Game theory model of exit selection in pedestrian evacuation considering visual range and choice firmness*

Wei-Li Wang(王维莉)[1,†], Fang-Fang Wan(万芳芳)[1], and Siu-Ming Lo(卢兆明)[2]

[1] Institute of Logistics Science and Engineering, Shanghai Maritime University, Shanghai 201306, China
[2] Department of Architecture and Civil Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China

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Exit choice is one of the most important pedestrian behaviors during evacuation. Distance to the exit is a generally recognized factor influencing expected moving time to the exit. Visual range determines how much information a pedestrian can perceive, thus the number of pedestrians within the visual field can be used to estimate waiting time at the exit. Besides, the choice firmness that reflects the degree to which a pedestrian would persist in his/her previous choice of exit is proposed. By integrating game theory into a cellular automata simulation framework, the pedestrian exit choice mechanism is investigated and explicitly modeled in this paper. A systematic analysis of the key factors influencing pedestrian evacuation is conducted, including visual radius and choice firmness of a pedestrian, initial crowd distribution of the room, exit layout as well as exit width. It is found that low choice firmness level can lead to unnatural pedestrian behavior such as wandering, which is adverse to evacuation. The longer the pedestrian's visual radius, the earlier the pedestrian can determine his/her final selection of the exit. Compared with the scenario where the pedestrians are randomly distributed, pedestrians clustered together in a corner of the room lead to high crowd density and imbalanced use of exits. Furthermore, the exit layout and exit width also have a certain influence on pedestrian evacuation process. The results of this paper may be of benefit to the formulation of behavioral rules in other pedestrian simulation models.

Keywords: game theory, exit selection, visual range, choice firmness

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1. Introduction

With social and economic development as well as the acceleration of urbanization, urban population expanded rapidly in recent years. The gathering of many people becomes a common phenomenon. However, due to the lack of effective management and control, crowd-related disasters such as stampede are frequently reported, which has brought serious threats to public safety. Therefore, pedestrian evacuation dynamics has attracted increasing attention of researchers in transportation, urban planning and other relevant fields. Establishing a simulation model and carrying out scenario-based simulations is one of the most effective ways to study pedestrian evacuation dynamics and evaluate performance of building layouts. The pedestrian evacuation models can usually be divided into two categories: macroscopic models and microscopic models. The macroscopic models treat the population as a whole, ignoring the interaction between individuals. They can describe pedestrian agglomeration characteristics, such as flow and speed, but may fail to demonstrate more detailed information about pedestrian flows. The pedestrian simulation model is usually microscopic, which takes each pedestrian as an individual, and focuses on the interactions between pedestrians and between the pedestrian and environment. Considering different divisions on space and time, microscopic models can further be divided into continuous and discrete ones, of which the social force model and cellular automata (CA) model are the most important representatives, respectively. The social force model was proposed by Helbing *et al.*,[1,2] which has been improved and applied in varied scenarios.[3,4] The CA model is greatly employed because of its simple rules and high computational efficiency.[5] Moreover, the original CA model can be combined, extended or improved according to the characteristics of each pedestrian and the surrounding environment.[6–11]

To better represent pedestrian evacuation dynamics, pedestrian behavior should be carefully considered and reasonably modeled. In terms of pedestrian behavior modeling, conflict over the moving space[12–18] and decision-making in exit selection[19–21] are the most widely studied topics. In fact, the exit selection is a complex decision-making process. Individuals would usually consider a number of factors in exit selection, and the distance to the exit is the most influencing one. Other import factors include crowd density around the exit, discomfort level, *etc*. In the modeling of pedestrian selection behavior, commonly used methods include random utility theory[22] and multi-logit model.[23] Game theory is a recognized tool to explain human behavior, especially when individuals face competing situation with each other. Ehtamo

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†Corresponding author. E-mail: wlwang@shmtu.edu.cn

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et al.[20] established a model of exit selection based on the game theoretic concept of best-response dynamics, where each player updates his/ her strategy periodically by optimally responding to other players' strategies. Lo *et al.*[21] proposed a game theory based exit choice model, in which the distance to the exit, crowd density and exit width are taken into account for calculating the payoff matrix, and the Nash equilibrium of the mixed strategy reflects the balance between evacuees and exit congestion. By introducing the floor field, the exit and the optimal path can be obtained. Xu and Huang[24] proposed a modified floor field model to simulate the multi-exit evacuation selection process, in which two cognitive coefficients of exit width and congestion around exit are explicitly considered. Yue *et al.*[25] combined distance-based strategy and time-based strategy with cognitive coefficients to establish a hybrid strategy of exit selection in pedestrian evacuation. Zhang *et al.*[11] defined a cost potential field, considering the effects of travel time and discomfort. As an optimal path-choice strategy was obtained, the model demonstrated a faster evacuation of pedestrians from a room and a shorter computation time than the classical floor field CA model. As visibility plays a vital role in pedestrian's decision-making, and only information within a certain range could be observed and used for selecting exits, an individual-based visual field was considered in some pedestrian evacuation models.[26–29] For example, Xu *et al.*[29] proposed a modified floor field model, by which the pedestrian evacuation dynamics in a room with multiple exits by considering the directional visual field is simulated. However, most of the existing researches used a unique value for representing the visual range, which usually varies from each other and lacks empirical evidence. Thus a systematic analysis of the influence of visual range on pedestrian evacuation behavior can be interesting and of importance. Furthermore, unnatural movements can occur if the evacuation model does not take a certain pedestrian behavior into consideration, such as choice firmness. It may not be an influencing factor for exit selection but could lead to a hesitation phenomenon that pedestrians wander back and forth between different exits, resulting in the inaccurate simulation of pedestrian evacuation dynamics.[30]

In this paper, the game theory is introduced into a cellular automata model to simulate pedestrian exit choice during evacuation. Meanwhile, the influence of visual range and choice firmness which may lead the pedestrians to hesitate between exits on pedestrian evacuation dynamics are explicitly considered and analyzed. The rest of the paper is organized as follows. In Section 2 we present the pedestrian evacuation model, in which the exit selection strategy is introduced. In Section 3, we describe the simulation environment and discuss the results in different simulation cases, where the effects of pedestrian visual radius, choice firmness level, crowd distribution, exit layout and exit width are especially considered. In the final section, we draw some conclusions from the present study.

2. Pedestrian evacuation model

It is assumed that every pedestrian knows the locations of all the exits. The space is divided into a number of equally sized cells in the proposed model. The cell may be occupied by one pedestrian at most, by the space boundary or empty. And the Moore neighborhood is used as shown in Fig. 1. In the simulation, a pedestrian occupies a cell, and he/she can make the exit choice and then move only one cell for each time step. When multiple pedestrians choose to move to the same cell, only a randomly selected pedestrian is allowed to enter into the cell.

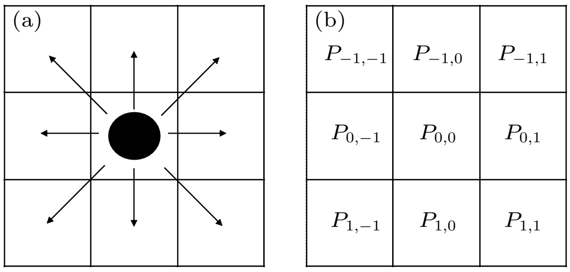

Fig. 1. Moore neighborhood: (a) possible moving directions for a pedestrian in the occupied cell, and (b) corresponding probabilities for the pedestrian in the occupied cell to update his/her position.

2.1. Visual field

In this paper, the pedestrian's visual field is a fan-shaped area with the current position of the pedestrian as the vertex, 2θ as the angle, R as the radius, and the line between the current position of the pedestrian and the center of the exit as the bisector, as shown in Fig. 2. Like the scenario in Ref. [28], as one cell can be occupied by, at most, one pedestrian, a cell or a pedestrian belongs to the visual field if and only if its center lies in it. In this paper, the visual angle is 120°. By referring to Ref. [31], the value of visual radius is set to be in a range between 3 m and 12 m.

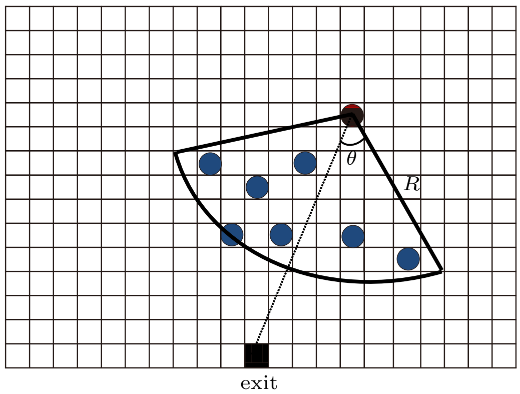

Fig. 2. Visual field of the pedestrian.

2.2. Exit selection model

In the process of evacuation, the exit selection strategy of a pedestrian will be affected by the strategies of other pedestrians. The relationship between pedestrians is competitive, and the goal is to evacuate the room as quick as possible. Assuming that all the pedestrians are rational, the game theory can be used to describe pedestrians' exit choice behavior. First, the multi-exit room evacuation problem is transformed into a standard game expression: (i) game players; (ii) strategy space; (iii) payoff function; and (iv) the update strategy. Suppose that there are N people in the room for being evacuated, and the number of exits is K. Thus the number of game players is N, the strategy space set is $S=\{S_1,\ldots,S_N\}$, the optional strategy set of player i is $S_i=\{e_1,\ldots,e_K\}$, where $S_i \in S$. The payoff function is used to estimate the evacuation time of a pedestrian under different strategies. At each time step, the estimated evacuation time of pedestrian i choosing exit e_k is $T_i(e_k)$. It is composed of $T_i^{\text{mov}}(e_k)$ and $T_i^{\text{que}}(e_k)$, which represent the estimated moving time and estimated queuing time towards exit e_k, respectively.

The estimated moving time of pedestrian i choosing exit e_k is calculated by Eq. (1), in which $d_i(e_k)$ is the distance from the position of individual i to the selected exit e_k, and v_i is his/her expected moving velocity.

$$T_i^{\text{mov}}(e_k)=\frac{d_i(e_k)}{v_i}. \tag{1}$$

The estimated queuing time of pedestrian i choosing exit e_k is calculated by Eq. (2), in which $\rho_i(e_k)$ represents the number of people in the pedestrian's visual field towards exit e_k, β_{e_k} represents the maximum number of people passing through exit e_k per unit time.

$$T_i^{\text{que}}(e_k)=\frac{\rho_i(e_k)}{\beta_{e_k}}. \tag{2}$$

When pedestrians move towards a certain exit, they usually continue heading for the same direction.[32] To reduce pedestrian's unnatural behavior of wandering between different exits in modeling, frequent change of the selected exit should be avoided. Consequently, choice firmness α $(0 \leqslant \alpha \leqslant 1)$ is introduced into the model, which represents the degree to which a pedestrian would persist in his/her previous choice of the exit. In particular, the pedestrian who selects an exit different from that in the last time step would bear a higher cost. And the greater the value of, the higher the cost of changing the exit is. The payoff function, that is, the estimated evacuation time of individual i is calculated from the following equation:

$$T_i(e_k)=\begin{cases} T_i^{\text{mov}}(e_k)+T_i^{\text{que}}(e_k), & S_i(t)=S_i(t-1), \\ (1+\alpha)\left(T_i^{\text{mov}}(e_k)+T_i^{\text{que}}(e_k)\right), & S_i(t)\neq S_i(t-1). \end{cases} \tag{3}$$

Pedestrians will update their strategies at each time step according to other pedestrians in the visual field and surrounding environment. Then, at time step $t+1$, individual i updates his/her strategy according to the following equation:

$$S_i(t+1)=\arg\{(T_i(e_k))\}. \tag{4}$$

2.3. Pedestrian movement model

In this paper, pedestrian movement is simulated based on the static field model. In particular, the movement of pedestrian i is determined by the static floor field value SF_i, which is measured as the reciprocal of the distance from pedestrian i to the exit e_k, and is shown below

$$SF_i=\frac{1}{\sqrt{(x_{e_k}-x_i)^2+(y_{e_k}-y_i)^2}}, \tag{5}$$

where (x_{e_k}, y_{e_k}) is the coordinate of the exit and (x_i, y_i) is the ordinate of the pedestrian i. At each time step, a certain number of empty grids may exist in the neighborhood of the pedestrian. Hence the probability of pedestrian i moving to grid j (in his/ her neighborhood) is calculated from the following equation.

$$P_{(i\to j)}=\frac{\exp(k_{\text{f}}SF_j)}{\sum_{l\in\Omega_i}\exp(k_{\text{f}}SF_l)}, \tag{6}$$

where k_{f} represents the pedestrian's familiarity with the exit and takes a constant value of 10 in this paper, and Ω_i denotes the set of all the accessible grids in the neighborhood of pedestrian i.

The simulation is performed using the algorithm as follows.

(I) Set the initial parameters including room layout and pedestrian distribution.

(II) At each time step, select an exit for each pedestrian and make all the pedestrians move towards their selected exits by the following rules.

(II)-1 Calculate the number of people in the visual field of each pedestrian, get the payoff function by Eq. (3), then select an exit by using optimal strategy through Eq. (4).

(II)-2 Calculate the floor field according to the selected exit by Eq. (5), determine the movement probability of grids in the neighborhood of each pedestrian by Eq. (6), and choose the grid with maximum probability as the target at next time step.

(II)-3 Record target grids of all the pedestrians. If two or more pedestrians choose the same grid, only one pedestrian is randomly selected to move to it.

(II)-4 Update the positions of all the pedestrians in parallel.

(III) Repeat steps (II)-1 to (II)-4 until all the pedestrians evacuate from the room.

3. Simulation and discussion

A series of simulations is performed in a rectangular room (12 m×12 m) with two exits. The cell size is set to be 0.4 m ×0.4 m, thus the average movement distance of pedestrians at each time step is 0.48 m (parallel movement is 0.4 m or diagonal movement is $0.4\sqrt{2}$ m).[32] Therefore, each time step in this model is 0.29 s, because the average movement speed of pedestrians under stress is 1.65 m/s.[2] Simulation results in this paper are the average values 30 runs, and the evacuation time is the time step when all the pedestrians evacuate from the room.

3.1. Simulation environment

Simulation cases are conducted by considering different pedestrian visual radii, choice firmness levels, crowd distributions and exit layouts. Rooms with two exits usually have three typical layouts, that is, two exits are on the same side, on the adjacent sides, and on the counter sides, as shown in Fig. 3. Two initial distributions of pedestrians are considered, that is, random distribution and cluster distribution as shown in Fig. 4. The number of people in the room is 225. Furthermore, three exit widths of 0.8 m, 1.2 m, and 1.6 m are considered.

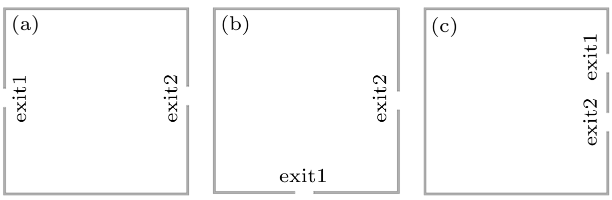

Fig. 3. Room exit settings: (a) on the counter sides, (b) on the adjacent sides, (c) on the same side.

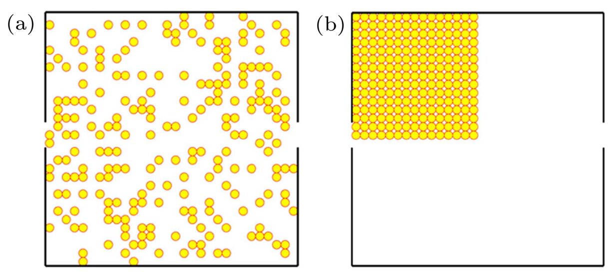

Fig. 4. Initial distributions of pedestrians: (a) random distribution and (b) cluster distribution.

3.2. Effect of choice firmness

In these simulation cases, 225 pedestrians are distributed randomly or in cluster in the room where two exits are on the counter sides and the exit width is 1.2 m. The range of choice firmness level α is set to be between 0 and 0.2. Specifically, the pedestrian hesitates to choose the exit when the value is small. On the contrary, the larger the value, the more the pedestrian tends to stick to his/her initial selection. When pedestrians are randomly distributed in the room, we can observe from Fig. 5(a) that the effect of choice firmness on pedestrian evacuation time is not obvious when visual radius is small. It is possible that the distance to the exit becomes the most important factor influencing the selection of the exit. When visual radius of the pedestrian is between 4.5 m and 7.5 m, evacuation times in these cases are longer than those in other cases, as queuing time has a significant influence on the exit choice. However, limited visual field of the pedestrian leads to insufficient environmental information, and queuing time changes dynamically with the movement of pedestrians, resulting in pedestrians hesitating between the two exits. In particular, when choice firmness is not considered ($\alpha = 0$), taking $R = 5.5$ m for example, the pedestrian will linger between the two exits for a long time as shown in Fig. 6(a). Contrarily, when choice firmness level is relatively high, such as $\alpha = 0.2$, the wandering time of pedestrians is greatly shortened. In other words, pedestrians tend to keep the exit choice and not change it frequently as shown in Fig. 6(b). Moreover, as visual radius continues to increase, pedestrians have a better grasp of the environmental information, choice firmness has no obvious influence on the pedestrians' evacuation performance (see Fig. 5(a)).

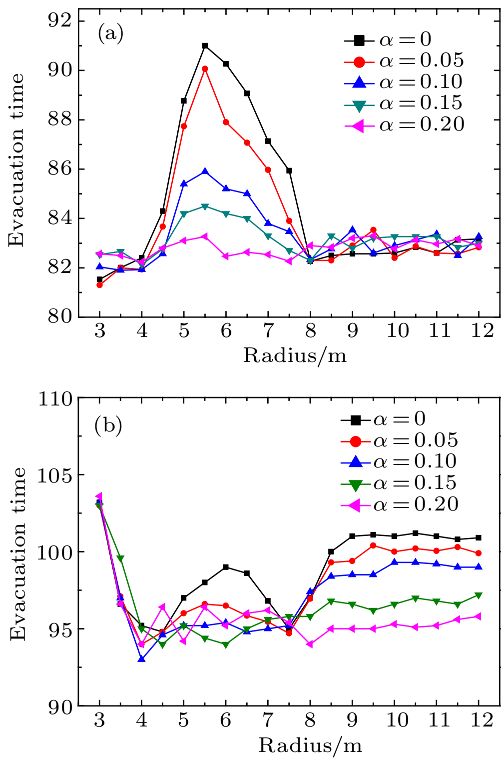

Fig. 5. Plots of evacuation time of pedestrians against visual radius under different choice firmness levels, showing (a) random distribution and (b) cluster distribution.

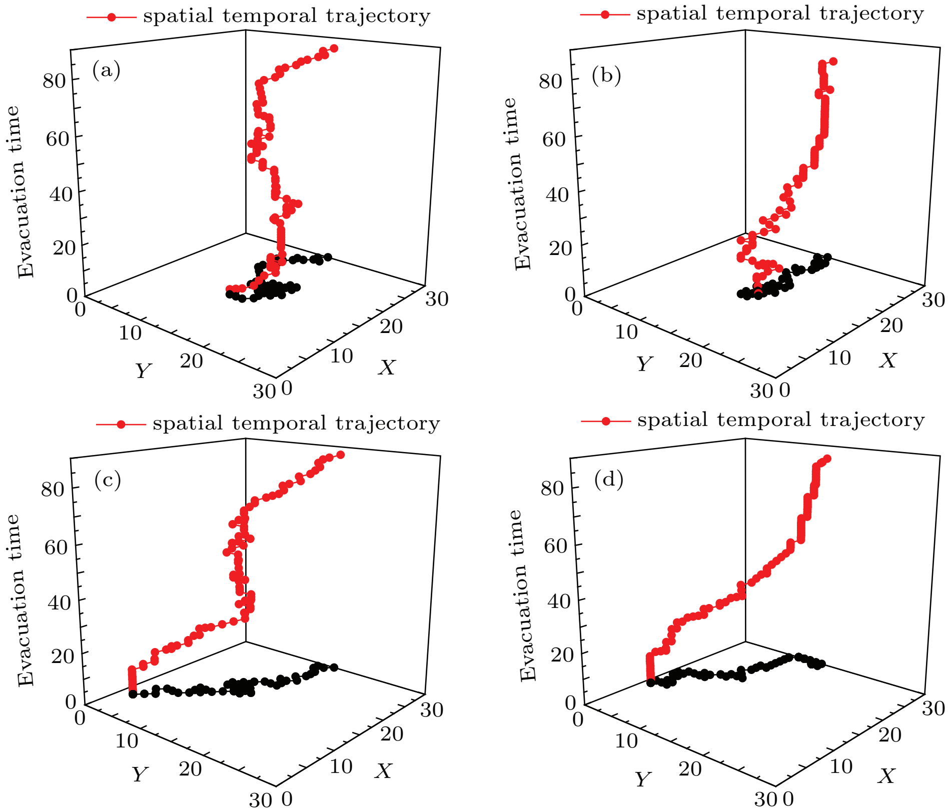

Fig. 6. Movement trajectories of pedestrian with $R = 5$ m under different choice firmness levels, showing (a) random distribution, $\alpha = 0$; (b) random distribution, $\alpha = 0.2$; (c) cluster distribution, $\alpha = 0$; and (d) cluster distribution, $\alpha = 0.2$.

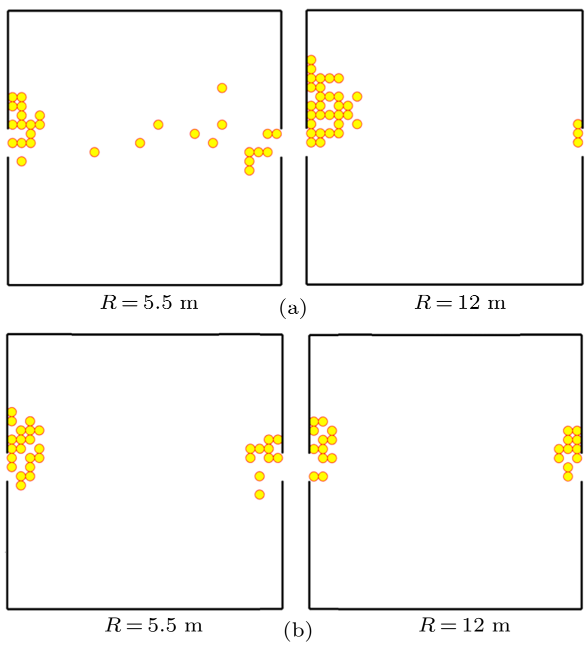

Fig. 7. Simulation snapshots of evacuation process under choice firmness level $\alpha = 0$ (a) and $\alpha = 0.2$ (b) at $T = 80$ when pedestrians initially distribute in cluster.

When pedestrians are initially distributed in cluster, the evacuation performance is different from that when randomly distributed. It can be seen from Fig. 5(b) that as pedestrian's visual radius increases from 3 m to 4 m, evacuation time decreases rapidly. When the visual radius is between 4 m and 8 m, pedestrians will also hesitate about the exit choice, which seems to be alleviated when choice firmness level is increased. Taking the case in which $\alpha = 0$ and $R = 5.5$ m for example, pedestrians wander in the middle of the room for a certain time as shown in Fig. 6(c). When $\alpha = 0.2$, pedestrians are more likely to stick to their earlier choices of exit as shown in Fig. 6(d). Furthermore, as pedestrians with larger visual radius have more information about the environment, the number of people choosing the two exits is more balanced in these cases. As shown in Fig. 7(b), when pedestrians are less likely to change the exit (such as $\alpha = 0.2$), the number of pedestrians in front of the two exits is more balanced in the case with $R = 12$ m than in the case with $R = 5.5$ m.

3.3. Effect of initial crowd distribution

In simulation,we consider the case where there are 225 pedestrians with choice firmness level $\alpha = 0.1$, located in the room where two exits are on the counter sides and the exit width is 1.2 m. Two different crowd distributions are considered, that is, pedestrians distributed randomly or in cluster. Figure 8(a) shows the evacuation time changing with visual radius in two initial crowd distributions. Obviously, the situation that pedestrians are clustered together in a corner of the room leads to relatively high local density and imbalanced use of exits, which is not benefit for the evacuation. Specifically, due to the short sight, more pedestrians will choose the nearer exit, the utilization of two exits is more imbalanced in the cluster simulation case where $R = 3$ m, leading to the longest evacuation time (the simulation case of $R = 0$ m is not considered here). With the increase of visual radius, more information can be obtained by the visual field, thus the evacuation time is gradually reduced. It seems that when $R = 4$ m the total evacuation time is the shortest in all the simulation cases where pedestrians are initially clustered together in a corner of the room. When R is between 4.5 m and 7.5 m, the evacuation time of the corresponding case is relatively short. However, with the further increase of the visual radius, pedestrians may perceive more environmental information, and a certain behavior of wandering between exits can happen when medium choice firmness level is considered ($\alpha = 0.1$). Accordingly, longer evacuation can be perceived in simulation cases where R is between 8 m and 12 m. In terms of the random distribution, pedestrians in the simulation cases where R is between 5 m and 7 m need longer evacuation time than in the other cases, as pedestrians may hesitate between the two exits.

Figures 8(b) and 8(c) are the percentages of pedestrians using each exit with different visual radii under random distribution and cluster distribution, respectively. For the random distribution, the numbers of people evacuating from both exits are almost the same, especially when visual radius of the pedestrian is larger than 9 m as shown in Fig. 8(b).

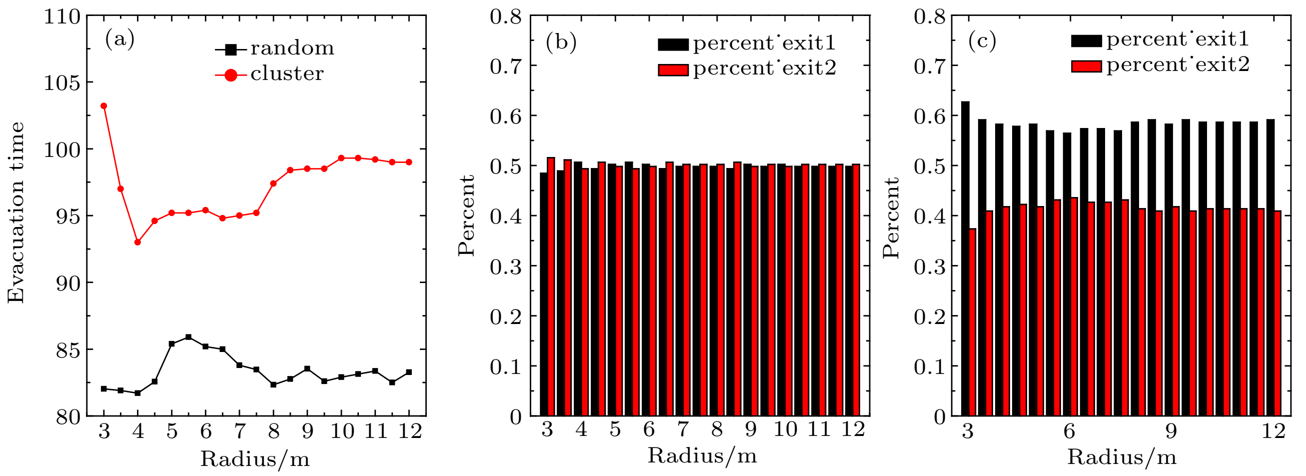

Fig. 8. Evacuation performances of pedestrians under different initial distributions, showing (a) evacuation times of pedestrians against the visual radius under different initial crowd distributions, (b) percentages of pedestrians using each exit in random distribution, and (c) percentages of pedestrians using each exit in cluster distribution.

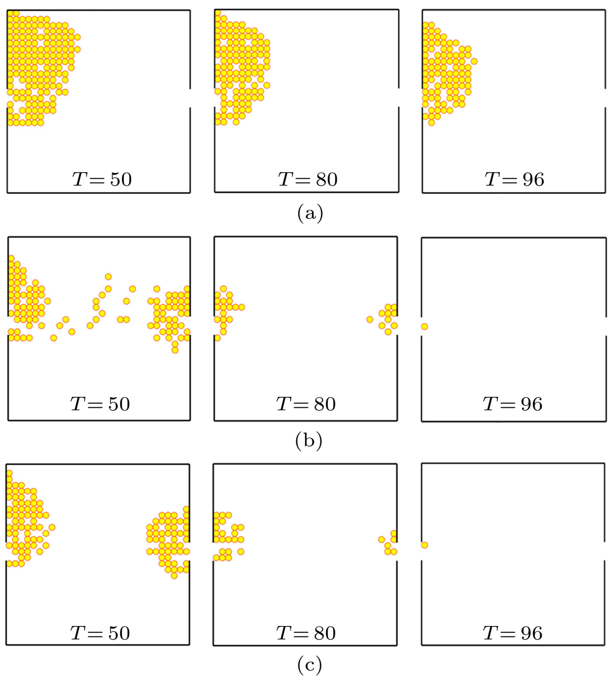

Fig. 9. Simulation snapshots of pedestrian evacuation process under visual radius $R = 0$ m (a), $R = 4$ m (b), and $R = 12$ m (c).

Figure 9(a) shows the case where $R = 0$ m and pedestrians are clustered together in a corner of the room, then distance to the exit becomes the only factor in making exit choice. Hence the exit far from the crowd is less used during evacuation. At $T = 96$, evacuation processes in two cases where $R = 4$ m and $R = 12$ m are almost completed as shown in Figs. 9(b) and 9(c). However, in the case where $R = 0$ m, a great number of pedestrians are still in the room as shown in Fig. 9(a). It can be seen from Figs. 9(b) and 9(c) that both exits are utilized in the evacuation process. Actually, the longer the visual radius, the earlier the pedestrian can determine his/her final exit. Accordingly, at $T = 50$ in the case where $R = 4$ m, we can observe some pedestrians still wander in the middle of the room (see Fig. 9(b)), while in the case where $R = 12$ m, all the pedestrians make choices and move to their selected exits (see Fig. 9(c)).

3.4. Effect of exit layout

Three exit layouts are considered, that is, two exits are on the same side, on the adjacent sides and on the counter sides of the room. In these simulation cases, exit width is 1.2 m and 225 pedestrians are randomly distributed in the room. As can be seen from Fig. 10(a), when exits are located on the adjacent sides of the room and pedestrian choice firmness level is low, the evacuation time fluctuates greatly when R is between 3 m and 5 m. With the increase of choice firmness level, the fluctuation is gradually attenuated. When exits are located on the same side of the room, the change of evacuation time is obvious neither with the variation of visual radius nor with the increase of choice firmness level. Moreover, when pedestrians stick to their initial exit choices, such as $\alpha = 0.2$, the simulation time of the case where exits are on the counter sides of the room is minimum, followed by the case where exits are on the same side, and pedestrians in the case where exits were on the adjacent sides experience the longest evacuation as shown in Fig. 10(d).

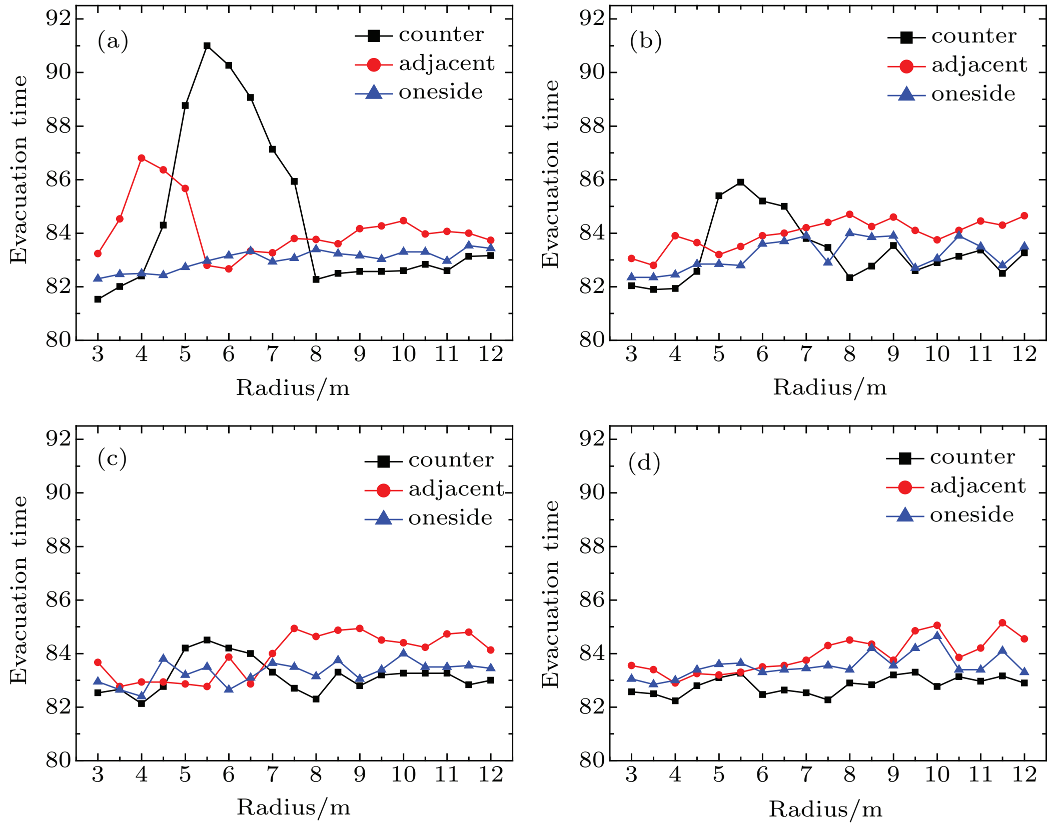

Fig. 10. Evacuation times of pedestrians against visual radius under different exit layouts: (a) $\alpha = 0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.15$, and (d) $\alpha = 0.2$.

3.5. Effect of exit width

In the following simulation cases (Fig. 11), 225 pedestrians are randomly distributed or clustered together in a corner of the room, with choice firmness level $\alpha = 0.1$. Two exits are on the counter sides, and the exit width is set to be 0.8 m, 1.2 m, and 1.6 m, separately. Whether pedestrians are randomly distributed or clustered together, the evacuation time is not notably changed with the variation of visual radius. When exit width increases from 0.8 m to 1.2 m, the evacuation time decreases significantly. However, the evacuation time can be observed to change less when the exit width increases from 1.2 m to 1.6 m. It indicates that the effect of the exit width on evacuation time is not linear, which is consistent with previous studies.

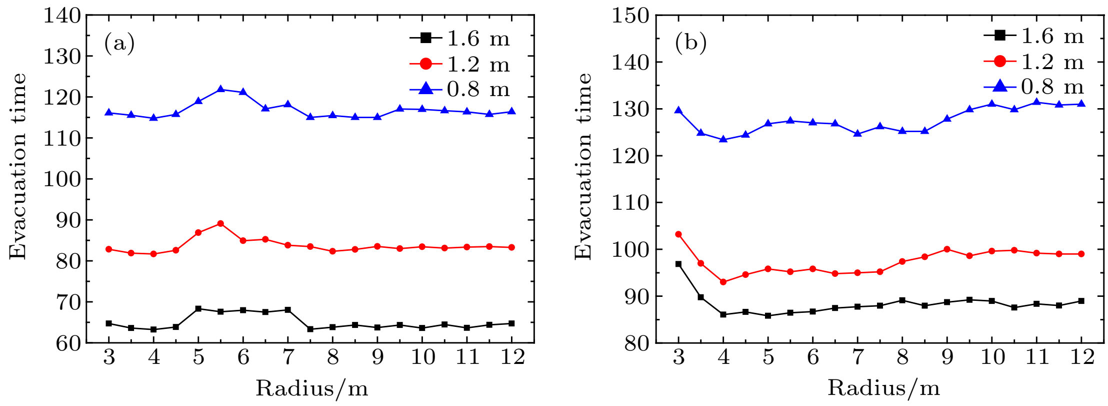

Fig. 11. Evacuation times of pedestrians against visual radius for different exit widths in the cases: (a) random distribution and (b) cluster distribution.

4. Conclusions

In this paper, we developed a game theory-based pedestrian exit choice model for pedestrian evacuation by using the cellular automata simulation framework. On the assumption that all the pedestrians are rational, the estimated evacuation time of a pedestrian through a specified exit is composed of estimated moving time and estimated queuing time towards it. Distance to the exit is used to obtain the expected moving time, and the number of pedestrians within the visual field of a pedestrian and exit flow volume are utilized in the calculation of expected queuing time. To reflect the hesitation phenomenon in evacuation, the parameter of choice firmness is introduced into the game theory model to better represent pedestrian's exit choice strategy. By designing a scenario of a rectangular room with two exits, a number of simulation cases are conducted. And influences of certain factors on pedestrian evacuation performance are investigated and analyzed, including visual radius and choice firmness level of the pedestrian, initial crowd distributions in the room, exit layout as well as exit width. The simulation results are indicated below. i) Choice firmness level reflects the pedestrian's persistence in choosing his/ her initial exit. Obviously, frequent change of target exit leads to unnatural pedestrian behavior such as wandering between different exits, which is adverse to evacuation. However, it could be avoided when choice firmness level is set to be a relatively large value in the model, such as $\alpha = 0.2$. ii) Compared with random distribution, pedestrians clustered together in a corner of the room leads to high crowd density and imbalanced use of exits, which is not beneficial to evacuation. iii) For visual radius, the longer the radius, the earlier the pedestrian can determine his/her final exit. Specifically, when $R = 0$ m, the distance to the exit becomes the only factor in making exit choice, therefore the exit far from the crowd is less used during evacuation. iv) In terms of exit layout, when pedestrian wandering behavior is not considered, the simulation time of the case where exits are on the counter sides of the room is minimum, followed by the case where exits are on the same side, and pedestrians in the case where exits are on the adjacent sides experience the longest evacuation. v) Generally, the exit with a larger width leads to a quicker evacuation, and the effect of exit width on evacuation time is not linear. To sum up, the proposed model is sound in theory and credible in simulation performances. The sensitivity analyses of choice firmness level and visual radius of this paper may benefit the formulation of behavioral rules in other pedestrian simulation models. Controlled experiments on pedestrian exit choice in a room with two exits are conducted, and detailed moving trajectories are obtained and analyzed, which is conducive to further investigating the choice firmness level in future work.

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