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A new car-following model with driver's anticipation effect of traffic interruption probability*

Guang-Han Peng(彭光含)†

College of Physical Science and Technology, Guangxi Normal University, Guilin 541004*, China*

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Traffic interruption phenomena frequently occur with the number of vehicles increasing. To investigate the effect of the traffic interruption probability on traffic flow, a new optimal velocity model is constructed by considering the driver anticipation term in the interruption case for car-following theory. Furthermore, the effect of driver anticipation in the interruption case is investigated via linear stability analysis. Also, the MKdV equation is obtained concerning the effect of driver anticipation in the interruption case. Moreover, numerical simulation states that the driver anticipation term in the interruption case contributes to the stability of traffic flow.

Keywords: traffic flow, interruption probability, optimal velocity model, numerical simulation

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1. Introduction

With the rapid growth of vehicle number, the traffic environment is becoming more and more deteriorated and traffic congestion tends to be more and more serious. Frequent traffic accidents lead to traffic interruption. To reveal the internal influence mechanism of traffic interruption factors, a few traffic models have been established by considering different accidents.[1–5] However, none of the models mentioned described car-following behavior explicitly. Recently, a macro model[6] was presented by integrating interrupt probability parameter. And a two-lane macro model[7] was established with consideration of the traffic interruption probability based on Ref. [6]. Especially, Tang *et al.*[8] proposed a car-following model involving the traffic interruption probability. In addition, a few scholars[9–49] have also studied other characteristics of traffic flow from different perspectives. In a real traffic case, traffic interruption is unpredictable. However, drivers often anticipate the traffic interruption with a certain probability, which has not been taken into account in previous models. Therefore, a new car-following model in this paper is proposed by taking into consideration the driver's anticipation of the traffic interruption probability.

2. OV model

Considering the driver's judgment of traffic interruption, a new driver anticipation of the traffic interruption probability optimal velocity model is proposed below:

$$\begin{aligned}\dot{x}_n(t+\tau) &= p_{n+1}V[\Delta x_n + T_1(-\dot{x}_n)]\\ &\quad + (1-p_{n+1})V(\Delta x_n + T_2\Delta\dot{x}_n),\end{aligned} \tag{1}$$

where p_{n+1} denotes the traffic interruption probability of leading vehicle $n+1$, T_1 the anticipation time to traffic interruption occurring, T_2 the reaction time to normal driving, $\Delta x_n = x_{n+1} - x_n$ the headway, $\Delta\dot{x}_n = \Delta v_n = v_{n+1} - v_n$ the velocity difference, $V(\cdot)$ the anticipation velocity function, and $V[\Delta x_n + T_1(-\dot{x}_n)]$ the anticipation term of traffic interruption. Here we choose the following form: $V[\Delta x_n + T_1(-\dot{x}_n)] = V(\Delta x_n) + T_1 V'(\Delta x_n)(-\dot{x}_n)$ and $V(\Delta x_n + T_2\Delta\dot{x}_n) = V(\Delta x_n) + T_2 V'(\Delta x_n)\Delta\dot{x}_n$. Then equation (1) can be rewritten as

$$\begin{aligned}\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} &= a[V(\Delta x_n) - v_n(t)] + \lambda_1 V'(\Delta x_n)p_{n+1}(-v_n)\\ &\quad + \lambda_2 V'(\Delta x_n)(1-p_{n+1})\Delta v_n.\end{aligned} \tag{2}$$

Here, $\lambda_1 = T_1/\tau$, $\lambda_2 = T_2/\tau$, and $V'(\Delta x_n) = \mathrm{d}V(\Delta x_n)/\mathrm{d}\Delta x_n$. The $V(\cdot)$ is chosen as follows:[10]

$$V(\Delta x) = v_{\max}/2[\tanh(\Delta x - h_c) - \tanh h_c], \tag{3}$$

where the safe distance $h_c = 4$ m and the maximum velocity $v_{\max} = 2$. For simplicity, the traffic interruption probability p_{n+1} is assumed to be a constant, *i.e.* $p_{n+1} = p_0$. After discretizing Eq. (2), we deduce the difference form as follows:

$$\begin{aligned}&\Delta x_n(t+2\tau)\\ &= \Delta x_n(t+\tau) + \tau[V(\Delta x_{n+1}(t)) - V(\Delta x_n(t))]\\ &\quad + \lambda_1 V'(\Delta x_n)p_0[-\Delta x_n(t+\tau) + \Delta x_n(t)]\\ &\quad + \lambda_2 V'(\Delta x_n)(1-p_0)\\ &\quad \times[\Delta x_{n+1}(t+\tau) - \Delta x_{n+1}(t) - \Delta x_n(t+\tau) + \Delta x_n(t)].\end{aligned} \tag{4}$$

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†Corresponding author. E-mail: pengguanghan@163.com

3. Anticipation of traffic interruption probability for linear stability condition

In this section, the linear stability analysis will be carried out with $V(b)$ at the uniform headway b to obtain the steady state solution as follows:

$$x_n^0(t) = bn + V(b)t \quad \text{with} \quad b = L/N. \tag{5}$$

Let $y_n(t)$ be a small perturbation and suppose, $x_n(t) = x_n^0(t) + y_n(t)$, then, equation (6) will be deduced from Eq. (4) as follows:

$$\begin{aligned}
&\Delta y_n(t+2\tau) \\
&= \Delta y_n(t+\tau) + \tau V'(b)[\Delta y_{n+1}(t) - \Delta y_n(t)] \\
&\quad + \lambda_1 V'(b) p_0[-\Delta y_n(t+\tau) + \Delta y_n(t)] \\
&\quad + \lambda_2 V'(b)(1-p_0) \\
&\quad \times [\Delta y_{n+1}(t+\tau) - \Delta y_{n+1}(t) - \Delta y_n(t+\tau) + \Delta y_n(t)],
\end{aligned} \tag{6}$$

where $\Delta y_n = y_{n+1}(t) - y_n(t)$. Assum $y_n(t) = \mathrm{e}^{(\mathrm{i}kn+zt)}$, then we will obtain

$$\begin{aligned}
&(\mathrm{e}^{zt} - 1)[\mathrm{e}^{zt} - \lambda_2(1-p_0)V'(b)(\mathrm{e}^{\mathrm{i}k} - 1) \\
&+ \lambda_1 p_0 V'(b)] - \tau V'(b)(\mathrm{e}^{\mathrm{i}k} - 1) = 0.
\end{aligned} \tag{7}$$

Let $z = z_1(\mathrm{i}k) + z_2(\mathrm{i}k)^2 + \cdots$, then we will obtain the first-order and second-order term of $\mathrm{i}k$ as follows:

$$\begin{aligned}
z_1 &= \frac{V'(b)}{1 + \lambda_1 V'(b) p_0}, \\
z_2 &= \frac{V'(b) + 2\lambda_2 V'(b)(1-p_0)z_1 - [3 + \lambda_1 V'(b) p_0]\tau z_1^2}{2[1 + \lambda_1 V'(b) p_0]}.
\end{aligned} \tag{8}$$

Thus, the uniform flow keeps stable when $z_2 > 0$. On the contrary, the uniform steady-state flow becomes unstable when $z_2 < 0$. Therefore, the neutral stability condition is gained as follows:

$$\begin{aligned}
a &= \frac{1}{\tau} \\
&= \frac{[3 + \lambda_1 V'(b) p_0]V'(b)}{[1 + \lambda_1 V'(b) p_0]^2 + 2\lambda_2 V'(b)(1-p_0)[1 + \lambda_1 V'(b) p_0]}.
\end{aligned} \tag{9}$$

Spontaneously, the stable condition is obtained below:

$$a > \frac{[3 + \lambda_1 V'(b) p_0]V'(b)}{[1 + \lambda_1 V'(b) p_0]^2 + 2\lambda_2 V'(b)(1-p_0)[1 + \lambda_1 V'(b) p_0]}. \tag{10}$$

Obviously, the stable condition is closely related to the driver's anticipation of the traffic interruption probability. Solid curves in Fig. 1 represent the neutral stability lines in the parameter space $(b; a)$ under $\lambda_1 = 0$ and 0.5, respectively, where $\lambda_2 = 0.2$. In Fig. 1(a), $\lambda_1 = 0$ implies that the anticipation term of traffic interruption is not under consideration. From Fig. 1(a), the stable region shrinks with the increase of traffic interruption probability p_0. However, the stable region becomes larger with the traffic interruption probability p_0 increasing according to Fig. 1(b). In view of Fig. 1, the anticipation term of traffic interruption is beneficial to the stability of traffic flow, which means that our consideration is necessary and reasonable.

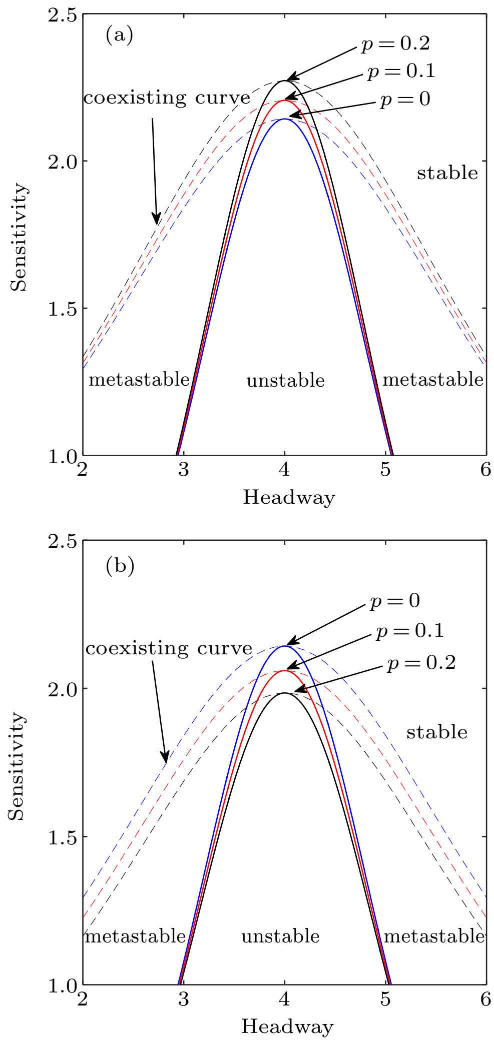

Fig. 1. Phase diagrams in the headway-sensitivity space for different traffic interruption probabilities.

4. Anticipation of traffic interruption probability for MKdV equation

In this section, to obtain the MKdV equation, nonlinear analysis is executed by introducing slow variable X and T with a small positive scaling parameter ε as follows:

$$X = \varepsilon(n + bt), \quad \text{and} \quad T = \varepsilon^3 t, \quad 0 < \varepsilon \leq 1, \tag{11}$$

$$\Delta x_n(t) = h_c + \varepsilon R(X, T). \tag{12}$$

Consequently, by expanding Eq. (4) to the fifth order of ε, the nonlinear partial differential equation is obtained as

$$\begin{aligned}
&\varepsilon^2[K_1 b - V'(h_c)]\partial_X R + \varepsilon^3 G_1 \partial_X^2 R \\
&\quad + \varepsilon^4(K_1 \partial_T R + G_2 \partial_X^3 R - G_3 \partial_X R^3)
\end{aligned}$$

$$+\varepsilon^5\{[3b\tau+\lambda_1 V'(h_c)p_0 b\tau - H]\partial_T\partial_X R + G_4\partial_X^4 R - G_5\partial_X^2 R^3\} = 0. \quad (13)$$

The coefficients G_i and K_i $(i = 1,2,\ldots,5)$ are displayed in Table 1 and Table 2, respectively. Let $b = -\rho_c^2 V'/G_1$ and $\tau/\tau_c = 1+\varepsilon^2$, then we will be able to eliminate the second order and third order term of ε, and the resulting equation is obtained as follows:

$$\varepsilon^4(\partial_T R - W_1\partial_X^3 R + W_2\partial_X R^3) + \varepsilon^5(W_3\partial_X^2 R + W_4\partial_X^4 R + W_5\partial_X^2 R^3) = 0. \quad (14)$$

The coefficients W_i $(i = 1,2,\ldots,5)$ are listed in Table 3. We perform the following transformations

$$T' = W_1 T, \quad R = \sqrt{\frac{W_1}{W_2}}R'. \quad (15)$$

Therefore, we obtain the MKdV equation with a correction term $O(\varepsilon)$ as follows:

$$\partial_T' R' - \partial_X^3 R' + \partial_X R'^3 + \varepsilon M[R'] = 0. \quad (16)$$

Here,

$$M[R'] = \sqrt{\frac{1}{W_1}}\left[W_3\partial_X^2 R' + \frac{W_1 W_5}{W_2}\partial_X^2 R'^3 + W_4\partial_X^4 R'\right]. \quad (17)$$

By ignoring the $O(\varepsilon)$, the kink–antikink soliton solution is derived for the MKdV equation as given below

$$R_0'(X,T') = \sqrt{c}\tanh\sqrt{c/2}(X - cT'). \quad (18)$$

In order to obtain the propagation velocity c, the following equation should be satisfied:

$$(R_0', M[R']) = \int_{-\infty}^{+\infty} \mathrm{d}X R_0'(X,T')M[R_0'(X,T')]. \quad (19)$$

Therefore, according to the method in Ref. [20], the propagation velocity c is described below:

$$c = 5W_2W_3/(2W_2W_5 - 3W_1W_4). \quad (20)$$

Subsequently, the headway for the MKdV equation is elaborated below:

$$\Delta x_n(t) = h_c + \sqrt{\frac{W_1 c}{W_2}\left(\frac{\tau}{\tau_c}-1\right)}\tanh\sqrt{\frac{c}{2}\left(\frac{\tau}{\tau_c}-1\right)} \times\left\{j + \left[1 - cW_1\left(\frac{\tau}{\tau_c}-1\right)\right]t\right\}. \quad (21)$$

Correspondingly, the amplitude is taken as

$$A = \sqrt{\frac{W_1 c}{W_2}\left(\frac{\tau}{\tau_c}-1\right)}. \quad (22)$$

Based on the above results, we can draw the coexisting curves in the $(\rho,\ a)$ plane by $\Delta x_n = h_c \pm A$ (see the dashed lines in Fig. 1). Then, the phase space is divided into three regions, which are called the stable region, the metastable region, and the unstable region, respectively. Clearly, the neutral and the coexisting curves ascend with the increase of the traffic interruption probability p_0 in the case without the anticipation term of traffic interruption in Fig. 1(a). But the neutral and the coexisting curves decrease with the traffic interruption probability p_0 increasing by considering the anticipation term of traffic interruption in Fig. 1(b). The result shows that the anticipation term of traffic interruption can improve traffic stability.

Table 1. Coefficients G_i of the model.

G_1	G_2	G_3
$\frac{3b^2\tau}{2}+\frac{\lambda_1 V'(h_c)p_0 b^2\tau}{2}-\frac{V'(h_c)}{2}-K_2 b$	$\frac{7b^3\tau^2+\lambda_1 V'(h_c)p_0 b^3\tau^2 - V'(h_c) - H(3b+3b^2\tau)}{6}$	$\frac{V'''(h_c)}{6}$
G_4	G_5	
$\frac{5b^4\tau^3}{8}+\frac{\lambda_1 V'(h_c)p_0 b^4\tau^3}{24}-\frac{V'(h_c)}{24}-K_2\frac{4b+6b^2\tau+4b^3\tau^2}{24}$	$\frac{V'''(h_c)}{12}$	

Table 2. Coefficients K_i of the model.

K_1	K_2	K_3	K_4	K_5
$1+\lambda_1 V'(h_c)p_0$	$\lambda_2 V'(h_c)(1-p_0)$	$3+\lambda_1 V'(h_c)p_0$	$7+\lambda_1 V'(h_c)p_0$	$15+\lambda_1 V'(h_c)p_0$

Table 3. Coefficients W_i of the model.

W_1	W_2	W_3	W_4	W_5
$-\frac{V'(h_c)}{6K_1^2K_3}\{K_4(K_1+2K_2)^3 - K_1K_3^2 - K_2[3K_3^2 + 3(K_1+2K_2)K_3]\}$	$-\frac{V'''(h_c)}{6}$	$-\frac{K_1+2K_2}{2K_1^2}V'(h_c)$	$-\frac{V'(h_c)}{24K_1^2K_3^3}\{K_5(K_1+2K_2)^3 - K_1K_3^3 - K_2K_3[4K_3 + 6(1+2K_2)K_3 + 4(K_1+2K_2)^2]\}$	$\frac{V'''(h_c)}{12K_1}$

5. Numerical simulation

Numerical simulation is executed with a periodic boundary condition, where $N = 200$ and $a = 1.96$. The initial disturbance is adopted as follows:

$$\begin{cases} \Delta x_n(0) = \Delta x_n(1) = 4, \ n \neq N/2, \ n = N/2+1, \\ \Delta x_n(0) = \Delta x_n(1) = 4+\delta, \ n \neq N/2, \\ \Delta x_n(0) = \Delta x_n(1) = 4-\delta, \ n = N/2+1. \end{cases} \tag{23}$$

Firstly, we observe the headway evolution after 10^4 s and the profile at the $t = 10300$ s in the case without the anticipation term of traffic interruption ($\lambda_1 = 0$) in Figs. 2 and 3, where $\delta = 0.1$ and $\lambda_2 = 0.2$. From Fig. 2, it can be seen that the stop-and-go traffic occurs and becomes worse with the increase of the traffic interruption probability p_0. From Fig. 3 it follows that the amplitude of headway becomes bigger and bigger with p_0 increasing. As a conclusion, the traffic interruption probability will destroy traffic stability when the anticipation term of traffic interruption is not considered.

Subsequently, figures 4 and 5 indicate the headway evolutions after 10^4 s and the profile at the $t = 10300$ s with consideration of the anticipation term of traffic interruption ($\lambda_1 = 0.5$). Obviously, the oscillating amplitude of the headway becomes smaller and smaller with the increase of the traffic interruption probability p_0 by considering the anticipation term of traffic interruption from Figs. 4(a)–4(c). Finally, the disturbance vanishes by increasing p_0 to an appropriate value in Fig. 4(d). More clearly, according to Fig. 5, the change amplitude of headway is dwindling gradually with p_0 increasing. The result signifies that the anticipation term of traffic interruption contributes to traffic stability as traffic interruption probability happens, which implies that the anticipation term of traffic interruption should be considered in car-following models.

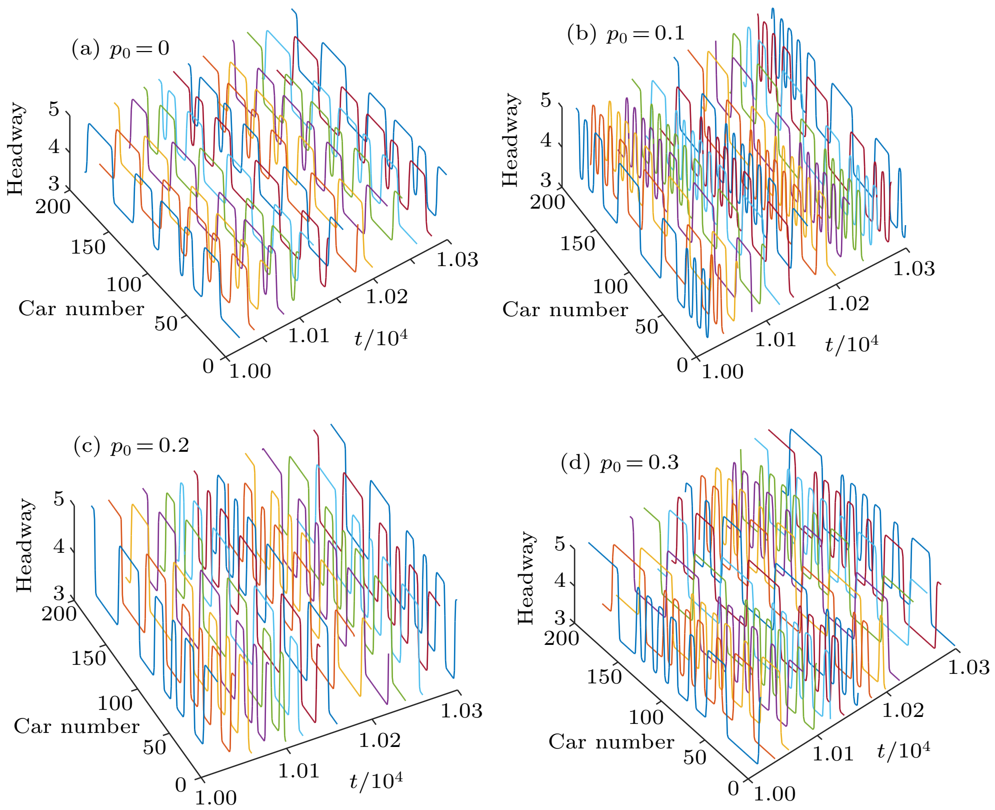

Fig. 2. Headway evolution after 10^4 steps.

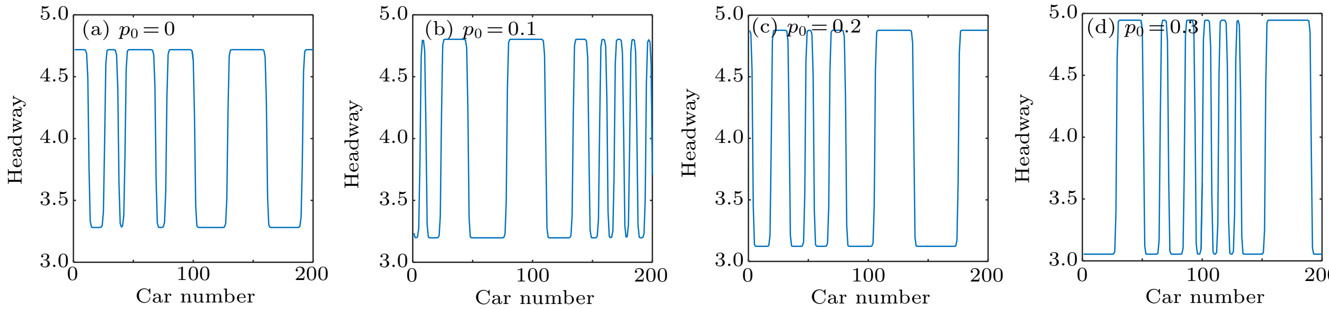

Fig. 3. Profiles of headway at time step 10300.

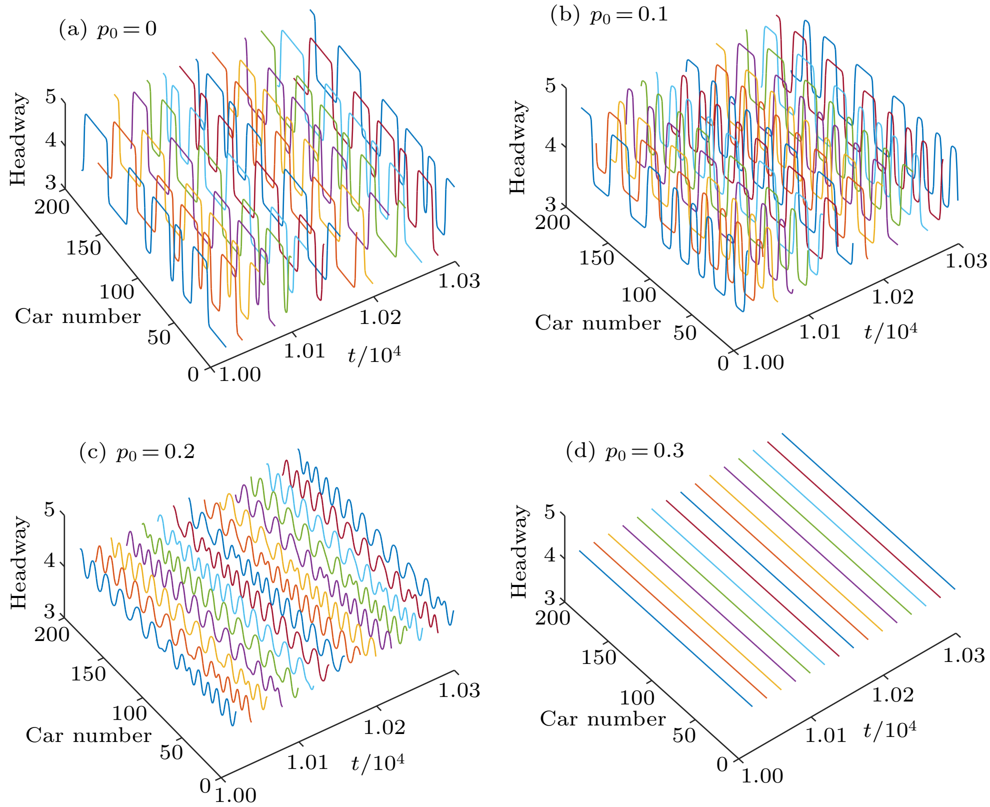

Fig. 4. Headway evolution after 10^4 steps.

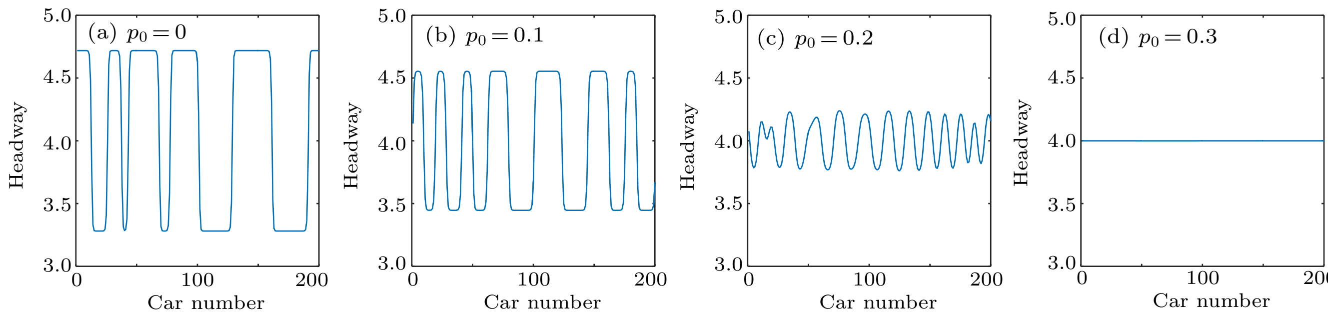

Fig. 5. Profile of headway at time step 10300.

6. Conclusions

As the modern traffic environment becomes more and more complex, traffic interruptions frequently occur. Therefore, it is worth studying the problem of traffic interruption. Therefore, a new car-following model in this paper is proposed with consideration of the anticipation term of traffic interruption. The criterion of stability is closely related to the anticipation of traffic interruption. Numerical simulation demonstrates that the anticipation term of traffic interruption has an important effect on traffic stability. Traffic interruption is a complex event in traffic flow. In this paper, we introduce a simple form as the anticipation term of traffic interruption probability. Therefore, it is worth investigating to calibrate the parameters by analyzing the traffic interruption events in the future.

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