Gain-induced large optical torque in optical twist settings*

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Inducing a significant optical torque remains a challenging task, since the law of angular momentum conservation implies that one has to harvest a lot of light. Such a problem was partially resolved by using optical twist via strong internal multiple scattering to recycle the photons, and one can induce a large torque per unit of radiation cross section. By using the Maxwell stress tensor and the generalized Lorentz–Mie scattering theory for multi-spheres, we investigate the influence of gain materials in further amplifying optical torque in the optical twist settings. It is found that, when combined with a gain layer, the optical torque of lossy (both in PT- and non-PT-symmetric structures) or lossless (low dielectric materials) clusters at resonance could be one order of magnitude larger than those of a single layer and previous studied plasmonic double layer structures. Moreover, the gain-enhanced large opposite rotations (*i.e.*, optical twist) of the two layers arise at resonances in these structures. In contrast, in the gain–gain double-layer cluster, optical torques on both layers have no significant increase and the two layers rotate in the same direction at resonances. This work provides an elaborate investigation on the gain media-induced optical twist, which offers more choices for optical micromanipulation.

Keywords: optical torque, optical twist, gain medium, double-layer clusters

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1. Introduction

Angular momentum carried by light can induce optical torque via scattering or absorption,^[1–10] which was first demonstrated by Beth,^[11] and provides applications in DNA unfolding, sequencing, and binding,^[12–15] biological molecular motors,^[16–18] nano-electromechanical systems,^[3,19–23] and so on. However, harvesting light for a large torque is difficult. The angular momentum must conserve, while the angular momentum carried by incident light is limited. Several ways to obtain a large optical torque have been demonstrated. Researchers can use birefringent particle to enhance the torque by reversing the photon spin angular momentum of the incident circularly polarized light,^[24] or use plasmonic motor to increase the extinction cross section via plasmonic resonance,^[3] or use double-layer clusters to enhance the torque per unit light energy extinction and absorption rate,^[6] *etc*.

Double-layer clusters recycle light by resonance light exchange between the two layers,^[6] which enhance internal multiple scattering. Thus significant difference between opposite optical torques called optical twist in double layer structures can be induced without extracting a large amount of light from the incident beam.^[6] In this paper, we use gain materials in optical twist to shape the beam to rotate other material clusters.^[25–31] Systematic studies on how gain materials, which amplify optical radiation, can affect the torque exerted on other material clusters like lossy (in *PT*- or non-*PT*symmetric structures), lossless (low dielectric materials), or even gain material clusters are carried out. This theoretical work extends the previous studies on optical twist, and provides more detailed basis in free optical micromanipulation.

This paper is organized as follows. The theoretical formalism and computational details used in the calculation are discussed in Section 2. In Section 3, we present numerical results on optical torque exerted on different optical twist settings. Section 4 is the conclusion and perspectives.

2. Theoretical formalism and computational details

In this paper, the time averaged optical torque acting on the lower and upper particle clusters constituted by identical spheres within the same layer are computed using the same equations as those in Ref. [6]

$$\Gamma_{\text{lower cluster}} = \sum_{i(\text{over lower particle})} \tau_i + \sum_{i(\text{over lower particle})} (r_i \times F_i),$$
$$\Gamma_{\text{upper cluster}} = \sum_{i(\text{over upper particle})} \tau_i + \sum_{i(\text{over upper particle})} (r_i \times F_i), \qquad (1)$$

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where r_i is the position vector of the *i*-th sphere measured from the center of mass of each layer, $F_i = \iint_{\sigma_i} T \cdot \hat{n} dS$ is the time averaged optical force that acts on the *i*-th sphere, and $\tau_i = \iint_{\sigma_i} \hat{n} \cdot (r_i \times T) dS$ is the torque acting on the *i*-th sphere about its own center. The time averaged Maxwell stress tensor T is solved by utilizing the generalized Lorentz–Mie scattering theory,^[32–34] and in this work, the series expansion was truncated at angular momentum $L_{\text{max}} = 65$ (further increase in L_{max} does not change the results much). Our algorithm subject only to numerical truncation error and can be considered exact within classical electrodynamics.^[6]

Throughout this paper, the clusters in air are illuminated by a z-propagating plane wave with a modest intensity 1 mW/ μ m², wavelength $\lambda = 532$ nm, and a spin angular momentum of \hbar per photon. The refractive index of constituted spheres is n = n' + in'',^[28] in which n' and n'' are the corresponding real and imaginary parts, respectively. $n'' \ge 0$ corresponds to a passive sphere made of lossy or lossless materials, while n'' < 0 is related to an active sphere made of gain media. In general, gain media should be modeled by a complex permittivity with an active imaginary part.^[35] For simplicity and validity (discussed in Fig. 1), we fix the refractive index at a moderate value n = 2 - 0.005i. The studied structures in this paper serve as the prototype for more realistic structures. Exact torques' values can be different between our structure and the realistic ones, the aim here is to demonstrate the role of gain medium in amplifying optical torques.

3. Results and discussion

In order to establish a preliminary understanding of gain involved optical torque, we firstly investigate the case of a single gain (n'' < 0) or lossy (n'' > 0) sphere [see Fig. 1]. As shown in Fig. 1(b), the lossy sphere obtains positive torque while the gain sphere gets negative torque (relative to the incident spin angular momentum). The trend is more obvious with the increase of the absolute value of n''. Physically, the optical torque exerted on a lossy sphere originates from the absorption of incident angular momentum which is defined as positive in this paper. Thus, the larger n'', the more positive angular momentum are absorbed and hence the larger positive torque can be generated. However, the situation is different for the gain sphere, which is subject to torque that is due to stimulated emission of radiation. As the gain sphere emits photons carrying positive angular momentum larger than the incident one, it gains negative torque due to the conservation of angular momentum. The larger the gain (large absolute value of n''), the greater the negative torque are acquired. It is worthy to note that, the peaks are more prominent when the whispering gallery modes (WGM)^[36] are excited at large radius [for example, pointed by the black arrow in Fig. 1(b)], which are also applicable for following multi-spheres settings. The peaks of the optical torque on the lossy sphere are wider (due to absorption) than those of the gain sphere. Since the gain material compensates part of loss, the optical torque on it has sharper peaks, especially for larger gain values [see the green line in Fig. 1(b)]. Corresponding angular velocities are also calculated from the classical equation of motion^[3] at steady state $(t \to \infty)$, $I(d\omega/dt) + \gamma \omega = \Gamma$, where $I = 2/5ma^2$ is the momentum of inertia of a sphere, $\gamma = 8\pi\eta a^3$ is a damping term^[37] originating from air viscosity, ω is the rotation velocity, Γ is the optical torque, m and a are the mass and radius of the sphere, respectively, and $\eta = 1.85 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$ is the viscosity coefficient of air in the case of temperature 25 °C and density 1.29 kg/m³. From Fig. 1(c), one can clearly see that, single lossy and gain spheres have opposite rotations. Positive rotations (denoted by the red and blue lines) share the same rotation direction of the polarization of the incident light, and negative rotations are denoted by the black and green lines. Compared to the lossy sphere, optical torques (speeds) of the gain sphere are enhanced more significantly (rapidly) when the absolute value of n'' is five times larger (from |n''| = 0.001 to |n''| = 0.005), as evident from Figs. 1(b) and 1(c). Therefore, a speedy and counter-rotating motor can be achieved by using a gain sphere. In principle, the optical torque of a gain sphere is expected to be magnified to infinity with the increase of gain, while it drops rapidly above gain threshold (n'' = 0.0065) in Fig. 1(d). Because the gain becomes "loss" when n'' is larger than its threshold value. The paradoxical symmetry between amplification and absorption is an artifact due to the unphysical assumption of a finite output in solving the time-independent wave equation. This is not the true behavior of the system with large gain or system sizes.^[38] The value of gain threshold varies with many factors like the wavelength of the incident light, size of structures, geometries, and so on. For these reasons, we choose moderate gains with $n_{\text{gain}} = 2 - 0.005i$ in the following calculations.

From the results of single gain and lossy spheres, we wish to increase the optical torque of lossy spheres by building a gain involved optical twist setting. First of all, a double-layer cluster consisting of one gain and one lossy spheres is studied [see Fig. 2(a)]. It is expected that the optical torque of the lossy sphere is supposed to be increased due to the multiple scattering between two layers. Indeed, optical twists (torques with opposite signs exerted on the two layers) appear in this setting as shown in Figs. 2(b) and 2(c). However, due to the reduced quality factor of the system, the optical torque of the lossy sphere is not apparently increased compared to that of one single lossy sphere in Fig. 1(b). Thus, in the following calculations, we use multiple spheres to constitute a larger structure to attract more light and enhance multiple scattering between the two layers. Since there is no significant difference when the gain and lossy layers are exchanged due to the small absolute value of n'' [see Figs. 2(b) and 2(c)], we just consider one configuration in following.

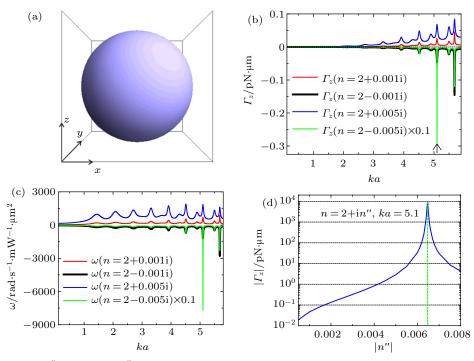


Fig. 1. (a) A single gain (n'' < 0) or lossy (n'' > 0) sphere in air impinged by a plane wave propagating along *z* direction. The corresponding optical torques and rotation speeds of the sphere as a function of *ka* are plotted in panels (b) and (c), respectively. Here *k* is the wavenumber and *a* is the radius of the sphere. For illustration purpose, Γ_z and ω denoted by green lines are multiplied by 0.1 in panels (b) and (c). (d) Absolute value of optical torque *versus* |n''| is shown for a gain sphere with ka = 5.1 pointed out by the black arrow in panel (b). The dashed vertical green line is inserted to highlight the paradoxical symmetry between amplification and absorption at gain threshold.

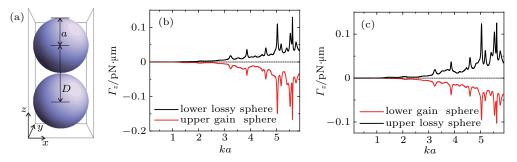


Fig. 2. (a) A double-layer cluster made of a gain ($n_{gain} = 2 - 0.005i$) and a lossy ($n_{lossy} = 2 + 0.005i$) spheres with the separation between spheres $D = 2a + 0.002 \mu m$. Optical torques for the upper gain (lossy) sphere and the lower lossy (gain) sphere are shown in panels (b) [(c)], respectively.

Optical torques exerted on a PT-symmetric double-layer cluster with multiple spheres [Fig. 3(a)] are shown in Fig. 3(c). At first glance, optical torques on both lossy and gain layers have sharp peaks at resonances. To get the detailed information at resonances, the partial view are presented in the insert figure of Fig. 3(c). One can see that large optical twists are formed when the WGM^[36] are excited. Furthermore, we wonder whether a gain medium can enhance the optical torque on a lossy structure. Compared to torques on a single lossy cluster [the blue line in Fig. 3(d), and the structure is shown in Fig. 3(b)], optical torques acting on the lossy cluster in the gain-lossy double-layer cluster [the black line in Fig. 3(c)] is one order of magnitude larger at resonances. Moreover, the black line in Fig. 3(c) is reploted in Fig. 3(d) to compare the detailed information away from resonances with the blue line. We found that, the magnitude of both positive and negative torques exerted on the lossy layer in the double-layer structure are in general greater than those on the single lossy structure.

In order to figure out the physical reason behind the amplification of torques away from resonances, we calculate optical torques exerted on a lossless-lossless double-layer cluster (with the same real part of refractive index as that of the PT-symmetric one) shown in Figs. 3(d) and 3(e). It is found that, values of the green and black lines (the purple and red lines) in Fig. 3(d) [Fig. 3(e)] have little difference without resonances. Therefore, the amplification of optical torques are mainly due to multiple scattering originally from the incident light when the resonances are not excited. The minor differences can also be realized like this, gain media plays an important role in compensating the absorption loss of the lossy spheres and makes a lossy sphere gain nearly the same optical torque as the lossless one. While the gain media manifests itself via emitting numerous light at resonances that induce a strong internal multiple scattering, and sharp peaks of optical twist exist as a result [see Fig. 3(c)].

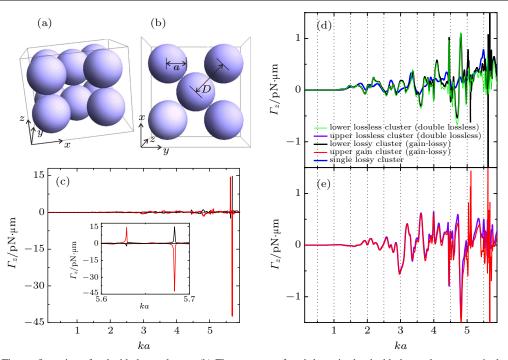


Fig. 3. (a) The configuration of a double-layer cluster. (b) The structure of each layer in the double-layer cluster or a single layer. The separation between adjacent spheres is $D = 2a + 0.002 \mu m$. (c) Optical torques exerted on the *PT*-symmetric double-layer structure with the refractive indexes of the lower lossy and upper gain layers are $n = 2 \pm 0.005i$, respectively. The insert figure displays the opposite torques of the two layers at resonance. (d) Optical torques exerted on a single lossy layer ($n_{lossy} = 2 + 0.005i$), the lower lossless layer ($n_{lossless} = 2$) in a lossless–lossless double-layer cluster, and a partial view of the black line in panel (c) are shown for comparison. Optical torques obtained by the upper lossless layer ($n_{lossless} = 2$) in a lossless–lossless double-layer cluster, and a partial view of the red line in panel (c) are also shown in panel (e) for comparison. Black dotted vertical lines in panels (d) and (e) are guides of eyes to highlight the optical twist.

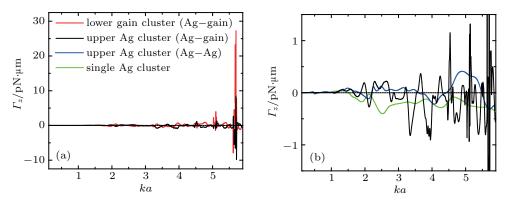


Fig. 4. Structures are the same as Figs. 3(a) and 3(b) except the separation between adjacent spheres $D = 2a + 0.0008 \,\mu\text{m}$. The refractive index of Ag is modeled by Drude model, [6,39,40] and $n_{gain} = 2 - 0.005i$. (a) Optical torques exerted on the upper Ag layer and the lower gain layer in a gain–Ag double-layer cluster. (b) Optical torques exerted on a single Ag layer cluster and that on the upper Ag layer in a Ag–Ag double-layer cluster are plotted compared to the partial view of the black line in panel (a).

Can resonances only be excited in *PT*-symmetric doublelayer clusters? In the following, we will focus on the non-*PT*symmetric clusters. Figure 4(a) shows optical torques acting on a Ag–gain double-layer cluster, in which the optical torque of the Ag layer is greatly enhanced at resonance. Sharp peaks at resonances are manifested compared to wider peaks of optical torques on a single Ag layer cluster [green line in Fig. 4(b)] or on the Ag layer in a Ag–Ag double-layer cluster^[6] [blue line in Fig. 4(b)]. Furthermore, both negative and positive torques of Ag layer in gain–Ag double-layer cluster are largely improved even away from resonances, which are also better than the effect of previous studied Ag to Ag^[6] as shown in Fig. 4(b), especially for larger spheres. This is because larger Ag has severe absorption which is bad for multiple scattering.

Usually, optical torques exerted on low dielectric material structures (such as glass) are quite small due to the weak scattering. Here, we will show that the gain media cluster can also greatly enhance optical torques of glass [see Fig. 5]. Compared to the optical torques on a single glass layer [blue line in Fig. 5(d)] and the glass layer in Ag–glass doublelayer cluster^[6] [green line in Fig. 5(d)], optical torques on the glass layer in the gain–glass double-layer cluster [black line in Fig. 5(c)] can be one order of magnitude larger at resonances. Apparently, even away from resonances, the magnitude of both positive and negative torques on glass are generally enhanced by the gain medium [see Fig. 5(d)]. Furthermore, the partial view of optical torques at resonances are presented in the insert figure of Fig. 5(c). One can see that large optical twists are also formed when the WGM are excited.

Finally, we investigate a gain–gain double-layer cluster to see if optical torques on gain layer itself can be further increased. As shown in Fig. 6(a), there is no obvious advantage over that on a single gain layer at resonances. The slightly enlarged torques when the resonances do not occur are due to internal multiple scattering between the two layers [see Fig. 6(b)]. While the most distinguishing feature is optical torques of the two layers in the gain–gain double-layer cluster rotate in the same direction at resonance [see the insert figures in Fig. 6(a)] compared to the opposite rotations of the two layers in gain-lossy or gain-lossless double-layer clusters [see Figs. 3(c) and 5(c)]. This is because the gain cluster is unlike a passive cluster which typically extinct the incident light and experience a positive torque. Therefore, the lower layer in a gain–gain double-layer cluster structure enhances the incident light and rotates backward, then the enhanced incident light is also enhanced by the upper layer, and the upper layer also recoils backward.

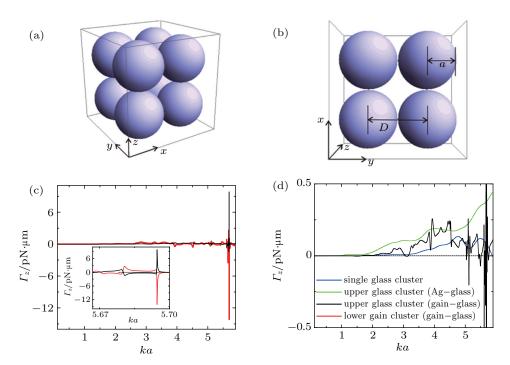


Fig. 5. (a) The configuration of a double-layer cluster. (b) The structure of each layer in the double-layer cluster or a single layer. The separation between adjacent spheres is $D = 2a + 0.002 \mu m$. Optical torques of the upper glass layer ($n_{glass} = 1.33$) and the lower gain layer ($n_{gain} = 2 - 0.005i$) in the gain–glass double-layer cluster are shown in panel (c), and the inset exhibits optical twist at resonances. Optical torques exerted on a single glass layer and that on the upper glass layer in a Ag–glass double-layer cluster are shown in panel (d) compared to the partial view of the black line in panel (c).

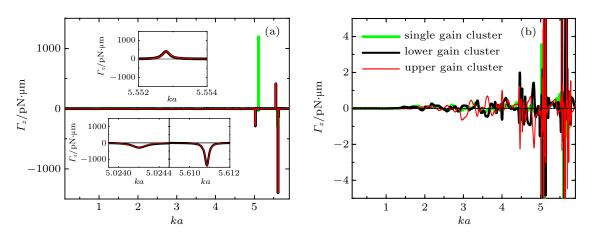


Fig. 6. Structures are the same as Figs. 3(a) and 3(b), and the refractive index of gain spheres is $n_{gain} = 2 - 0.005i$. (a) Optical torques exerted on the gain–gain double-layer cluster (red and black lines) and on a single gain structure (green line). The inserts show the same rotation direction of the two layers in gain–gain double-layer cluster at resonances. The partial view of torques in panel (a) is shown in panel (b).

4. Conclusion and perspectives

In summary, investigation of gain-enhanced optical torques is carried out by using the Maxwell stress tensor and the generalized Lorentz-Mie scattering theory. Since single lossy (gain) sphere has positive (negative) torques originating from the absorption of incident positive angular momentum (the emission of photons carrying larger positive angular momentum), we design gain involved double-layer optical twist settings. By doing so, one can effectively enhance the internal multiple scattering especially at resonances, which result in large enhancement of optical torques on the lossy (lossless) layer in gain-lossy (gain-lossless) double-layer clusters. Optical torques exerted on gain involved double layer clusters present as sharp peaks when WGM are excited, because the gain materials compensate loss. Away from resonances, the magnitudes of both positive and negative torques are also enhanced, which provides free opposite rotations. It is worth to note that the two layers in the gain-gain double layer cluster rotate in the same direction at resonances while opposite rotations (i.e., optical twist) of the two layers arise in gain-lossy or gain-lossless double-layer clusters. Elaborated investigations on gain enhanced optical torques in this work provide more choices and flexibilities in optical micromanipulation.^[41-44]

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