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Electromagnetic field of a relativistic electron vortex beam*

Changyong Lei(雷长勇)[1] and Guangjiong Dong(董光炯)[1,2,†]

[1] *State Key Laboratory of Precision Spectroscopy, School of Physics and Electronics, East China Normal University, Shanghai* 200241, *China*
[2] *Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan* 030006, *China*

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Electron vortex beams (EVBs) have potential applications in nanoscale magnetic probes of condensed matter and nanoparticle manipulation as well as radiation physics. Recently, a relativistic electron vortex beam (REVB) has been proposed [*Phys. Rev. Lett.* **107** 174802 (2011)]. Compared with EVBs, except for orbital angular momentum, an REVB has intrinsic relativistic effect, i.e., spin angular momentum and spin–orbit coupling. We study the electromagnetic field of an REVB analytically. We show that the electromagnetic field can be separated into two parts, one is only related to orbital quantum number, and the other is related to spin–orbit coupling effect. Exploiting this separation property, the difference between the electromagnetic fields of the REVB in spin-up and spin-down states can be used as a demonstration of the relativistic quantum effect. The linear momentum and angular momentum of the generated electromagnetic field have been further studied and it is shown that the linear momentum is weakly dependent on the spin state; while the angular momentum is evidently dependent on the spin state and linearly increases with the topological charge of electron vortex beam. The electromagnetic and mechanical properties of the REVB could be useful for studying the interaction between REVBs and materials.

Keywords: relativistic electron vortex beam, electromagnetic vortex field, spin–orbit coupling, orbital angular momentum

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1. Introduction

It is well known that the wavefunction for a free electron can be a plane wave. In 2007, a vortex wavefunction for the electron was predicted[1] and was demonstrated soon.[2–5] An electron vortex beam (EVB) carries orbital angular momentum and has magnetic moment. These features have been actively explored as chiral-dependent energy-loss spectroscopy in ultramicroscopy for detecting magnetic properties in condensed matter and spintronics[6–8] at nano scale or even down to atomic size. Using the electron's orbital angular momentum to generate spin-polarized electron beams has been proposed.[9] Moreover, the mechanical properties of an EVB[10] can be used for nanoparticle manipulation.[11,12] When EVBs are investigated for probing materials,[13] it is found that transition radiation induced by giant magnetic moment could be detected,[14–16] especially quantum correction to Cherenkov radiation could be detected[17,18] and could be further developed as a macroscopic diagnostic tool for electron vortex beams[19] or Cherenkov concentrator.[20,21] Moreover, optical vortex[22–25] can be generated by undulation of twisted electrons.[26,27]

In electron energy-loss spectroscopy (EEL),[3,28–32] a relativistic electron beam of 100–200 keV is typically used, calling for relativistic description using the Dirac equation.[33] Recently, Bliokh *et al.*[34] have found a relativistic Bessel-type solution to the Dirac equation. It was shown that compared with its nonrelativistic counterpart, the relativistic electron vortex beam (REVB) has an intrinsic spin angular momentum and spin–orbit coupling.[34] How to demonstrate the relativistic quantum effect of REVB is yet to be investigated. Further, to study the interaction of an electron beam with materials, it is essential to study the feature of the electromagnetic field generated by an EVB. The electromagnetic field generated by the relativistic EVB and its mechanical properties is pending for investigation.

In this paper, we investigate the electric and magnetic fields generated by a relativistic EVB.[34] It is shown that the electric field and the magnetic field can be separated into two parts, one is only related to the orbital quantum number as a demonstration of the nonrelativistic quantum effect, and the other is related to the spin–orbit coupling effect intrinsic in the REVB. Thus we propose to measure the relativistic quantum effect through the electromagnetic field of the REVB. We find that the electromagnetic field of the REVB is a vortex field. We further study mechanical properties (the linear and angular momentums) of the electromagnetic vortex field.

The paper is organized as follows. We develop the theory of electromagnetic field of a relativistic Bessel beam in Section 2, give the numerical results in Section 3, and finally present the inclusions.

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†Corresponding author. E-mail: gjdong@phy.ecnu.edu.cn

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2. Theory of electromagnetic field of a relativistic Bessel electron beam

Our analysis starts from a Bessel-type relativistic EVB of the topological charge l with the radial and axial wave vectors $k_\perp$ and k_z, given by[34]

$$\psi_{l,s}(\boldsymbol{r},t) = \frac{N_l\,\mathrm{e}^{\mathrm{i}\Phi(z,t)}}{\sqrt{2}}\left[\begin{pmatrix} G_1 w_s \\ G_2\sigma_z w_s\end{pmatrix} J_l(k_\perp\rho)\,\mathrm{e}^{\mathrm{i}l\varphi} + \sqrt{\Delta}\begin{pmatrix}\tilde{0}\\ \sigma_y w_s\end{pmatrix} J_{l+2s}(k_\perp\rho)\,\mathrm{e}^{\mathrm{i}(l+2s)\varphi}\right], \quad (1)$$

with the phase $\Phi(z,t) = k_z z - Et/\hbar$, the first kind of Bessel function J_l of the order l, $G_1 = \sqrt{1+m_\mathrm{e}c^2/E}$, $G_2 = \sqrt{E_k/E}\cos\alpha$, $\tilde{0} = \binom{0}{0}$, and spin–orbit coupling strength $\Delta = c^2\hbar^2k^2\sin^2\alpha/\left[E\left(E+m_\mathrm{e}c^2\right)\right]$, in which

$$\alpha = \tan^{-1}(k_\perp/k_z) \quad (2)$$

is the opening angle and $E_k = E - m_\mathrm{e}c^2$ is the kinetic energy of the EVB, and the eigenenergy $E = c\sqrt{\hbar^2k^2+m_\mathrm{e}^2c^2}$ ($k^2 = k_\perp^2 + k_z^2$). The spin state w_s is the eigenstate of the Pauli operator σ_z with eigenvalue $s = \pm 1/2$. Here σ_y is the Pauli matrix in y direction. We assume a truncated relativistic Bessel beam, i.e., the effective region of a Bessel vortex beam cross section only spans $\mathfrak{L}_\mathrm{R}$ and the axial range of the beam is $\mathfrak{L}_z$. The normalization factor is given by $N_l = \sqrt{k_\perp/(2\mathfrak{L}_\mathrm{R}\mathfrak{L}_z)}$.[35]

Bliokh *et al.*[34] have obtained the probability density of an REVB as follows:

$$\rho_{l,s}(\boldsymbol{r}) = |N_l|^2\left[\left(1-\frac{\Delta}{2}\right)J_l^2(k_\perp\rho) + \frac{\Delta}{2}J_{l+2s}^2(k_\perp\rho)\right], \quad (3)$$

and the current density of the REVB[34] $\boldsymbol{j}_{l,s}(\boldsymbol{r}) = j_{l,s,\varphi}(\boldsymbol{r})\boldsymbol{e}_\varphi + j_{l,z}(\boldsymbol{r})\boldsymbol{e}_z$ with

$$j_{l,s,\varphi}(\boldsymbol{r}) = |N_l|^2\frac{\hbar kc^2}{E}J_l(k_\perp\rho)J_{l+2s}(k_\perp\rho)\sin\alpha, \quad (4)$$

$$j_{l,z}(\boldsymbol{r}) = |N_l|^2\frac{\hbar kc^2}{E}J_l^2(k_\perp\rho)\cos\alpha. \quad (5)$$

In this paper, we note that the probability density and current density can be written as a sum of two parts. One is related to nonrelativistic quantum effect and the other corresponds to the relativistic quantum effect. For the probability density, $\rho_{l,s}(\boldsymbol{r}) = \rho_l^{(\mathrm{nr})}(\boldsymbol{r}) + \rho_{ls}^{(\mathrm{r})}(\boldsymbol{r})$ (hereafter, superscripts (nr) and (r) are used to represent nonrelativistic quantum effect and relativistic quantum effect, respectively) with $\rho_l^{(\mathrm{nr})}(\boldsymbol{r}) = |N_l|^2\left(1-\frac{\Delta}{2}\right)J_l^2(k_\perp\rho)$, which is irrelevant to the spin state and thus comes from the nonrelativistic quantum effect, and $\rho_{ls}^{(\mathrm{r})}(\boldsymbol{r}) = |N_l|^2\Delta J_{l+2s}^2(k_\perp\rho)/2$, in which the quantum number l and s cannot be separated, thus coming from the relativistic quantum effect characterized by the spin–orbit coupling. For the current density, we note that $2s = \pm 1$. Using the recurrence relation of the Bessel function,[37] equation (4) can be rewritten as $j_{l,s,\varphi}(\boldsymbol{r}) = j_l^{(\mathrm{nr})}(\boldsymbol{r}) + j_{ls}^{(\mathrm{r})}(\boldsymbol{r})$, in which

$$j_l^{(\mathrm{nr})}(\boldsymbol{r}) = \frac{2l|N_l|^2\hbar c^2}{E}\frac{J_l^2(k_\perp\rho)}{\rho},$$

$$j_{\mathrm{ls}}^{(\mathrm{r})}(\boldsymbol{r}) = -\frac{|N_l|^2\hbar kc^2}{E}J_l(k_\perp\rho)J_{l-2s}(k_\perp\rho)\sin\alpha. \quad (6)$$

The term $j_l^{(\mathrm{nr})}(\boldsymbol{r})$ is independent of spin and comes from a nonrelativistic quantum effect, while $j_{ls}^{(\mathrm{r})}(\boldsymbol{r})$ is dependent on quantum numbers l and s, thus arises from the relativistic quantum effect, spin–oibit coupling.[36] The separation of the nonrelativistic quantum effect and relativistic quantum effect could give us a way to measure relativistic quantum effect intrinsic in the REVB through the electromagnetic field of the REVB as follows.

Using Gauss's law $\nabla\cdot\boldsymbol{E}_{l,s}(\boldsymbol{r}) = -e\rho_{l,s}(\boldsymbol{r})/\varepsilon_0$ with the electric charge e and vacuum dielectric constant ε_0 for electric field $\boldsymbol{E}_{l,s}(\boldsymbol{r})$, we obtain the electric field of the REVB, $\boldsymbol{E}_{l,s}(\boldsymbol{r}) = [E_{l,s}^{(\mathrm{r})}(\rho) + E_l^{(\mathrm{nr})}(\rho)]\boldsymbol{e}_\rho$ with

$$E_{l,s}^{(\mathrm{r})}(\rho) = s\frac{e\hbar}{\varepsilon_0}\frac{j_{l,s,\varphi}(\boldsymbol{r})}{E+m_\mathrm{e}c^2},$$

$$E_l^{(\mathrm{nr})}(\rho) = -\frac{e|N_l|^2}{2\varepsilon_0}\rho\left[J_l^2(k_\perp\rho) - J_{l-1}(\xi)J_{l+1}(k_\perp\rho)\right]. \quad (7)$$

$E_{l,s}^{(\mathrm{r})}(\rho)$ is a purely relativistic effect, arising from spin-dependent current density, while $E_l^{(\mathrm{nr})}(\rho)$ has been found for a nonrelativistic EVB,[38] coming from the nonrelativistic effect. To separate the relativistic and nonrelativistic quantum effects, we can use the difference between the electric fields respectively in spin-up and spin-down states, given by

$$E_{l,1/2}(\rho) - E_{l,-1/2}(\rho) = \frac{e|N_l|^2E_k}{2\varepsilon_0kE}\sin\alpha J_l(k_\perp\rho)\left[J_{l+1}(k_\perp\rho) - J_{l-1}(k_\perp\rho)\right]. \quad (8)$$

Using $J_l(\rho) \sim \frac{1}{\sqrt{2\pi l}}\left(\frac{\tilde{e}\rho}{2l}\right)^l$ (Ref. [37]) ($\tilde{e}$ is the natural constant) for $\tilde{e}\rho \ll l$, thus using high topological charge l, $E_{l,1/2}(\rho) - E_{l,-1/2}(\rho)$ is to be reduced. In this situation, the influence of the spin-angular momentum can be reduced. Similarly, we can show that when the topological charge is sufficiently large, $E_l^{(\mathrm{nr})}(\rho)$ is dominant over $E_{l,s}^{(\mathrm{r})}(\rho)$, i.e., the relativistic effect is weakened by increasing the topological charge.

Further using Ampere's circuit law $\nabla\times\boldsymbol{B}_{l,s}(\boldsymbol{r}) = -\mu_0 e\boldsymbol{j}_{l,s}(\boldsymbol{r})$ with vacuum magnetic permeability μ_0 for the magnetic field $\boldsymbol{B}_{l,s}(\boldsymbol{r})$, we obtain the magnetic field $\boldsymbol{B}_{l,s}(\boldsymbol{r})$ generated by the EVB, $\boldsymbol{B}_{l,s}(\boldsymbol{r}) = B_{l,\varphi}\boldsymbol{e}_\varphi + B_{l,s,z}\boldsymbol{e}_z$ with

$$B_{l,\varphi} = -\frac{e|N_l|^2\hbar k}{2\varepsilon_0 E}\rho\cos\alpha \times\left[J_l^2(k_\perp\rho) - J_{l-1}(k_\perp\rho)J_{l+1}(k_\perp\rho)\right], \quad (9)$$

and $B_{l,s,z} = B_{l,z}^{(\mathrm{nr})} + B_{l,s,z}^{(\mathrm{r})}$ in which

$$B_{l,z}^{(\mathrm{nr})} = \frac{2|N_l|^2e\hbar l}{E\varepsilon_0}\int_\rho^\infty\frac{J_l^2(k_\perp\rho')}{\rho'}\mathrm{d}\rho',$$

$$B^{(\mathrm{r})}_{l,s,z} = -\frac{|N_l|^2 e\hbar k}{E\varepsilon_0}\sin\alpha\int_{\rho}^{\infty} J_l\left(k_\perp\rho'\right)J_{l-2s}\left(k_\perp\rho'\right)\mathrm{d}\rho'. \quad (10)$$

The magnetic field $B_{l,\varphi}$ is independent of spin state of the EVB, and thus its nonrelativistic limit is consistent with that in Ref. [38] for a nonrelativistic EVB. In the low velocity limit, $B^{(\mathrm{nr})}_{l,z}$ approaches to its nonrelativistic result in Ref. [38]. Spin-state-dependent $B^{(\mathrm{r})}_{l,s,z}$ arises from the relativistic quantum effect. The difference between the magnetic fields $B_{l,s,z}$ in spin-up and spin-down states, i.e.,

$$B_{l,1/2,z}(\rho) - B_{l,-1/2,z}(\rho) = \frac{e\hbar|N_l|^2 k\sin\alpha}{\varepsilon_0 E k_\perp} J_l^2(k_\perp\rho) \quad (11)$$

could be used to demonstrate the relativistic quantum effect.

We also note that the electric field and the magnetic field in the transverse plane are zero at the vortex center $\rho = 0$ ($E_{l,s}(0)$, $B_{l,\varphi}(0) = 0$). Thus the transverse electric field and magnetic field are vortex fields.

Finally, we explore the mechanical properties of REVB-induced electromagnetic field, such as the linear momentum and the angular momentum. The electromagnetic momentum density[10] $\tilde{p}_{l,s} = \varepsilon_0 \boldsymbol{E}_{l,s}(\boldsymbol{r})\times\boldsymbol{B}_{l,s}(\boldsymbol{r})$. The linear momentum $\boldsymbol{P}_{l,s} = \int \tilde{p}_{l,s}\mathrm{d}V$ of the electromagnetic fields is given by

$$\boldsymbol{P}_{l,s} = (P^{(\mathrm{nr})}_l + P^{(\mathrm{r})}_{l,s})\boldsymbol{e}_z, \quad (12)$$

with

$$P^{(\mathrm{nr})}_l = P_{\mathrm{A}}\int_0^{\infty}\left[J_l^2(\tilde{\rho}) - J_{l-1}(\tilde{\rho})J_{l+1}(\tilde{\rho})\right]^2\tilde{\rho}^3\,\mathrm{d}\tilde{\rho}, \quad (13)$$

$$P^{(\mathrm{r})}_{l,s} = -P_{\mathrm{A}}\frac{2sE_k}{E}\sin^2\alpha\int_0^{\infty}J_l(\tilde{\rho})J_{l+2s}(\tilde{\rho}) \times\left[J_l^2(\tilde{\rho}) - J_{l-1}(\tilde{\rho})J_{l+1}(\tilde{\rho})\right]\tilde{\rho}^2\,\mathrm{d}\tilde{\rho}, \quad (14)$$

and $P_{\mathrm{A}} = \pi\mathrm{e}^2|N_l|^4\hbar k_z\mathfrak{L}_z/(2\varepsilon_0 k_\perp^4 E)$. $P^{(\mathrm{nr})}_l$ is only dependent on the orbital quantum number l and agrees with the nonrelativistic result in the nonrelativistic limit.[10] $P^{(\mathrm{r})}_{l,s}$ is related to spin–orbit coupling, having no nonrelativistic correspondence. However, $P^{(\mathrm{r})}_{l,s}$ is negligibly weak, as will be shown in the numerical simulation in Fig. 2.

The angular momentum $\boldsymbol{L}_{l,s} = \int \boldsymbol{r}\times\tilde{p}_{l,s}\mathrm{d}V$ of the electromagnetic field reads

$$\boldsymbol{L}_{l,s} = \left(L^{(\mathrm{nr})}_l + L^{(\mathrm{r})}_{l,s}\right)\boldsymbol{e}_z, \quad (15)$$

with

$$L^{(\mathrm{nr})}_l = 2lL_{\mathrm{A}}\int_0^{\infty}\int_{\tilde{\rho}}^{\infty}\mathrm{d}\tilde{\rho}'\,\mathrm{d}\tilde{\rho}\frac{J_l^2(\tilde{\rho}')}{\tilde{\rho}'}\tilde{\rho}^3 \times\left[J_l^2(\tilde{\rho}) - J_{l-1}(\tilde{\rho})J_{l+1}(\tilde{\rho})\right], \quad (16)$$

$$L^{(\mathrm{r})}_{l,s} = L_{\mathrm{A}}\left\{\int_0^{\infty}\int_{\tilde{\rho}}^{\infty}\mathrm{d}\tilde{\rho}'\,\mathrm{d}\tilde{\rho}J_l(\tilde{\rho}')J_{l-2s}(\tilde{\rho}')\tilde{\rho}^3 \times\left[J_l^2(\tilde{\rho}) - J_{l-1}(\tilde{\rho})J_{l+1}(\tilde{\rho})\right] + \frac{2sE_k}{E}\sin^2\alpha \times\int_0^{\infty}\int_{\tilde{\rho}}^{\infty}\mathrm{d}\tilde{\rho}'\,\mathrm{d}\tilde{\rho}J_l(\tilde{\rho}')J_{l+2s}(\tilde{\rho}')J_l(\tilde{\rho})J_{l+2s}(\tilde{\rho})\tilde{\rho}^2\right\}, \quad (17)$$

and $L_{\mathrm{A}} = \pi\mathrm{e}^2|N_l|^4\hbar\mathfrak{L}_z/(\varepsilon_0 k_\perp^4 E)$. $L^{(\mathrm{nr})}_l$ is only dependent on orbital angular momentum, in consistence with the nonrelativistic results in Ref. [10] in the low velocity limit. $L^{(\mathrm{r})}_{l,s}$ corresponds to the relativistic effect.

3. Numerical results

In Section 2, we have shown that the differences of the electric fields and z-component of magnetic fields between spin-up and spin-down states, $E_{l,1/2}(\rho) - E_{l,-1/2}(\rho)$ and $B_{l,1/2,z}(\rho) - B_{l,-1/2,z}(\rho)$ can be used to demonstrate the relativistic quantum effect. Therefore we numerically investigate how they could be controlled with the topological charge l, the beam kinetic energy E_k and the opening angle α, using radial range $\mathfrak{L}_{\mathrm{R}} = 0.23$ nm, $\mathfrak{L}_z = 0.03$ m in the calculation. The numerical results are presented in Fig. 1. Figure 1(a), plotted with $\alpha = 0.7$ rad and kinetic energy $E_k = 0.2$ MeV, shows that with the increasing topological charge, the differences of both the electric and magnetic fields between two spin states can be reduced. Thus it is better to use the REVB with small topological charge for testing the relativistic quantum effect intrinsic in the REVB. Figure 1(b), plotted with $\alpha = 0.7$ rad and $l = 1$ for kinetic energy $E_k = 0.2$ MeV, 0.6 MeV, and 1 MeV, shows that with the increasing kinetic energy, the difference of electric field between the two spin states can be increased. In contrast, the difference of the magnetic field strength between the two spin states is not changed, but the peak positions have been shifted. Figure 1(c), plotted with $l = 1$, kinetic energy $E_k = 0.2$ MeV for $\alpha = 0.2$ rad, 0.7 rad, and 1.2 rad, shows that the increasing opening angle α can enhance the difference between the electric or magnetic field strengths in the two spin states. Equations (8) and (11) show that the strengths of both $E_{l,1/2}(\rho) - E_{l,-1/2}(\rho)$ and $B_{l,1/2,z}(\rho) - B_{l,-1/2,z}(\rho)$ are proportional to $\sin\alpha$, while the dependence of the peak positions of both $E_{l,1/2}(\rho) - E_{l,-1/2}(\rho)$ and $B_{l,1/2,z}(\rho) - B_{l,-1/2,z}(\rho)$ on α arises from $k_\perp = k\sin\alpha$.

We further study the relation of the linear and angular momentums of the electromagnetic vortex fields in Fig. 2 to topological charge l, opening angle α, kinetic energy E_k as well as the spin state. Figure 2 shows that the linear momentum of the vortex field is weakly dependent on spin state, while the difference between the angular momentums in spin-up and spin-down states is remarkable. With the increasing topological charge l, the linear momentum decreases. In contrast, the angular momentum increases, especially the difference of the angular momentums between the two spin states gets pronounced, as a result that the spin–orbit coupling effect increases.[36] Equations (12)–(17) shows that both the linear momentum and the angular momentum decrease with $k_\perp$. Thus figures 2(b) and 2(c) show that both the linear momentum and angular momentum decrease with increasing the opening angle ($k_\perp = k\sin a$) and the incident kinetic energy E_k.

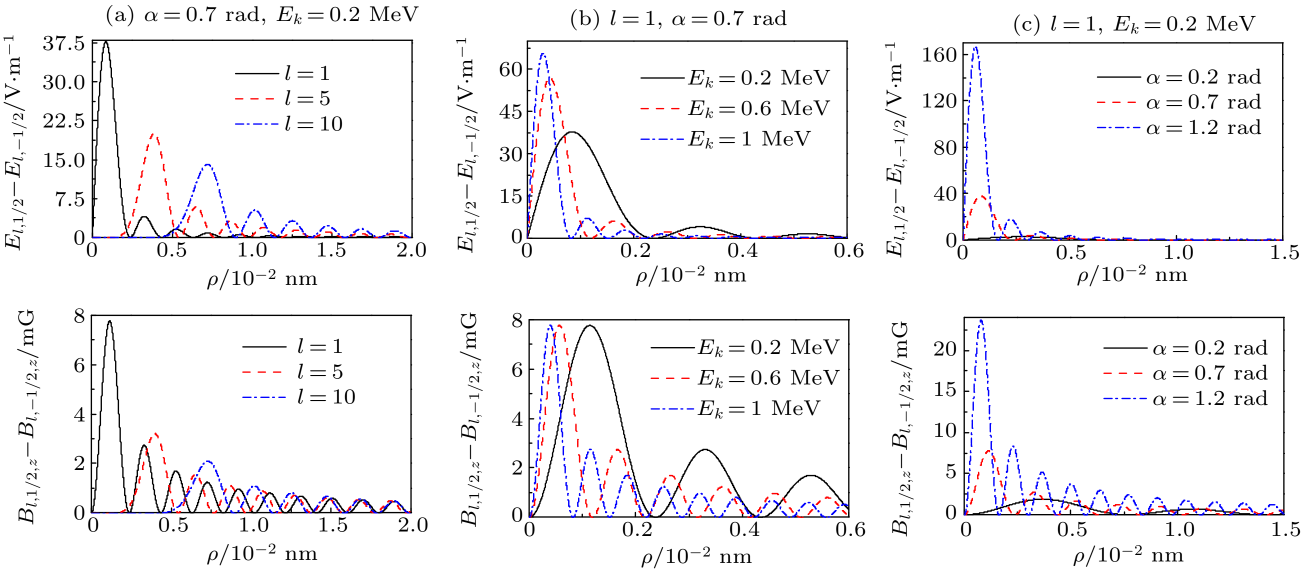

Fig. 1. The differences of electric fields (left) and magnetic fields (right) of the REVB between the spin-up ($s = 1/2$) and spin-down ($s = -1/2$) states, $E_{l,1/2}(\rho) - E_{l,-1/2}(\rho)$ and $B_{l,1/2,z}(\rho) - B_{l,-1/2,z}(\rho)$, vs. ρ for three cases: (a) $l = 1$, 5, 10, $\alpha = 0.7$ rad, $E_k = 0.2$ MeV; (b) $E_k = 0.2$ MeV, 0.6 MeV, 1 MeV, $l = 1$, $\alpha = 0.7$ rad; (c) $\alpha = 0.2$ rad, 0.7 rad, 1.2 rad, $l = 1$, $E_k = 0.2$ MeV.

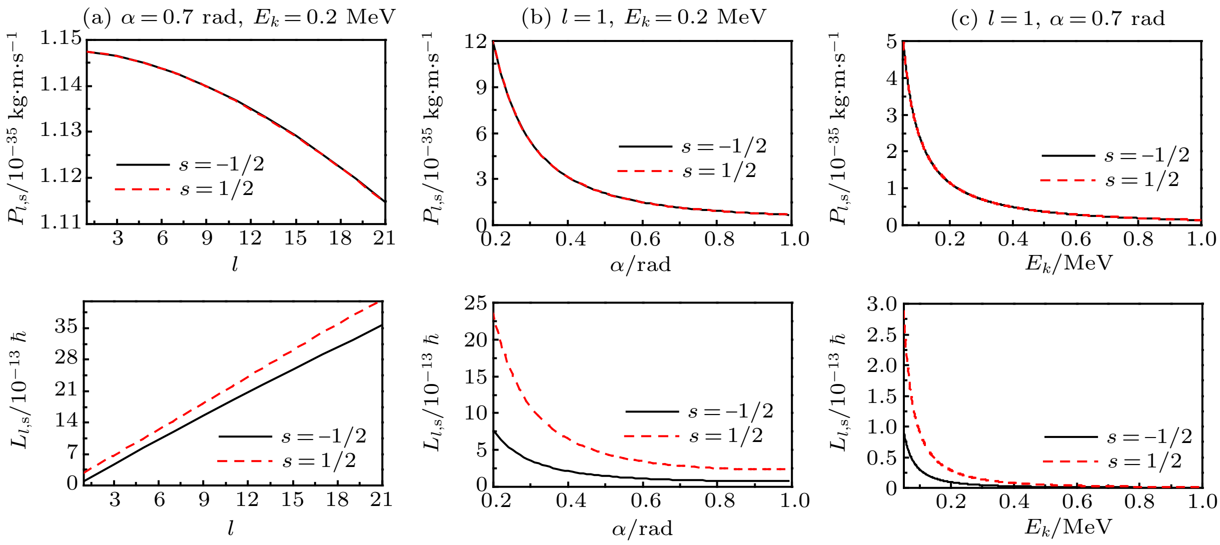

Fig. 2. Linear momentum and angular momentum of the electromagnetic field of the relativistic EVBs, respectively, in spin-up ($s = 1/2$) and spin-down ($s = -1/2$) states vs (a) topological charge l for opening angle $\alpha = 0.7$ rad, kinetic energy $E_k = 0.2$ MeV, (b) opening angle α for $l = 1$ rad, $E_k = 0.2$ MeV, (c) kinetic energy E_k for $\alpha = 0.7$ rad, and $l = 1$.

4. Conclusions

In summary, we have deduced the formula for electromagnetic field generated by a truncated relativistic electron vortex beam, and show that the electric and magnetic fields are vortex fields in the transverse plane. Moreover, they are dependent on spin-state and vortex topological charge. The electric field includes two parts, i.e., the relativistic effect and the nonrelativistic effect. The magnetic field has azimuth component related to the nonrelativistic quantum effect and part longitudinal component corresponding to a relativistic effect. We further study the mechanical properties (the linear momentum and angular momentum) of the electromagnetic field. We find that the linear momentum of the vortex electromagnetic field are weakly dependent on the spin state of the electron vortex beam, while its angular momentum is remarkably dependent on the spin state. The topological charge of the electron vortex beam has strong influence on the linear and angular momentums, i.e., the linear momentum is reduced with the increasing topological charge, while the angular momentum linearly increases with the topological charge. The electromagnetic field and its linear/angular momentum can be controlled by the opening angle and kinetic energy.

The electromagnetic field of the REVB can be used to explore the relativistic quantum effect, since the difference between electromagnetic fields in spin-up and spin-down states is purely related to spin–orbit coupling effect in the REVB. Moreover, the electromagnetic and mechanical properties of the REVB could be useful for studying the interaction between the REVB and materials.

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