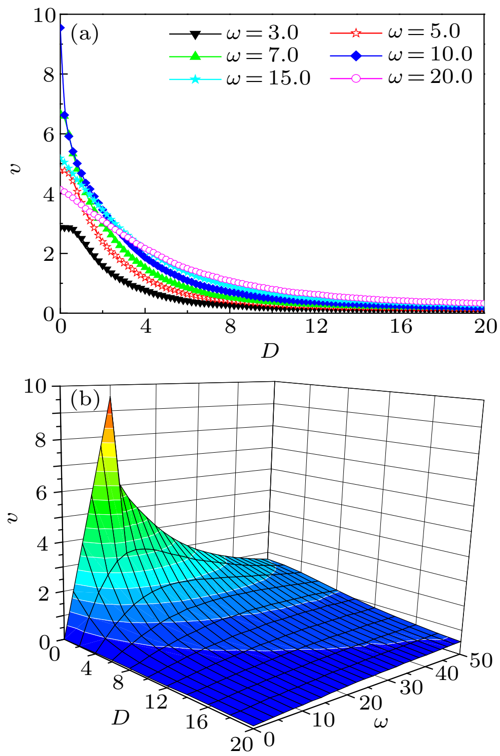

Fig. 6. The current υ versus (a) the noise intensity D for different values of the angular frequency ω, and (b) the noise intensity D and the angular frequency ω, with other parameters being $\gamma = 1.0$, k= 3.0, $a = 5.0$, $V_0 = 3.0$, $\lambda = 3.0$, and $\varepsilon = 0.2$.

4. Conclusions

In this paper, the directed transport properties of two elastically coupled Brownian motors in a two-dimensional traveling-wave potential are investigated. The effects of the traveling wave parameters (the angular frequency and the wavelength), the system parameters (the coupling strength and the free length of the spring), and the noise intensity on the directed transport are discussed respectively. The results show that the traveling wave is the essential condition of the directed transport. With the increase of the angular frequency and the wavelength, the current of the system varies nonmonotonically, and has its maximum at an optimal angular frequency or wavelength. In terms of the system, the coupling between motors is not necessary to achieve the directed transport, but the coupling strength and the free length of the spring can locally modulate the current. In the range of low-to-moderate coupling strength, the current obviously varies. In particular, at small angular frequencies, the current can reach a local maximum at an optimal coupling strength. But at strong coupling, the current is hardly affected by the coupling strength. In addition, at small angular frequencies, the current oscillates periodically with the free length of the spring. When the free length is an integral multiple of the wavelength, the current also has a local maximum. But at very large angular frequencies, the dependence of the current on the free length a decreases. Besides, when there is no noise, the current has a maximum at an optimal angular frequency. While in the case of noise, the current decreases rapidly with the increase of the noise intensity, indicating the interference effect of noise on the directed transport.

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