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Detection and quantification of entanglement with measurement-device-independent and universal entanglement witness*

Zhi-Jin Ke(柯芝锦)[1,2], Yi-Tao Wang(王轶韬)[1,2], Shang Yu(俞上)[1,2], Wei Liu(刘伟)[1,2], Yu Meng(孟雨)[1,2], Zhi-Peng Li(李志鹏)[1,2], Hang Wang(汪航)[1,2], Qiang Li(李强)[1,2], Jin-Shi Xu(许金时)[1,2], Ya Xiao(肖芽)[3], Jian-Shun Tang(唐建顺)[1,2,†], Chuan-Feng Li(李传锋)[1,2,‡], and Guang-Can Guo(郭光灿)[1,2]

[1] *CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei* 230026, *China*
[2] *CAS Center For Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei* 230026, *China*
[3] *Department of Physics, Ocean University of China, Qingdao* 266100, *China*

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Entanglement is the key resource in quantum information processing, and an entanglement witness (EW) is designed to detect whether a quantum system has any entanglement. However, prior knowledge of the target states should be known first to design a suitable EW, which weakens this method. Nevertheless, a recent theory shows that it is possible to design a universal entanglement witness (UEW) to detect negative-partial-transpose (NPT) entanglement in unknown bipartite states with measurement-device-independent (MDI) characteristic. The outcome of a UEW can also be upgraded to be an entanglement measure. In this study, we experimentally design and realize an MDI UEW for two-qubit entangled states. All of the tested states are well-detected without any prior knowledge. We also show that it is able to quantify entanglement by comparing it with concurrence estimated through state tomography. The relation between them is also revealed. The entire experimental framework ensures that the UEW is MDI.

Keywords: entanglement witness, entanglement detection, entanglement quantification, measurement-device-independent

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1. Introduction

Entanglement plays a key role in quantum information processing, from quantum computation[1] to quantum teleportation.[2] In particular, entanglement can provide more security for quantum cryptography.[3] In such applications, we need to quantify the entanglement of a quantum system. There have been several well-known entanglement measures, such as concurrence,[4] entanglement of formation,[5] negativity[6–8] and random robustness.[9] All of these require estimating a large number of density matrix elements, which is difficult work for bipartite and multipartite quantum states. If we only need to verify if a state has entanglement or not, the entanglement witness (EW) is a good choice. By using a proper EW operator, one can quickly receive a "yes" or "no" result, which is much easier to achieve.

However, several studies show that conventional EW (CEW) is measurement-device-dependent, because errors and misalignments of the measurement devices can lead to incorrect estimations of the quantities, incurring erroneous conclusions.[10–12] Moreover, another problem of CEW is that[13] one should have some information about the tested states first, then the proper EW operator to detect the state can be found or designed, otherwise one has to try the EW operators randomly.

The recent work by Shahandeh *et al.*[14] has introduced a measurement-device-independent (MDI) and universal entanglement witness (UEW) theory based on semiquantum game framework,[15] which can be used to deal with the drawbacks of CEW. A schematic diagram of the game framework is shown in Fig. 1. Alice and Bob (players) share state $\hat{\rho}_{AB}$. Charlie (referee) prepares quantum question states $\{\hat{\tau}_i^{A_0}\}$ and $\{\hat{\omega}_i^{B_0}\}$ and sends to Alice and Bob respectively during each round. Alice and Bob, respectively, measure the question state and their own qubit, and then send their measurement outcomes x, y to Charlie. Local operations and classical communication (LOCC) are allowed during the game. Charlie then calculates the reward function from their classical responses. If the outcome of the reward function is larger than 0, Alice and Bob win the game and get the payoff (equal to the outcome) from Charlie. Thus the aim of Alice and Bob is to

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†Corresponding author. E-mail: tjs@ustc.edu.cn
‡Corresponding author. E-mail: cfli@ustc.edu.cn

make the payoff as large as they can. In this game, whether the payoff will be larger than 0 is connected to whether there is entanglement in two players' shared state. And how large the payoff can be is connected to how much entanglement in their shared state. As referee, Charlie should ensure that two players cannot win the game by cheating, i.e., they will never win the game if their shared state does not have entanglement.

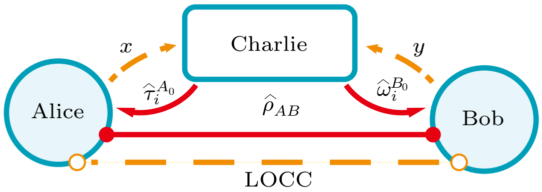

Fig. 1. The scheme of extremal semiquantum nonlocal game. Players (Alice and Bob) share state $\hat{\rho}_{AB}$, and then, they receive quantum questions $\hat{\tau}_i^{A_0}$, $\hat{\omega}_i^{B_0}$ from referee (Charlie). After performimg local POVM on their own part of shared state and the quantum question state, they return classical answers x and y to Charlie. LOCC is allowed in this game. If the shared state is entangled, then Alice and Bob can always win this game (achieve payoff value larger than 0) as long as they perform suitable measurements.

To prevent two players from cheating, there are two requirements for Charlie's quantum question states $\{\hat{\tau}_i^{A_0}\}$ and $\{\hat{\omega}_i^{B_0}\}$. First, the question states should be nonorthogonal to each other. Second, they should be able to form an extremal decomposable entanglement witness operator $\hat{W}_{\rm sq}^{\rm de}$.[16] Then, only if Alice and Bob share negative-partial-transpose (NPT)[17] entangled states, will they possibly be able to achieve payoff larger than 0. Otherwise, no matter what measurements they perform, they will never win, i.e., there will never be positive outcomes. This is the MDI characteristic, which prevents Alice and Bob from cheating. In other words, two players cannot get positive rewards by cheating (e.g., performing some special measurements) if they do not share a pair of entangled states.

To achieve the maximum positive outcome, two players should also try to find the optimal positive operator-valued measure (POVM). It should be stressed here that, the maximum payoff that the players possibly achieve is proportional to the amount of NPT entanglement in the shared state. Therefore, if we replace the shared state with unknown states that need to be witnessed, upon this semiquantum game framework, we are able to design an MDI and universal entanglement witness (MDI-UEW).

In this paper, we present and experimentally demonstrate an MDI-UEW that has the ability to detect and quantify entanglement at the same time. We first show that NPT entangled states should be classified into different collections based on their optimal EW operators. Without loss of generality, we choose entangled states from four representative collections (including both pure and mixed states) as examples to explain the MDI-UEW in detail. Our results demonstrate the universality of MDI-UEW in witnessing NPT entanglement of a low-dimensional bipartite system and verify the ability of its outcome to quantify entanglement, which is comparable to concurrence. Furthermore, we perform a theoretical analysis and a numerical simulation to reveal that the outcome of MDI-UEW for an entangled state is less than or equal to its concurrence.

2. Theoretical framework

2.1. Limitation of CEW and classification of NPT entangled state

In CEW theory, the canonical way to design an effective EW operator for a given entangled state is based on positive-partial-transpose (PPT) criterion,[18] which is as follow:[16]

1. Take the partial transpose $\hat{\rho}^{\mathrm{T}_B}$ of state $\hat{\rho}$ (T_B denotes that the second qubit B takes transposition);

2. Obtain the eigenstate $|\psi\rangle$ of $\hat{\rho}^{\mathrm{T}_B}$, whose eigenvalue is negative;

3. $W = -2|\psi\rangle\langle\psi|^{\mathrm{T}_B}$ is the desired optimal EW operator.

In this paper, we change the sign of the EW operator by multiplying it by -1 for the sake of consistency. Then we get $\mathrm{Tr}(W\hat{\rho}) > 0$ for entangled states (corresponding to the definition of payoff in the semiquantum game) and $\mathrm{Tr}(W\hat{\rho}) \leq 0$ for separable states.

Let us take Bell state $|\Psi^-\rangle$ as an example; the eigenstate with negative eigenvalue of its partial transpose is $|\Phi^+\rangle$, then the corresponding EW is $\hat{W} = -2|\Phi^+\rangle\langle\Phi^+|^{\mathrm{T}_B}$. Simply check the result of $\mathrm{Tr}(\hat{W}|\Psi^-\rangle\langle\Psi^-|)$ and it shows that it is larger than 0. We can know that $\hat{W}$ is the effective EW operator. However, if we replace the state with the other three Bell states, e.g., $|\Psi^+\rangle$, $\mathrm{Tr}(\hat{W}|\Psi^+\rangle\langle\Psi^+|)$ is less than 0, that means $|\Psi^+\rangle$ is judged as separable state by $\hat{W}$. Becasue of this limitation of CEW, we have to divide the NPT entangled states into different collections.

Definition Given an entangled state $\hat{\rho}_{AB}$, we say that it is in the collection $C_{|\psi\rangle}$ if and only if the eigenvector with negative eigenvalue of its partial transposition $\hat{\rho}_{AB}^{\mathrm{T}_B}$ is $|\psi\rangle$, where $|\psi\rangle$ is an entangled state.

Then, we should know which collection a state is in before we choose an EW operator. The collection information is what we call prior information of a state in this work.

In our experiment, we will choose input states from four representative collections. According to CEW theory, it is impossible to pick out all entangled states by just one EW operator. In previous experiments on CEW, such as Refs. [19,20], though there are lots of input states for testing, actually all the states belong to one collection.

2.2. MDI-UEW

In the semiquantum game scheme, Charlie's reward function is

$$R_{\rm NPT}^{\rm MDI}\left(\hat{\rho}_{AB};\hat{W}_{\rm sq}^{\rm de}\right) = \max_{\hat{Z}^{\tilde{A}\tilde{B}}} \sum_i \beta_i \bar{R}\left(\hat{\rho}_{AB}, \hat{Z}^{\tilde{A}\tilde{B}} | \hat{\tau}_i^{A_0}, \hat{\omega}_i^{B_0}\right), \quad (1)$$

where

$$\hat{W}_{\rm sq}^{\rm de} = \sum_i \beta_i \hat{\tau}_i^{A_0{\rm T}} \otimes \hat{\omega}_i^{B_0{\rm T}} \tag{2}$$

is an arbitrary extremal decomposable EW operator that is decomposed into a combinations of quantum question states (T denotes the transposition operation) with coefficient $\{\beta_i\}$ and

$$\bar{R}\left(\hat{\rho}_{AB}, \hat{Z}^{\tilde{A}\tilde{B}} | \hat{\tau}_i^{A_0}, \hat{\omega}_i^{B_0}\right) = {\rm Tr}\left[\hat{Z}^{\tilde{A}\tilde{B}}\left(\hat{\tau}_i^{A_0} \otimes \hat{\rho}_{AB} \otimes \hat{\omega}_i^{B_0}\right)\right] \tag{3}$$

is the conditional probability. $\bar{R}$ equals to the expectation value of POVM $\hat{Z}^{\tilde{A}\tilde{B}}$ when they receive question states $\hat{\tau}_i^{A_0}$ and $\hat{\omega}_i^{B_0}$ respectively. $\tilde{A}$ ($\tilde{B}$) is a joint Hilbert space consisting of AA_0 (BB_0).

The $R_{\rm NPT}^{\rm MDI}\left(\hat{\rho}_{AB}; \hat{W}_{\rm sq}^{\rm de}\right)$ is the maximum among all possible combinations of $\bar{R}$, and measurement basis $\hat{Z}^{\tilde{A}\tilde{B}}$ decides whether this maximum can be achieved or not. The coefficient β_i is decided by the EW operator. The detailed calculations of β_i are in Appendix A.

Equation (1) gives an MDI-UEW for NPT entanglement (reference [21] provides detailed proof for MDI). Only one EW operator (that is equivalent to the combination of quantum question states) is used by Charlie in this scheme, regardless of what states Alice and Bob possess. As long as they perform an optimal measurement basis ($\hat{Z}_{\rm opt}^{\tilde{A}\tilde{B}}$), they always obtain $R_{\rm NPT}^{\rm MDI}\left(\hat{\sigma}_{AB}; \hat{W}_{\rm sq}^{\rm de}\right) = 0$ for any separable state $\hat{\sigma}_{AB}$ and $R_{\rm NPT}^{\rm MDI}\left(\hat{\rho}_{AB}; \hat{W}_{\rm sq}^{\rm de}\right) > 0$ for any NPT entangled state $\hat{\rho}_{AB}$.[14] No assumptions should Charlie make here that specific measurements $\hat{Z}^{\tilde{A}\tilde{B}}$ have been correctly performed by Alice and Bob according to MDI characteristic, because there are two necessary conditions for Eq. (1) to have positive outcome. One is that the two players perform the optimal measurements, and the other is that the states they share have NPT entanglement.

2.3. Entanglement-witness measure

The maximal measurement outcome of MDI-UEW is defined as EWM, that is,

$$E(\hat{\rho}) = R_{\rm NPT}^{\rm MDI}(\hat{\rho}; \hat{W}_0), \tag{4}$$

where $\hat{W}_{\rm sq}^{\rm de}$ in Eq. (1) is specified as $\hat{W}_0 = -2|\Phi^+\rangle\langle\Phi^+|^{{\rm T}_B}$. $E(\hat{\rho})$ is convex and has a range from 0 to 1. $E(\hat{\rho}) = 0$ for any separable state, while $E(\hat{\rho}) \in (0, 1]$ for any NPT entangled state. Thus, $E(\hat{\rho})$ can be used as an entanglement measure.[14] $\hat{W}_0$ is the best choice for definition of $E(\hat{\rho})$, which significantly simplifies our experiment. Besides, $E(\hat{\rho})$ is equal to the negativity of the state in two-qubit system.[22,23] Details can be found in discussion section.

3. Experimental setup

Our experimental setup for MDI-UEW is shown in Fig. 2, where four photons are used. In the source part, two 2-mm β-barium-borate (BBO) crystals produce the original entangled photon pairs by type-I spontaneous parametric down conversion (SPDC). The states of the photon pairs have the form of $\frac{1}{\sqrt{2}}\left(|{\rm HV}\rangle + {\rm e}^{{\rm i}\phi}|{\rm VV}\rangle\right)$.

Then, with some probability, two photon pairs are respectively distributed into paths 2 and 3 and paths 1 and 4 with two non-polarizing beam splitters, NPBS1 and NPBS2. The photon pairs going through the NPBSes simultaneously to paths 2 and 3 are used as target bipartite states $\hat{\rho}_{AB}$. We name them photon pairs 2 and 3. While photon pairs are used to prepare the question states $\hat{\tau}_i^{A_0}$ and $\hat{\omega}_i^{B_0}$ if they are reflected by the NPBSes simultaneously, we name them photon pairs 1 and 4. Two polarizers make them separable because only one component of the original state $|{\rm HH}\rangle$ can pass through the polarizers.

In this experiment, the tested (input) states are chosen from four representative collections of entanglement, including both pure states and mixed states. Using phase compensators (PCs), time delay plates (TDPs), and half-wave plates (HWPs) in path 2 and path 3, we prepare all Bell states and several Bell diagonal states (mixture of any two Bell states in a different proportion).[24]

By carefully selecting the phase of PC1, $|\Phi^+\rangle$ or $|\Phi^-\rangle$ are prepared. When HWP2 is set at an angle of 45° and HWP1 is set at an angle of 0°, we can prepare state $|\Psi^+\rangle$ and $|\Psi^-\rangle$ as well.

NPBS3 and NPBS4 are used to prepare the second Bell state component of a Bell diagonal state. HWP3 is used to tune the polarization of photon 3 while PC2 is used to tune the relative phase between different polarizations. TDP is used to make the transmitting part and reflecting part of NPBS3 decoherent.

States $\cos\theta|{\rm HH}\rangle + \sin\theta|{\rm VV}\rangle$ are prepared from $|\Phi^+\rangle$ with the help of beam displacers BD1 and BD2. We change the ratio between H-polarization and V-polarization of photon 3 by adjusting the angle of HWP4 and HWP5. Then redundant amount of H- or V-polarization will be discarded when passing through BD2 and finally we obtain the desired states with $\theta \in [0, \pi/2]$.

The HWP-QWP (quarter-wave plate) combinations in path 1 and path 4 are used to prepare the question states in the collection of $|\pm x\rangle, |\pm y\rangle, |\pm z\rangle$ (eigenstates of Pauli matrices). There are a total of 12 pairs of $\{\hat{\tau}_i^{A_0}, \hat{\omega}_i^{B_0}\}$ (see Appendix A).

A complete set of POVM $\{\hat{Z}^{\tilde{A}\tilde{B}}\}$, used in our experiment, can be expanded into combinations of $\sigma_x^{\otimes 4}$, $\sigma_y^{\otimes 4}$, $\sigma_z^{\otimes 4}$, and $I^{\otimes 4}$ (the Pauli matrix and identity matrix). Detailed information is provided in Appendix B. Four QWP-HWP-PBS (polarizing beam splitter) combinations in the measurement part are sufficient to perform these measurements. By carefully adjusting the time delay between photon pairs 2 and 3 and pairs 1 and 4, we collect four-photon events and calculate the rewards for Alice and Bob according to Eq. (1).

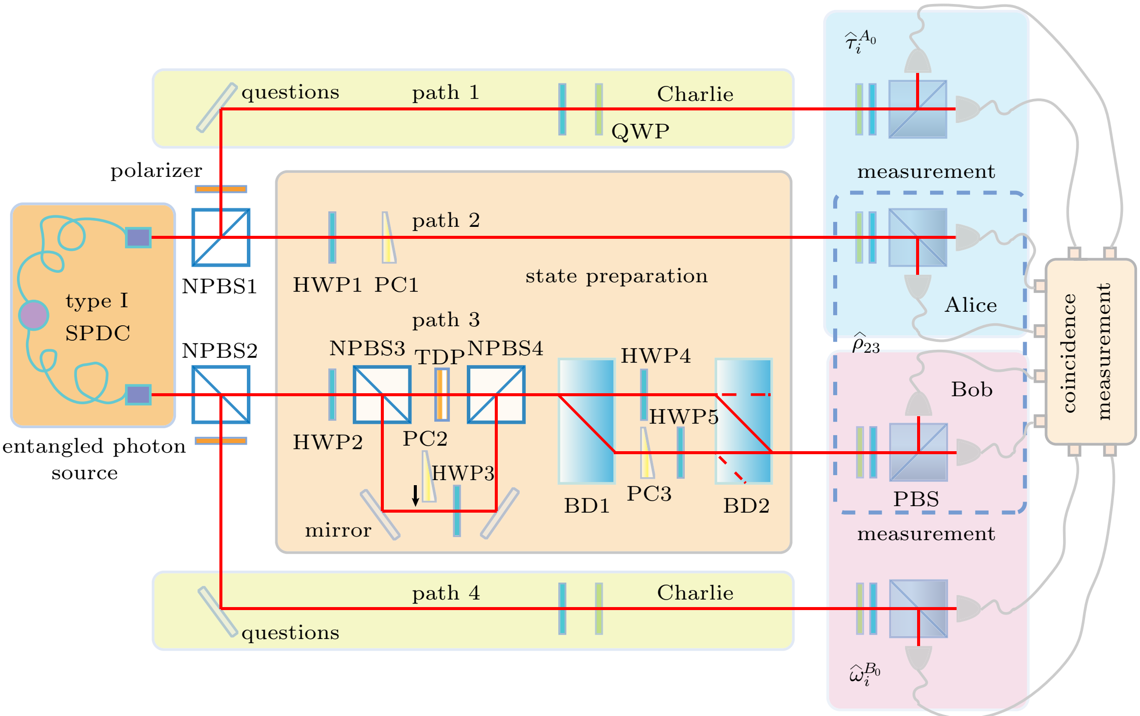

Fig. 2. Experimental setup for MDI-UEW. The original entangled photon pairs are generated by type-I SPDC in two 2-mm BBO crystals with the form of $\frac{1}{\sqrt{2}}\left(|\mathrm{HV}\rangle + \mathrm{e}^{\mathrm{i}\phi}|\mathrm{VV}\rangle\right)$. The reflected parts of non-polarizing beam splitters NPBS1 and NPBS2 are used as question states.The transmitted parts of NPBS1 and NPBS2 are used to prepare $\hat{\rho}_{AB}$. By selecting a suitable phase compensator PC1, $|\Phi^+\rangle$ or $|\Phi^-\rangle$ can be prepared. When half wave-plate HWP2 is set at an angle of 45°, and HWP1 is set at an angle of 0°, $|\Psi^+\rangle$, $|\Psi^-\rangle$ can be prepared. The bypath of NPBS3 is used to prepare the second Bell state component in the Bell diagonal state. HWP4 and HWP5 are used to change the ratio between H-polarization and V-polarization in $|\Phi^+\rangle$, then a redundant amount of H- or V-polarization will be discarded when passing through beam displacer BD2. Then states $\cos\theta|\mathrm{HH}\rangle + \sin\theta|\mathrm{VV}\rangle$ are prepared. Wave-plate combinations in path 1 and path 4 are used to rotate the polarizations to encode photons 1 and 4 to the desired question states, $\hat{\tau}_i^{A_0}$ and $\hat{\omega}_i^{B_0}$. The measurement part consists of four QWP-HWP-PBS combinations, where QWP and PBS denoting quarter wave-plate and polarizing beam splitter, respectively. All the four photons are separately detected by single-photon avalanche diodes, but the four-photon events are coincidently counted together and analyzed by a picosecond time analyzer with well-tuned delays.

4. Analysis of experimental results

The experimental results are shown in Fig. 3 and Table 1. The error bars represent one standard deviation, which is derived by using the Monte Carlo method. There are two comparisons in Fig. 3. One is the entanglement-detection ability between CEW and MDI-UEW, and the other is the entanglement-quantification ability between entanglement-witness measure $E(\hat{\rho})$ and concurrence. Here, concurrence is used as a reference and to make a uniform standard to judge whether a state is entangled or not.

In Fig. 3(a), four Bell states are tested by both CEW and MDI-UEW. MDI-UEW successfully witnesses the entanglement in these states (orange bars) because their values are all larger than 0, while CEW only witnesses the entanglement in state $|\Psi^-\rangle$ (green bars) because only one bar is larger than 0.

The EW operator in our experiment is $W_0 = -2|\Phi^+\rangle\langle\Phi^+|^{\mathrm{T}_B}$, which is only designed for state $|\Psi^-\rangle$ and its related collections $C_{|\Phi^+\rangle}$ in CEW. According to the analysis in Subsection 2.1, four Bell states come from four different collections, thus one EW operator is not sufficient to correctly detect entanglement in them based on CEW theory. Therefore, the other three Bell states are inevitably recognized as seperable states, where their outcomes are less than 0. The design of an MDI-UEW is based on a specific EW operator, but it does not depend on it. Any decomposable extremal EW operator is available. Figure 3(a) shows that entangled states in different collections can all be detected by an MDI-UEW. This demonstrates the universality of MDI-UEW.

In Fig. 3(b), the tested states are $\cos\theta|\mathrm{HH}\rangle + \sin\theta|\mathrm{VV}\rangle$, where $\theta \in [0, \pi/2]$. They are entangled states and in collection $C_{|\Psi^-\rangle}$ when $\theta \neq 0$ or $\pi/2$. The EW operator W_0 is unable to detect their entanglement in CEW theory. The amount of entanglement in this series of states varies according to the value of θ. The black square points are the experimental values of their concurrence and the black solid line is the theoretical value. Concurrence is used here as reference. The blue triangle points are the experimental results of CEW. We can find that, no matter how much entanglement is in the states, it always gives out -1; in other words, these states are all recognized as separable states by CEW, which is incorrect. The blue solid line is the theoretical value. The red circular points are experimental results of MDI-UEW and the red solid line shows their theoretical values. When θ equals 0 or $\pi/2$, the corresponding state is $|\mathrm{HH}\rangle$ or $|\mathrm{VV}\rangle$. They are separable states, thus the values of them are 0, which is a correct detection. With the increase of θ, the states become entangled and the MDI-UEW gives out values larger than 0 for them. Moreover, these values are associated with the amount of entanglement in the states (referred to concurrence), which indicates that the outcome of MDI-UEW is possible to be used as a kind of entanglement measure.

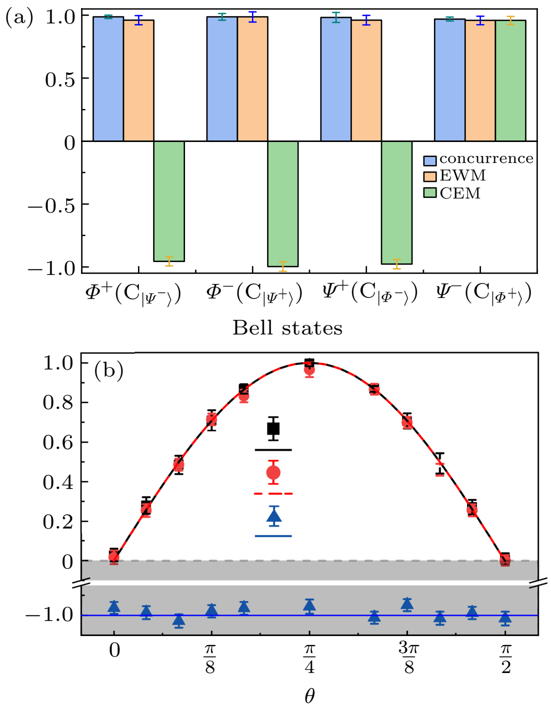

Fig. 3. Comparison between the MDI-UEW and CEW on entanglement detection ability and comparison between the EWM and concurrence on entanglement quantification ability. CEW is calculated by $\mathrm{Tr}(\hat{W}_0\hat{\rho})$. EWM represents $E(\hat{\rho})$, which is the witness outcome of MDI-UEW. (a) Experimental results for the Bell states. The related collections of Bell states are labelled in the parentheses. The orange bars represent the experimental results of the EWM, the green bars represent the CEW, and the blue bars represent the estimated concurrence. (b) Experimental results of $\cos\theta|HH\rangle + \sin\theta|VV\rangle$ ($\theta \in [0, \pi/2]$). The theoretical values of the EWM ($\mathrm{EWM_{th}}$, red dashed line) and concurrence ($\mathrm{Con._{th}}$, black solid line) are both $\sin 2\theta$, while the theoretical value of CEW (blue solid line) is -1. Red circular points represent the experimental results of EWM ($\mathrm{EWM_{exp}}$), whereas the black square points represent the estimated concurrence ($\mathrm{Con._{exp}}$). Blue triangle points represent the results of CEW without any ability for entanglement detection or quantification for the target states.

Figure 3(b) also shows an interesting fact that, once the EW operator is mismatched to the states in CEW, the expect value will always be -1, no matter whether the states are maximally entangled, partially entangled, or even separable. In other words, negative value of CEW provides no information.

MDI-UEW also works for mixed states. Table 1 shows the experimental results for the Bell diagonal states. The EW operator W_0 only witnesses entangled states in collections $C_{|\Phi^+\rangle}$ according to CEW theory. State 1 is in this collection, and thus it can be correctly detected. However, state 2 and state 3 are in the collections $C_{|\Psi^-\rangle}$ and $C_{|\Psi^+\rangle}$, respectively. Therefore, it is not surprising that the outcomes of CEW for these two are negative and they are judged as separable. Meanwhile, MDI-UEW successfully overcomes this problem, because its outcomes for these three states are positive.

The analysis above has confirmed the advantage of MDI-UEW in detecting entanglement over CEW. Next we turn to the comparison between EWM $E(\hat{\rho})$ and concurrence in the entanglement-quantification ability. Concurrence is a well-defined entanglement measure, and if EWM $E(\hat{\rho})$ can provide similar quantification for entangled states, this can guarantee the ability of EWM to be a kind of entanglement measure.

Concurrences of states in Fig. 3 are achieved by performing full state tomography. The blue bars in Fig. 3(a) are the experimental estimated concurrences of Bell states, which are maximally entangled states with theoretical values as 1. The orange bars are the experimental results of EWM, and all of them have reached the theoretical value 1. In Fig. 3(b), the theoretical values of both EWM and concurrence are $\sin 2\theta$, and the experimental results achieve perfect agreement with the theoretical predictions. The states in Table 1 are mixed states. The concurrence of them is $|2p-1|$ in theory, so does the EWM. Summarizing the experimental results, the ability of EWM as a kind of entanglement measure has been confirmed. However, we should stress that, the evidence here is not sufficient to claim that EWM is the same as concurrence.

Table 1. Experimental results for the Bell diagonal states.

States	p	$\mathrm{EWM_{th.}}$	$\mathrm{EWM_{exp.}}$	$\mathrm{CEW_{th.}}$	$\mathrm{CEW_{exp.}}$
State 1	0.7	0.4	0.393 ± 0.029	0.4	0.393 ± 0.033
State 2	0.55	0.1	0.076 ± 0.031	-1	-0.980 ± 0.020
State 3	0.4	0.2	0.173 ± 0.031	-0.2	-0.211 ± 0.027

$\text{State 1} = p|\Psi^-\rangle\langle\Psi^-| + (1-p)|\Phi^+\rangle\langle\Phi^+|,$

$\text{State 2} = p|\Phi^+\rangle\langle\Phi^+| + (1-p)|\Phi^-\rangle\langle\Phi^-|,$

$\text{State 3} = p|\Psi^-\rangle\langle\Psi^-| + (1-p)|\Phi^-\rangle\langle\Phi^-|.$

5. Discussion

The values of the concurrence and EWM are equal for the tested states, however, they are actually two different entanglement measures, and EWM $E(\hat{\rho})$ of a state is less than or equal to its concurrence. Negativity is the bridge for these two measures, which is also an entanglement measure.

According to Theorem 23 and Lemma 6 in Ref. [23], the definition of negativity of a two-qubit state is as follows:

$$N(\hat{\rho}) = 2|\lambda_0| = -2\lambda_0, \tag{5}$$

where λ_0 is the minimum negative eigenvalue of $\hat{\rho}^{\mathrm{T}_B}$. At the same time, EWM reads

$$E(\hat{\rho}) = R^{\mathrm{MDI}}_{\mathrm{NPT}}(\hat{\rho}; \hat{W}_0) = -2\lambda_0. \tag{6}$$

The detailed calculation is in Appendix D.

Comparing Eq. (5) with Eq. (6), it is obvious that the EWM is equivalent to the negativity of the states in a two-qubit system.

Then, it has been proved that the negativity of a state is less than or equal to its concurrence. Therefore, the same is true for the EWM in a two-qubit system. And the negativity/EWM will equal concurrence if the eigenstate with negative eigenvalue of $\hat{\rho}^{\mathrm{T}_B}$ is a Bell state (up to local unitary transformation) (see Theorem 25 in Ref. [23]).

The states we used in our experiment are from the four collections $C_{|\Psi^-\rangle}$, $C_{|\Psi^+\rangle}$, $C_{|\Phi^-\rangle}$, and $C_{|\Phi^+\rangle}$, thus their eigenstates with negative eigenvalues are Bell states. Note that, the related eigenstates with negative eigenvalues are labelled at the subscript of each collection. For example, any state in collection $C_{|\Psi^-\rangle}$ has common eigenstate with negative eigenvalue $|\Psi^-\rangle$. Therefore, the EWM of the input states is theoretically equal to the concurrence as shown in Fig. 3. In other cases, EWM is always less than concurrence.

Figure 4 shows numerical simulation of EWM and concurrence for a series of asymmetric states (which are useful in one-way steering[25,26])

$$\hat{\rho}_{\mathrm{asym.}} = p|\psi\rangle_{AB}\langle\psi| + (1-p)\hat{\rho}_A \otimes \frac{I_B}{2}, \tag{7}$$

where $|\psi\rangle_{AB} = \cos\theta|\mathrm{HH}\rangle + \sin\theta|\mathrm{VV}\rangle$ and $\hat{\rho}_A$ is reduced density matrix of $|\psi\rangle_{AB}\langle\psi|$. $p \in [0,1]$ while $\theta \in [0,\pi]$. These states are separable when $p \leq 1/3$ or $\theta = \pi/2$. Otherwise, they are entangled. Generally speaking, for entangled states in Eq. (7), their eigenstates with negative eigenvalues are not Bell states, therefore, the EWMs of them are always less than concurrence. The inset of Fig. 4 shows that EWM is quite different from concurrence.

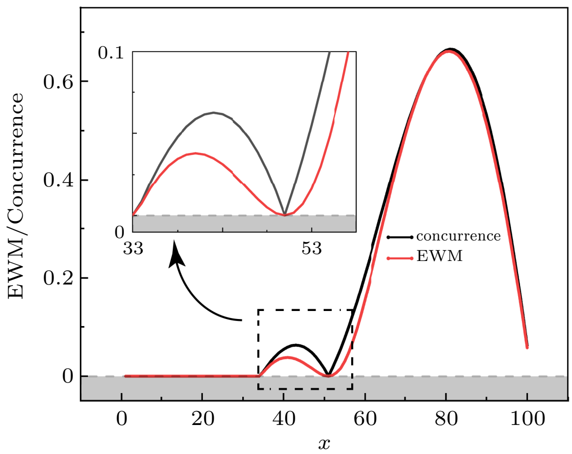

Fig. 4. Concurrence and EWM for a series of asymmetric states. States are defined in Eq. (7), the two parameters (p,θ) are united as x, which is $p = 0.01x$ and $\theta = 0.01x\pi$. EWM of a state is less than or equal to its concurrence.

6. Conclusion

We introduce MDI-UEW to solve the two drawbacks of CEW, which is measurement-device-dependent and needs prior information about the tested states. The MDI characteristic of MDI-UEW removes the trust from the measurement devices (including measurement bases and measurement agents). And the characteristic of universality enables us to witness NPT entanglement of unknown bipartite states with limited dimensions. Four representative collections of NPT entangled states are used as examples in our experiment, and MDI-UEW successfully witnesses all of them while CEW is only able to detect one collection. Moreover, we find EWM $E(\hat{\rho})$ (the outcome of MDI-UEW) can directly quantify the amount of entanglement. This gives MDI-UEW another advantage over CEW. Concurrence of the target states is estimated as a reference in our experiment. We have demonstrated that EWM has sufficient ability to quantify entanglement comparable to concurrence and theoretically prove that EWM is always less than or equal to concurrence, followed with a numerical demonstration for a series of asymmetric states.

Appendix A: Entanglement witness operator

The EW operator used in the experiment is $\hat{W}_0 = -2|\Phi^+\rangle\langle\Phi^+|^{\mathrm{T}_B}$. According to Ref. [21], one possible expansion of $\hat{W}_0$ is as below:

1. $\beta_{s,t} = -\delta_{s_2,t_2}\frac{3\delta_{s_1,t_1}-1}{3}$, where $s = (s_1,s_2)$ and $t = (t_1,t_2)$, with $s_1,t_1 = 0,1$ and $s_2,t_2 = 1,\ldots,3$; $\delta_{i,j}$ is the Kronecker delta;

2. $\tau_s = \frac{\mathbb{I}+(-1)^{s_1}\sigma_{s_2}}{2}$, $\omega_t = \frac{\mathbb{I}+(-1)^{t_1}\sigma_{t_2}}{2}$.

Then, $\hat{W}_0 = \sum_{s,t}\beta_{s,t}\hat{\tau}_s^{A_0\mathrm{T}} \otimes \hat{\omega}_t^{B_0\mathrm{T}}$ holds. According to the values of s and t, only 12 combinations of them give out non-zero $\beta_{s,t}$.

Appendix B: Measurement basis

If the EW operator is $\hat{W}_0 = -2|\Phi^+\rangle\langle\Phi^+|^{\mathrm{T}_B}$ and the target state is in collection $C_{|\Phi^+\rangle}$, the measurement basis has the form below: $\hat{Z}^{\tilde{A}\tilde{B}} = |\Phi^+\rangle_{\tilde{A}}\langle\Phi^+| \otimes |\Phi^+\rangle_{\tilde{B}}\langle\Phi^+| + |\Phi^-\rangle_{\tilde{A}}\langle\Phi^-| \otimes |\Phi^-\rangle_{\tilde{B}}\langle\Phi^-| + |\Psi^+\rangle_{\tilde{A}}\langle\Psi^+| \otimes |\Psi^+\rangle_{\tilde{B}}\langle\Psi^+| + |\Psi^-\rangle_{\tilde{A}}\langle\Psi^-| \otimes |\Psi^-\rangle_{\tilde{B}}\langle\Psi^-|$. However, full Bell state measurements (BSMs) are impossible to be implemented with linear optical system.[27] Luckily, in our method, it does not matter to Charlie what measurement is being done and how it has been performed by Alice and Bob.[14] That means, BSM is not the necessary part in the scheme. Thus, we expand $\hat{Z}^{\tilde{A}\tilde{B}}$ into combinations of $\sigma_x^{\otimes 4}$, $\sigma_y^{\otimes 4}$, $\sigma_z^{\otimes 4}$, and $I^{\otimes 4}$, and obtain $\hat{Z}^{\tilde{A}\tilde{B}} = \frac{1}{4}\left(\sigma_x^{\otimes 4} + \sigma_y^{\otimes 4} + \sigma_z^{\otimes 4} + I^{\otimes 4}\right)$, which is able to be performed with linear optical system. The other three POVMs used in the experiment are $\frac{1}{4}\left(-\sigma_x^{\otimes 4} + \sigma_y^{\otimes 4} - \sigma_z^{\otimes 4} + I^{\otimes 4}\right)$, $\frac{1}{4}\left(\sigma_x^{\otimes 4} - \sigma_y^{\otimes 4} - \sigma_z^{\otimes 4} + I^{\otimes 4}\right)$, and $\frac{1}{4}\left(-\sigma_x^{\otimes 4} - \sigma_y^{\otimes 4} + \sigma_z^{\otimes 4} + I^{\otimes 4}\right)$, which are the optimal measurements for states in collections $C_{|\Psi^-\rangle}$, $C_{|\Psi^+\rangle}$, and $C_{|\Phi^-\rangle}$, respectively. These four bases form a complete set of POVM.

Appendix C: Classification of the Bell diagonal states

For state $p|\Phi^+\rangle\langle\Phi^+| + (1-p)|\Phi^-\rangle\langle\Phi^-|$, the eigenvalues of its partial transposition are $1/2$, $(1-2p)/2$, and $(-1+2p)/2$. When $p < 1/2$, $(-1+2p)/2$ is the negative eigenvalue, and its corresponding eigenvector is $|\Psi^+\rangle$. According to the classification rule of NPT entanglement described in the main text, the state is in collection $C_{|\Psi^+\rangle}$. When $p > 1/2$, the negative eigenvalue is $(1-2p)/2$ with the eigenvector $|\Psi^-\rangle$, which means that the state is in collection $C_{|\Psi^-\rangle}$. This classification rule is suitable for the other two Bell diagonal states in Table 1 of the main text.

Appendix D: The reason to choose $\hat{W}_0$

According to the canonical EW-design method mentioned in the main text, for each two-qubit NPT entangled states $\hat{\rho}$, suppose its optimal EW is $\hat{W}_{\rm opt} = -D|\psi\rangle\langle\psi|^{\rm T_B}$ ($D = 2$ for two-qubit system), then

$$\mathrm{Tr}(\hat{W}_{\rm opt}\hat{\rho}) = -D\lambda_0 = -2\lambda_0, \tag{D1}$$

where λ_0 is the minimum negative eigenvalue of $\hat{\rho}^{\rm T_B}$.

Suppose that Charlie prepares a game $J^{\rm de}_{\rm sq}$ corresponding to the Schmidt rank-2 decomposable extremal EW (EEW) $\hat{J}^{\rm de} = -2|\xi\rangle\langle\xi|^{\rm T_B}$, which is not matched to $\hat{\rho}$. Then

$$R^{\rm MDI}_{\rm NPT}(\hat{\rho};\hat{J}^{\rm de}) = -2q\lambda_0, \tag{D2}$$

where $q \in (0,1]$ is the success probability that $|\xi\rangle$ can be transformed into $|\psi\rangle$ via a stochastic LOCC (SLOCC) map.[28,29] Equation (D2) gives a unique NPT entanglement measure corresponding to game $J^{\rm de}_{\rm sq}$.

If q in Eq. (D2) can be a constant, then there is only one parameter in the equation, which will make the experiment much easier. Luckily, $\hat{W}_0$ can be transformed into any other Schmidt rank-2 decomposable EEW by using LOCC (deterministically) according to the proof of Theorem 2 in the supplemental material of Ref. [14]. Because the transformation is deterministic, q will equal 1, and we obtain

$$R^{\rm MDI}_{\rm NPT}(\hat{\rho};\hat{W}_0) = -2\lambda_0, \tag{D3}$$

There are three advantages if $\hat{W}_0$ is used as fixed EW operator in our experiment:

- Compared to Eq. (D2), equation (D3) has only one parameter determined by $\hat{\rho}$.
- Generally, Eq. (D3) $\geq$ Eq. (D2), because $0 < q \leq 1$. If q is too small, the outcome will be more difficult to be measured.
- LOCC is much easier to be performed than SLOCC.

Thus, we would like to choose $\hat{W}_0$ as the fixed operator to simplify our experiment.

Appendix E: Method to search optimal $\hat{Z}$

Given the witness operator $\hat{W}_0$, the optimal measurement basis has a general form of

$$\hat{Z}_{\rm general} = 4\times\begin{pmatrix}\cos^2\theta & 0 & 0 & \cos\theta\sin\theta\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \cos\theta\sin\theta & 0 & 0 & \sin^2\theta\end{pmatrix} \otimes |\text{Bell states}\rangle\langle\text{Bell states}|, \tag{E1}$$

where $\theta \in \left(0, \frac{\pi}{2}\right)$ (determined by the target states and it can be left unknown) and $|\text{Bell states}\rangle$ represents one of the four Bell states $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, or $|\Psi^-\rangle$. Therefore, there are four forms of $\hat{Z}_{\rm general}$, and they form a complete POVM set.

$\hat{Z}_{\rm general}$ can be decomposed into combinations of Pauli matrices as follows:

$$\begin{pmatrix}\cos^2\theta & 0 & 0 & \cos\theta\sin\theta\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \cos\theta\sin\theta & 0 & 0 & \sin^2\theta\end{pmatrix} = \frac{1}{16}\Big(\sin 2\theta\left(\sigma_x^{\otimes 2} - \sigma_y^{\otimes 2}\right) + \left(I^{\otimes 2} + \sigma_z^{\otimes 2}\right) + 2\left(\cos^2\theta - \frac{1}{2}\right)\left(\sigma_z\otimes I + I\otimes\sigma_z\right)\Big), \tag{E2}$$

$$|\Phi^+\rangle\langle\Phi^+| = \frac{1}{4}(I^{\otimes 2} + \sigma_x^{\otimes 2} - \sigma_y^{\otimes 2} + \sigma_z^{\otimes 2}). \tag{E3}$$

The other three Bell states have a form similar to $|\Phi^+\rangle$, the only difference is the sign of each term.

Here are the steps for players in the semiquantum nonlocal game to try every possible measurement basis for the unknown state:

- Perform the measurement basis of $\hat{Z}_{\rm general}$, e.g. $\sigma_x^{\otimes 2} \otimes \sigma_y^{\otimes 2}$, one by one;
- Record the coincidence counts of four $\hat{Z}_{\rm general}$;
- Send all the coincidence counts back to Charlie (referee).

For Charlie, the only thing to do is to calculate $R^{\rm MDI}_{\rm NPT}$ based on coincidence counts of those four $\hat{Z}_{\rm general}$ and find the maximum over θ in $(0,\pi/2)$ among them. The maximum is exactly the amount of NPT entanglement of the target state.

Note that, there is no need for the players to know the target states beforehand. Our method is equivalent to the process that the players transform Charlie's EW into another that is suitable for the target states through changing the measurement bases, but they actually do not know which EW they have acquired. It is the ergodic process of measurement bases that finds the suitable EW. In our method, this ergodic process only appears in the data-calculation step done by Charlie. This is practical in two-qubit system.

In our experiment, the tested entangled states are chosen from four representative collections. Because they have good symmetry, the θ in $\hat{Z}_{\mathrm{general}}$ for them equals $\pi/4$, and their optimal measurements can be expressed in a much simpler form presented in Section 3 in our manuscript.

References

[1] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[2] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett*. **70** 1895
[3] Ekert A K 1991 1991 *Phys. Rev. Lett.* **67** 661
[4] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
[5] Bennett C H, DiVincenzo D P, Smolin J A and Wootters W K 1996 *Phys. Rev. A* **54** 3824
[6] Zyczkowski K, Horodecki P, Sanpera A and Lewenstein M 1998 *Phys. Rev. A* **58** 883
[7] Vidal G and Werner R F 2002 *Phys. Rev. A* **65** 032314
[8] Nakano T, Piani M and Adesso G 2013 *Phys. Rev. A* **88** 012117
[9] Vidal G and Tarrach R 1999 *Phys. Rev. A* **59** 141
[10] van Enk S J, Lutkenhaus N and Kimble H J 2007 *Phys. Rev. A* **75** 052318
[11] Skwara P, Kampermann H, Kleinmann M and Bruss D 2007 *Phys. Rev. A* **76** 012312
[12] Moroder T, Guehne O, Beaudry N, Piani M and Luetkenhaus N 2010 *Phys. Rev. A* **81** 052342
[13] Seevinck M and Uffink J 2007 *Phys. Rev. A* **76** 042105
[14] Shahandeh F, Hall M J W and Ralph T C 2017 *Phys. Rev. Lett.* **118** 150505
[15] Chruscinski D and Sarbicki G 2014 *J. Phys. A: Math. Theor.* **47** 483001
[16] Chruscinski D and Sarbicki G 2014 *J. Phys. A: Math. Theor.* **47** 483001
[17] Horodecki M, Horodecki P and Horodecki R 1998 *Phys. Rev. Lett.* **80** 5239
[18] Horodecki M, Horodecki P and Horodecki R 1996 *Phys. Rev. A* **223** 1
[19] Xu P, Yuan X, Chen L K, Lu H, Yao X C, Ma X F, Chen Y A and Pan J W 2014 *Phys. Rev. Lett.* **112** 140506
[20] Verbanis E, Martin A, Rosset D, Lim C C W, Thew R T and Zbinden H 2016 *Phys. Rev. Lett.* **116** 190501
[21] Branciard C, Rosset D, Liang Y C and Gisin N 2013 *Phys. Rev. Lett.* **110** 060405
[22] Eisert J, Brandao F G S L and Audenaert K M R 2007 *New J. Phys.* **9** 46
[23] Verstraete F 2002 *A study of entanglement in quantum information theory*, Ph.D. Dissertation (Leuven: Katholieke Universiteit)
[24] Sun K, Ye X J, Xu J S, Xu X Y, Tang J S, Wu Y C, Chen J L, Li C F and Guo G C 2016 *Phys. Rev. Lett.* **116** 160404
[25] Bowles J, Hirsch F, Quintino M T and Brunner N 2016 *Phys. Rev. A* **93** 022121
[26] Xiao Y, Ye X J, Sun K, Xu J S, Li C F and Guo G C 2017 *Phys. Rev. Lett.* **118** 140404
[27] Vaidman L and Yoran N 1999 *Phys. Rev. A* **59** 116
[28] Nielsen M A 1999 *Phys. Rev. Lett.* **83** 436
[29] Vidal G 1999 *Phys. Rev. Lett.* **83** 1046