# Optical absorption in asymmetrical Gaussian potential quantum dot under the application of an electric field ${ }^{*}$ 

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#### Abstract

We theoretically investigate the optical absorption coefficient（OAC）in asymmetrical Gaussian potential quantum dots subject to an applied electric field．Confined wave functions together with energies of electron energies in an effective mass approximation framework are obtained．The OAC is expressed according to the iterative method and the compact－ density－matrix approach．Based on our results，OAC is sensitively dependent on external electric field together with the incident optical intensity．Additionally，peak shifts into greater energy as the quantum dot radius decrease．Moreover，the parameters of Gaussian potential have a significant influence on the OAC．


Keywords：electric field，optical absorption，quantum dot

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## 1．Introduction

Recently，many low－dimensional semiconductor systems including quantum dots（QDs）and other confined nanostruc－ tures have been studied．Due to the discrete energy levels and unique optical characters，much attention has been focused on both colloid QDs ${ }^{[1-4]}$ and self－assembly QDs．${ }^{[56]}$ Compared with bulk materials，confined nanostructures have significantly different nonlinear optical properties．${ }^{[7-10]}$ Particularly，elec－ tric field application has been extensively studied for the semi－ conductor nanostructures．${ }^{[11-13]}$ In recent years，tremendous efforts are made to examine those optical as well as electronic characteristics for QDs，where carrier motion is restricted at each spatial direction．External static electric field can modify electron optical properties and transport within QDs．The elec－ tric field applied can result in carrier distribution polarization， along with quantum state energy shift，where the intensity of optoelectronic devices can be controlled and modulated effec－ tively．Therefore，the electric field effect on the carrier within QDs deserves investigation．

The optical absorption coefficient（OAC）with semicon－ ductor nanostructures has aroused wide attention over the last few years．${ }^{[14-16]}$ Many researchers considered the electric field effect when investigating the OAC．For instance，Mandal et al．${ }^{[17]}$ have studied the OAC of QDs under noise，electric and magnetic fields，confinement potential．Zhang et al．${ }^{[18]}$ have investigated the OAC and the refractive index changes within the asymmetrical Gaussian potential quantum well in the presence of an applied external electric field．Çakır et al．${ }^{[19]}$ reported the magnetic field impacts on linear and non－ linear OACs in spherical QDs．Ghajarpour et al．${ }^{[20]}$ examined

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hydrogenic impurity，magnetic and electric fields on the OAC within QDs．Al et al．${ }^{[21]}$ have analyzed the electric field im－ pacts on the binding energy and the OAC within the Tietz－Hua quantum well．In all the above－mentioned articles，authors proposed electric and magnetic fields，the external perturba－ tion on the OAC．In this paper，we mainly investigate the OAC within an asymmetrical Gaussian potential QD when an exter－ nal electric field is applied，although extreme nonlinear optics is another important research hotspot of nonlinear optics and a lot of important research results have been obtained．${ }^{[22-25]}$

The numerical research on the OAC within the asymmet－ rical Gaussian potential QD when an external electric field is applied is presented in this study．To be specific，a theoretical framework is presented in Section 2．Section 3 presents the numerical discussion of findings，and Section 4 presents the conclusions．

## 2．Model and analysis

In the effective mass approximation framework，the Schrödinger equation of the system is given by ${ }^{[26,27]}$

$$
\begin{equation*}
\hat{H} \Psi=\left[\frac{\boldsymbol{P}^{2}}{2 m^{*}}+\frac{1}{2} m^{*} \omega_{0}^{2} \rho^{2}+V(z)+e F z\right] \Psi, \tag{1}
\end{equation*}
$$

where $m^{*}$ stands for effective mass of the electron in the con－ duction band，and $\omega_{0}=\hbar / m^{*} R^{2}$ represents electron frequency， $R$ indicates QD radius，$F$ is electric field magnitude．The asymmetrical Gaussian potential $V(z)$ is

$$
V(z)= \begin{cases}-V_{0} \exp \left(-\frac{z^{2}}{2 L^{2}}\right), & z \geq 0  \tag{2}\\ \infty, & z<0\end{cases}
$$

[^0]where $V_{0}$ stands for the Gaussian potential barrier height, and $L$ indicates the confinement potential range. First of all, the related system eigenfunctions are shown as follows:
\[

$$
\begin{align*}
\Psi= & f(\rho, \varphi) \chi(z),  \tag{3}\\
f(\rho, \varphi)= & \frac{1}{a^{1+|m|}} \sqrt{\frac{(n+|m|)!}{2 \pi 2^{|m|} n!}} \frac{1}{m!} \exp (\mathrm{i} m \varphi) \rho^{|m|}  \tag{9}\\
& \times \exp \left(-\frac{\rho^{2}}{4 a^{2}}\right) F\left(-n, 1+|m|, \frac{\rho^{2}}{2 a^{2}}\right),  \tag{4}\\
\chi_{n_{z}}(z)= & \sqrt{\frac{\alpha}{2^{n_{z} n_{z}!\sqrt{\pi}}}} \exp \left(-\frac{\alpha^{2} z^{2}}{2}\right) \\
& \times H_{n_{z}}\left[\alpha\left(z-\frac{e F L^{2}}{V_{0}}\right)\right] \tag{5}
\end{align*}
$$
\]

where $\alpha=\frac{\left(m^{*} V_{0}\right)^{1 / 4}}{(\hbar L)^{1 / 2}}, a=\sqrt{\frac{\hbar}{2 m^{*} \omega_{0}}}$ stands for scale of effective length, $F(a, b, x)$ represents the confluent hypergeometric function, $n_{z}$ is the number of quantum, $n$ represents the number of radial quantum, whereas $H_{n_{z}}(z)$ stands for Hermite polynomials.

In addition, electron eigenenergies $E$ is determined according to the equation

$$
\begin{align*}
E= & 2 \hbar \omega_{0}\left(n+\frac{1}{2}\right)+\left(2 n_{z}+\frac{3}{2}\right) \frac{\hbar}{L} \sqrt{\frac{V_{0}}{m^{*}}} \\
& -V_{0}-\frac{e^{2} F^{2} L^{2}}{2 V_{0}} . \tag{6}
\end{align*}
$$

By adopting the iterative method and the compact-densitymatrix approach, the OAC formula within the QD is calculated. Assume that our system can be triggered via an applied electromagnetic field, whose electric field vector can be expressed as

$$
\begin{equation*}
E(t)=\widetilde{E} \mathrm{e}^{-\mathrm{i} \omega t}+\widetilde{E}^{*} \mathrm{e}^{\mathrm{i} \omega t} \tag{7}
\end{equation*}
$$

where $\omega$ stands for external incident field frequency. Thereafter, the density matrix operator $\rho$ evolution is determined by the Schrödinger equation dependent on time

$$
\begin{equation*}
\frac{\partial \rho_{i j}}{\partial t}=\frac{1}{\mathrm{i} \hbar}\left[H_{0}-q z E(t), \rho\right]_{i j}-\Gamma_{i j}\left(\rho-\rho^{(0)}\right)_{i j} \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\alpha^{(3)}(\omega, I)=-\omega \sqrt{\frac{\mu}{\varepsilon_{\mathrm{R}}}}\left(\frac{I}{2 \varepsilon_{\mathrm{R}} n_{\mathrm{r}} c}\right) \times \frac{\left|M_{21}\right|^{2} \sigma_{v} \hbar \Gamma_{12}}{\left[\left(E_{21}-\hbar \omega\right)^{2}+\left(\hbar \Gamma_{12}\right)^{2}\right]^{2}}\left[4\left|M_{21}\right|^{2}-\frac{\left|M_{22}-M_{11}\right|^{2}\left[3 E_{21}^{2}-4 E_{21} \hbar \omega+\hbar^{2}\left(\omega^{2}-\Gamma_{12}^{2}\right)\right]}{E_{21}^{2}+\left(\hbar \Gamma_{12}\right)^{2}}\right] \tag{14}
\end{equation*}
$$

For the above-mentioned equations, $\mu$ stands for the system permeability, $\varepsilon_{\mathrm{R}}$ represents the actual permittivity, $\varepsilon_{0}$ suggests the free space permittivity, $I$ indicates incident optical intensify, $c$ indicates light speed, $n_{\mathrm{r}}$ represents refractive index, $\left.M_{i j}=\left|\left\langle\Psi_{i}\right| e z\right| \Psi_{j}\right\rangle \mid(i, j=1,2), E_{i j}=E_{i}-E_{j}$ suggests an energy interval between two distinct electronic states. Moreover, the overall coefficients of optical absorption are expressed as

$$
\begin{equation*}
\alpha(\omega, I)=\alpha^{(1)}(\omega)+\alpha^{(3)}(\omega, I) \tag{15}
\end{equation*}
$$

where $H_{0}$ indicates the system Hamiltonian in the absence of light field $E(t), \Gamma_{i j}$ represents relaxation time, and $\rho^{(0)}$ stands for the unperturbed density matrix. Eqation (8) can be determined based on the iterative approach as follows:

$$
\rho(t)=\sum_{n} \rho^{n}(t)
$$

with

$$
\begin{aligned}
\frac{\partial \rho_{i j}^{(n+1)}}{\partial t}= & \frac{1}{\mathrm{i} \hbar}\left\{\left[H_{0}, \rho^{(n+1)}\right]_{i j}-\mathrm{i} \hbar \Gamma_{i j} \rho_{i j}^{(n+1)}\right\} \\
& -\frac{1}{\mathrm{i} \hbar}\left[q z, \rho^{(n)}\right]_{i j} E(t)
\end{aligned}
$$

The QD electric polarization is expanded as follows:

$$
\begin{align*}
P(t)= & \varepsilon_{0} \chi^{(1)} \widetilde{E} \mathrm{e}^{-\mathrm{i} \omega t}+\varepsilon_{0} \chi_{0}^{(2)}|\widetilde{E}|^{2}+\varepsilon_{0} \chi_{2 \omega}^{(2)} \widetilde{E}^{2} \mathrm{e}^{-\mathrm{i} 2 \omega t} \\
& +\varepsilon_{0} \chi_{3 \omega}^{(3)} \widetilde{E}^{3} \mathrm{e}^{-\mathrm{i} 3 \omega t}+\text { c.c. }, \tag{11}
\end{align*}
$$

where $\chi^{(1)}, \chi_{0}^{(2)}, \chi_{2 \omega}^{(2)}$, and $\chi_{3 \omega}^{(3)}$ represent the linear susceptibility, optical rectification, coefficient of second-harmonic generation, as well as susceptibility of third-harmonic generation, respectively. In addition, $\varepsilon_{0}$ indicates vacuum dielectric constant. Moreover, the $n$-th order electronic polarization can be calculated by

$$
\begin{equation*}
P^{(n)}(t)=\frac{1}{V} \operatorname{Tr}\left(\rho^{(n)} e z\right), \tag{12}
\end{equation*}
$$

where $V$ stands for interaction volume, whereas Tr is summation or trace over matrix diagonal elements $\rho^{(n)} e z$.

Within the iterative and compact-density matrix approaches, the analytical expressions for the linear and thirdorder nonlinear susceptibilities are calculated by ${ }^{[28,29]}$

$$
\begin{equation*}
\alpha^{(1)}(\omega)=\omega \sqrt{\frac{\mu}{\varepsilon_{\mathrm{R}}}} \frac{\left|M_{21}\right|^{2} \sigma_{v} \hbar \Gamma_{12}}{\left(E_{21}-\hbar \omega\right)^{2}+\left(\hbar \Gamma_{12}\right)^{2}} \tag{13}
\end{equation*}
$$

## 3. Results and discussion

This section introduces the third-order nonlinear, linear, as well as total OAC within a GaAs/AlGaAs QD in the presence of an applied external electric field. The following parameters are used for calculation, $m^{*}=0.067 m_{0}$, (where $m_{0}$ is the electron mass), $n_{\mathrm{r}}=3.2, T_{12}=0.2 \mathrm{ps}, \Gamma_{12}=1 / T_{12}$, $\sigma_{v}=5 \times 10^{24} \mathrm{~m}^{-3}, \mu=4 \pi \times 10^{-7} \mathrm{Hm}^{-1} .{ }^{[30,31]}$

Figure 1 shows the third-order nonlinear, linear, together with total OAC according to the incident photon energy.

Clearly, the third-order nonlinear, linear, as well as total OAC get a resonant peak at the same location. The linear part of the OAC occupies a prominent position. However, it is affected by the third-order nonlinear terms. Thus, the total OAC is reduced. It can also be found based on Figs. 2-6 that the total OAC can be adequately adjusted by the external electric field, parameters of Gaussian potential, the QD radius, and incident optical intensity.


Fig. 1. The third-order nonlinear, linear, as well as total OAC based on incident photon energy at the parameters of $R=10.0 \mathrm{~nm}, L=10.0 \mathrm{~nm}$, $V_{0}=100 \mathrm{meV}, F=100 \mathrm{kV} / \mathrm{cm}, I=5.0 \mathrm{MW} / \mathrm{cm}^{2}$.

Figure 2 displays the total OAC based on the incident photon energy with four distinct $F$ values. As is observed, the OAC resonant peak elevates with the increase in $F$, suggesting the huge impact of electric field magnitude on the OAC resonant peak. For this feature, its physical origin is interpreted: when $z \ll L$, the Gaussian potential is the same as parabolic potential. For parabolic potential, the energy interval of the initial excited state compared with the system ground state does not change as the electric field changes.


Fig. 2. The total OAC based on the incident photon energy at $R=$ $10.0 \mathrm{~nm}, L=10.0 \mathrm{~nm}, V_{0}=100 \mathrm{meV}, I=5.0 \mathrm{MW} / \mathrm{cm}^{2}$ and four different values of $F$.

The total OAC based on the incident photon energy with four distinct $L$ values is displayed in Fig. 3. Obviously, a broader range of the confinement potential leads to a larger resonant peak value. Additionally, as $L$ elevates, the OAC resonant peak location remains unchanged.

For investigating quantum dot radius effect on the total OAC, we plot the total OAC based on the incident photon energy for three distinct $R$ values. It can be known from Fig. 4
that the total OAC magnitude increases as the radius of QD decreases. The position of the resonant peak is another property, which obviously red shifts as the QD radius elevates, which is due to the weaker quantum confinement when QD radius increases. Therefore, difference in energy $E_{21}$ between the system ground state and the initial excited state declines.


Fig. 3. The total OAC based on the incident photon energy at $R=$ $10.0 \mathrm{~nm}, V_{0}=100 \mathrm{meV}, F=100 \mathrm{kV} / \mathrm{cm}, I=5.0 \mathrm{MW} / \mathrm{cm}^{2}$ and four different values of $L$.


Fig. 4. The total OAC based on the incident photon energy at $L=$ $10.0 \mathrm{~nm}, V_{0}=100 \mathrm{meV}, F=100 \mathrm{kV} / \mathrm{cm}, I=5.0 \mathrm{MW} / \mathrm{cm}^{2}$ and three different values of $R$.


Fig. 5. The total OAC based on the incident photon energy at $R=$ $10.0 \mathrm{~nm}, L=10.0 \mathrm{~nm}, F=100 \mathrm{kV} / \mathrm{cm}, I=5.0 \mathrm{MW} / \mathrm{cm}^{2}$ and three distinct values of $V_{0}$.

Figure 5 presents the total OAC according to the incident photon energy for three distinct $V_{0}$ values. It clearly seems that with the depth of Gaussian potential $V_{0}$ decreasing, the peak value of total OAC increases. As is observed, the total OAC peak position remains unchanged as the $V_{0}$ magnitude changes.

Total OAC based on the incident photon energy with three distinct $I$ values is illustrated in Fig. 6. From the figure, it can be clearly noticed that, a higher incident optical intensity $I$ results in a smaller total OAC. In addition, those resonant peaks of the total OAC are divided to two peaks at an incident optical intensity $I$ of $15 \mathrm{MW} / \mathrm{cm}^{2}$. The reason is that the third-order OAC exerts an important part in the total OAC.


Fig. 6. The total OAC based on the incident photon energy at $R=$ $10.0 \mathrm{~nm}, L=10.0 \mathrm{~nm}, V_{0}=100 \mathrm{meV}, F=100 \mathrm{kV} / \mathrm{cm}$ and three distinct values of $I$.

## 4. Conclusions

In this paper, the OAC in the QD is effectively investigated. Our calculations mainly concentrate on the OAC dependency on the electric field $F$, the QD radius $R$, Gaussian potential $V_{0}$ and $L$ parameters, and incident optical intensity $I$. According to our findings, the OAC theoretical value markedly elevates because of the Gaussian potential range and the electric field applied, but the peak value will decrease as the $R$, $V_{0}$ and $I$ increase. Moreover, with the QD radius increasing, the resonant peaks of the OAC shift toward lower energy area. Our results are more remarkable than those in the previous work. ${ }^{[19]}$ Hopefully, this research can provide certain foundation for experimental and theoretical work on optical devices.

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