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Direct immune-SCIR public-opinion propagation model based on real-time online users*

Yun-Ming Wang(王运明)[1,2], Tian-Yi Guo(郭天一)[1,2,†], Wei-Dong Li(李卫东)[1,2,‡], and Bo Chen(陈波)[3]

[1] *School of Electrical and Information Engineering, Dalian Jiaotong University, Dalian* 116028, *China*
[2] *Liaoning Railway Logistics Network Engineering Technology Research Center, Dalian* 116028, *China*
[3] *College of Mechanical and Electronical Engineering, Lingnan Normal University, Zhanjiang* 524048, *China*

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Current public-opinion propagation research usually focused on closed network topologies without considering the fluctuation of the number of network users or the impact of social factors on propagation. Thus, it remains difficult to accurately describe the public-opinion propagation rules of social networks. In order to study the rules of public opinion spread on dynamic social networks, by analyzing the activity of social-network users and the regulatory role of relevant departments in the spread of public opinion, concepts of additional user and offline rates are introduced, and the direct immune-susceptible, contacted, infected, and refractory (DI-SCIR) public-opinion propagation model based on real-time online users is established. The interventional force of relevant departments, credibility of real information, and time of intervention are considered, and a public-opinion propagation control strategy based on direct immunity is proposed. The equilibrium point and the basic reproduction number of the model are theoretically analyzed to obtain boundary conditions for public-opinion propagation. Simulation results show that the new model can accurately reflect the propagation rules of public opinion. When the basic reproduction number is less than 1, public opinion will eventually disappear in the network. Social factors can significantly influence the time and scope of public opinion spread on social networks. By controlling social factors, relevant departments can analyze the rules of public opinion spread on social networks to suppress the propagate of negative public opinion and provide a powerful tool to ensure security and stability of society.

Keywords: public opinion propagation model, direct immunization, real-time online users, basic reproduction number

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1. Introduction

When analyzing online public opinion, it is useful to consider the social network as the carrier and the social event as the core. This model presents a collection of expressions, communications, and interactions comprising the majority of netizens' emotions, attitudes, opinions, and subsequent influences. Internet public opinion is highly subjective, and it is directly conveyed via the internet in various forms without verification or packaging by relevant government departments. At present, the number of social-network users has spanned into the hundreds of millions, and social networks have provided the means of spreading public sentiment. This spread has revealed characteristics of fast speed and wide range. Thus, whenever negative public opinion erupts on the network, it can pose a very serious challenge to well-being and security. Therefore, studying the mechanism of public-opinion propagation in social networks, constructing a more reasonable public-opinion propagation model, and proposing an agile and effective public-opinion control strategy are of great significance for maintaining social stability.

In recent years, the theory of complex networks has received widespread attention from scholars, who have used complex theoretical networks to reveal the characteristics and evolution of many real networks. Consequently, complex network theory has provided the mechanism to elaborate on the spread of public opinion. It has been found that the spread of public opinion in social networks is very similar to the spread of infectious disease. Thus, those propagation models have been very useful. Classic infectious disease propagation models include susceptible–infected (SI), SI–susceptible (SIS), and SI–refractory (SIR) models.[1–3] Based on the SIR model, Zhou *et al.* analyzed the influence of topology and node-degree distribution on the spread of social-network rumors and concluded that the number of communicators in the network was related to the topology. The final number of communicators in the scale-free network was found to be larger than that in the random network.[4] Ding *et al.* considered direct immunity for the propagation of information in the Weibo

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†Corresponding author. E-mail: guotianyi960531@gmail.com
‡Corresponding author. E-mail: li@djtu.edu.cn

network and proposed an improved SIR propagation model.[5] Moreno *et al.* explored the process of rumor-spreading in scale-free networks and found that those networks could suppress the spread.[6] Xia *et al.* introduced the attractiveness and ambiguity of rumors into the susceptible–exposed–infected–refractory (SEIR) model, and proposed a hesitation mechanism, finding that ambiguity had a greater impact on the threshold of rumor propagation and its final scope.[7] Zhang *et al.* proposed a network generation model with a community structure as the goal, and studied the influence of uneven community strength and scale on the spread of rumors.[8] Yin *et al.* proposed a multiple-information susceptible–discussing–immune model to characterize social-network public-opinion information propagation, simulating it on the Weibo network. Thus, they more accurately predicted the propagation trends of public sentiment.[9] Chen *et al.* considered user interactions during public-opinion propagation and proposed a two-level-mixing rumor propagation model based on a dynamic trust network. Their simulation showed that the trust between users could control the spread.[10] Li *et al.* considered the influence of positive and negative social reinforcement effects on the second spread of public opinion and the role of relevant government departments in the supervision of public sentiment, proposing a direct-reinforcement susceptible, contacted, infected, and refractory (DR-SCIR) model, which more accurately described the propagation and immune process of public opinion in social networks.[11] Yao *et al.* proposed the susceptible–exposed–infected–incubation–refractory (SE2IR) rumor propagation model with a hesitation mechanism and found that a target immune strategy was more effective than an random immune strategy for this model.[12]

For the research of propagation control, there are random, target, and acquaintance control strategies based on the different methods of selecting immune nodes. Based on the SIS epidemic model, Pastor *et al.* studied the above control strategies in uniform and scale-free networks, finding that random control strategies had better control effects in uniform networks. However, they performed poorly in scale-free networks. Target control strategies had good control effects in both.[13] Yang *et al.* proposed a temporal random control strategy that could effectively reduce the existing time and range of rumor spread.[14] Cohen *et al.* studied acquaintance-control strategies based on the traditional rumor propagation models and proposed an effective control strategy for scale-free networks, obtaining a critical threshold for complete immunity.[15] Saran *et al.* improved acquaintance-control strategies and leveraged respondent-driven sampling to provide an economic and efficient strategy for intervention and control of invisible people.[16] For the suppression of rumors in online social networks, Gu *et al.* proposed an improved important-acquaintance immunization strategy based on the SEIR model. Simulation analysis showed that the model conformed to the spread characteristics of real social-network rumors. Simultaneously, the important acquaintance immunization strategy had a good suppression effect.[17] Mehta *et al.* considered the effect of node importance on suppressing the spread of public opinion and proposed a new measure of node importance, finding that the spread of information could be effectively reduced by suppressing important nodes.[18] Guo *et al.* established a public-opinion communication model based on irrational game theory. Simulation analysis showed that increasing social deterrence, controlling the spread of opinion leaders, and providing positive news could suppress the spread of rumors.[19]

At present, most research on public-opinion propagation has focused on closed network topologies without considering fluctuations in the number of network users or the impact of social factors on propagation. However, in real social networks, user states can be divided into online and offline, such that the number of active users is not consistent. Meanwhile, during the process of controlling network sentiment, relevant departments, media services, and the public resolve to a benign and interactive three-way relationship. In this scenario, relevant departments increase their involvement to enhance the transparency of information, and the media releases information in a timely manner to meet demand. Ultimately, the goal is to achieve control over the spread of malignant sentiment across social networks. Based on the SCIR model, this paper considers the activity of network users, introduces the concepts of additional-user and offline rates, which causes the number of nodes in the network to fluctuate, and establishes a direct immune (DI)-SCIR public-opinion propagation model based on real-time online users. The direct-immunity control strategy of social factors includes interventional force, real information credibility, and interventional time of relevant departments. The activities cause the S-state node to directly change to the R state without secondary propagation after being exposed to public opinion. From the theoretical analysis of the balance point and basic reproduction amount of the DI-SCIR model, the boundary conditions of public-opinion propagation are obtained. Finally, the impact of different factors on public-opinion propagation in social networks is analyzed via simulation, and the direct-immunity rules of different factors are summarized to provide a reference for relevant departments when responding to rumors and maintaining network and social security.

The rest of the paper is organized as follows. Section 2 proposes the DI-SCIR public-opinion propagation model based on real-time online users and establishes its control strategy. Section 3 theoretically analyzes the equilibrium point and basic reproduction number of the DI-SCIR model. Section 4 simulates the feasibility and effectiveness of the model, analyzes its influence on the spread of public opinion by changing model parameters, and compares the control effects of different models. Section 5 provides the conclusion to the paper.

2. DI-SCIR public-opinion propagation model based on real-time online users

2.1. SCIR public-opinion propagation model

As a typical complex network, social networks support interactions among friends and followers and spread public-opinion information via comments, reposts, and other means. The spread model of infectious diseases in complex networks is the main theoretical tool used for studying the spread of public opinion on social networks. The classic SIR infectious-disease propagation model divides network nodes into three states: susceptible S, infected I, and recovered state R.[20–22] To study the propagation rule of public-opinion information, researchers have introduced contact state C on the basis of the SIR model, redefining the status of each state node to construct the SCIR propagation model.[23]

SCIR divides network users into four states: unknown S, hesitant C, propagating I, and immune R. Among them, the unknown state S represents the state in which the user has not been exposed to public-opinion information. However, it is still possible to spread the information. Hesitant state C represents the state in which the user has obtained the information from the neighboring node but has not yet decided whether to spread the information. Propagation state I represents the state in which the user has obtained public-opinion information and can disseminate the information. Immunity state R represents the state wherein the user has obtained the information but will not propagate it. The state-transfer rules of the public-opinion propagation process are defined as follows:

(i) After the unknown state S contacts the propagating state I, it will change to the hesitant state C, with a probability of P_{SC}.

(ii) After the hesitant state C contacts the propagation state I, it will either change to the propagation state I, with a probability of P_{CI}, or it will change to the immune state R, with a probability of P_{CR}.

(iii) Propagation state I transfers to the immune state R, with a probability of P_{IR}.

(iv) After the node reaches the immune state R, its state will not change.

The SCIR public-opinion propagation model is shown in Fig. 1.

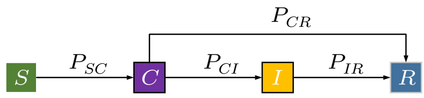

Fig. 1. SCIR public-opinion propagation mode.

In Fig. 1, P_{SC} represents the probability of being exposed to public opinion, but the node has not decided whether to spread the public opinion. This is the internal contact probability. P_{CI} represents the probability of contacting and spreading the public opinion. This is the probability of indirect forwarding. P_{CR} indicates the probability that the state of hesitation does not spread and becomes immune. This is the indirect immunization probability. P_{IR} indicates the probability that the propagator does not believe the public opinion for some reason and becomes an immunizer. This is the forwarding immunization probability. These probabilities meet the constraint of $0 \leqslant P_{SC}, P_{CI}, P_{CR}, P_{IR} \leqslant 1$.

The differential equation of the SCIR public-opinion propagation model is

$$\begin{cases} \dfrac{\mathrm{d}S_k}{\mathrm{d}t} = -P_{SC}S_kI_k, \\ \dfrac{\mathrm{d}C_k}{\mathrm{d}t} = P_{SC}S_kI_k - (P_{CI}+P_{CR})C_k, \\ \dfrac{\mathrm{d}I_k}{\mathrm{d}t} = P_{CI}C_k, \\ \dfrac{\mathrm{d}R_k}{\mathrm{d}t} = P_{CR}C_k + P_{IR}I_k. \end{cases} \tag{1}$$

2.2. DI-SCIR public-opinion propagation model based on real-time online users

Considering a user's active factors in social networks, some users may temporarily leave the network, causing the problem of the number of online users changing in real time. Thus, this article introduces the concept of additional rate and offline rate, in which the additional rate is defined as the percentage of new online users in the total number of users at a certain moment, and the offline rate is defined as the proportion of the number of offline users at a certain moment in the total number of users.

This article also considers the existence of direct immunization. Relevant departments can take certain measures, such as publishing real information, to supervise the spread of public opinion and guide its direction, thereby controlling the spread of public opinion in social networks. Specifically, direct immunization is applied to unknown users, such that when the relevant departments release real information, the unknown users will directly change to the immune state with probability P_{SR} when they first contact the public opinion information, thereby controlling the spread of public opinion and reducing the social impact of negative public opinion. The DI mechanism can be expressed by the following formula:

$$S+I \xrightarrow{P_{SR}} R+I. \tag{2}$$

Based on the above considerations, the DI-SCIR public-opinion propagation model based on real-time online users is established, as shown in Fig. 2.

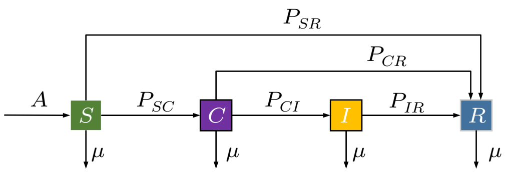

Fig. 2. DI-SCIR public-opinion propagation model based on real-time online users.

In Fig. 2, A represents the additional rate of users in unknown states in the network. μ represents the offline rate of various users. P_{SR} represents the probability that the public opinion will not be transmitted by unknown users after being exposed to it. That is, the direct immunity probability. These probabilities meet the constraints of $0 \leqslant A$, μ, P_{SC}, P_{SR}, P_{CI}, P_{CR}, $P_{IR} \leqslant 1$.

Because there is an additional rate, A, and an offline rate, μ, in the network, the number of real-time online users on the network is

$$|1+A-\mu|N(t)=S(t)+C(t)+I(t)+R(t). \tag{3}$$

Therefore, the differential equation of the DI-SCIR public-opinion propagation model based on real-time online users is

$$\begin{cases} \dfrac{\mathrm{d}S_k}{\mathrm{d}t}=A-(P_{SC}+P_{SR})S_kI_k-\mu S_k, \\ \dfrac{\mathrm{d}C_k}{\mathrm{d}t}=P_{SC}S_kI_k-(P_{CI}+P_{CR}+\mu)C_k, \\ \dfrac{\mathrm{d}I_k}{\mathrm{d}t}=P_{CI}C_k-(P_{IR}+\mu)I_k, \\ \dfrac{\mathrm{d}R_k}{\mathrm{d}t}=P_{CR}C_k+P_{IR}I_k+P_{SR}S_kI_k-\mu R_k. \end{cases} \tag{4}$$

The initial value of the model is

$$S_k(0)=1-C_k(0)-I_k(0)-R_k(0)>0, \tag{5}$$

where, $C_k(0), I_k(0), R_k(0) \geqslant 0$.

Because the first three equations of Eq. (4) do not contain state R, equation (4) can be simplified to

$$\begin{cases} \dfrac{\mathrm{d}S_k}{\mathrm{d}t}=A-(P_{SC}+P_{SR})S_kI_k-\mu S_k, \\ \dfrac{\mathrm{d}C_k}{\mathrm{d}t}=P_{SC}S_kI_k-(P_{CI}+P_{CR}+\mu)C_k, \\ \dfrac{\mathrm{d}I_k}{\mathrm{d}t}=P_{CI}C_k-(P_{IR}+\mu)I_k. \end{cases} \tag{6}$$

2.3. Control strategy of public-opinion propagation based on direct immunity

The primary means of a public-opinion propagation control strategy involves the increase of the number of immune nodes and the probability of direct immunity. At present, the common immunization strategies of complex network theory include random (i.e., uniform) immunization, target (i.e., selective) immunization, and acquaintance (i.e., nearest-neighbor) immunization. That is, some nodes are randomly selected from the network as target nodes. Then, the target node and its connecting edges are removed, which is equivalent to setting the target node as a permanent immune node. The chance of each node being immunized is equal, not different, to the risk of node infection.[24] It is an immune strategy designed for the non-uniform characteristics of scale-free networks. It mainly immunizes the largest nodes in the network. After the large nodes are immunized, the connected edges can be removed, which greatly reduces the possible paths of propagation. This method has a good immune effect, but it must know the global information of the network in advance. This is difficult to achieve in a huge social network.[25] Among all the nodes in the network, a certain proportion is completely and randomly selected as intermediate nodes. For each intermediate node, one neighbor node is randomly selected as the target node, and it is set as a permanent immune node, avoiding the need to know the global information for target immunity.[26]

Unfortunately, most of the current research on public-opinion propagation control strategies is based on the network topology, and little consideration has been given to the influence of social factors on the immune function. This article summarizes the social factors that affect direct immunity into the following categories.

2.3.1. Interventional force of relevant departments

The main purpose of the immunization strategy is to reduce the number of users who spread malignant public opinion on the network. When the relevant government departments take restrictive measures on the spread of public opinion, the greater the intervention and the greater the importance attached to public opinion. Then, the relevant departments will take a more powerful approach to reduce the impact of negative public opinion and promote more network users believing real information, ultimately creating more immune users at the initial stage of direct immunity. Simultaneously, because opinion leaders in social networks will have an important influence on the spread of sentiment,[27,28] increasing the involvement of relevant departments should limit the spreading ability of influential communicators in the network, thus suppressing the spread.

2.3.2. Credibility of real information

Social media not only passively provides information channels, it also changes modes of thinking. Official authoritative information held by the government and other relevant departments must be released to the public through the media. As an important means of countering malignant public opinion, mainstream media should grasp the initiative of information release as soon as possible, building a bridge between the masses and the government, striving to improve the credibility of the government and to increase the credibility of the real information, ultimately leading to malignant public opinion completely losing the market. Notably, earthquake warnings via the seismic network have higher credibility than many other agencies or media. Therefore, the credibility of the real information source will greatly affect immunity control.

2.3.3. Interventional time of relevant departments

Most mass incidents caused by negative public sentiment are related to failures of relevant departments to deal with them in a timely and effective manner. When public opinion begins to spread on the internet, relevant departments should quickly verify the authenticity of public opinion and estimate its possible impact. After the initial stages of outbreak are missed, negative public opinion will be fomented, increasing the difficulty of public opinion to control it. Therefore, the relevant departments must collect rumors and information in a timely and accurate manner and intervene quickly.

From the perspective of propagation dynamics, the information will inevitably be perturbed by noise during the propagation process, perhaps weakening the effect of the relevant departments' direct immunity. This degree of noise is related to the social factors embedded into the information itself. The lower the credibility of the real information, the greater the interference of noise. The wider the range of real information propagation, the stronger the interference. To reduce the spread of malignant public opinion and to reduce its harmfulness, we must try to increase the spread of real information released by the relevant departments and reduce the interference of noise for the direct immunization strategy.

To control the spread of public opinion, this paper uses social factors to intervene in the model, considering the influence of noise interference. The direct-immunity probability, P_{SR}, is defined as

$$P_{SR}=\begin{cases}0, & 0\leqslant t<T,\\ \alpha\beta(1-\mathrm{e}^{-\frac{\beta}{\alpha}}), & t\geqslant T,\end{cases} \tag{7}$$

where α represents the intervention strength, reflecting the density of nodes in the immune state at the initial moment of adopting the immunization strategy. β represents the credibility of the real information, $\beta\in[0,1]$. T represents the interventional time of relevant departments. $\mathrm{e}^{-\beta/\alpha}$ represents the noise interference, which is proportional to the interventional force, α, and inversely proportional to the credibility of the real information, β. The direct-immunity probability P_{SR} changes with the intervention force α, and the real information credibility β, as shown in Fig. 3.

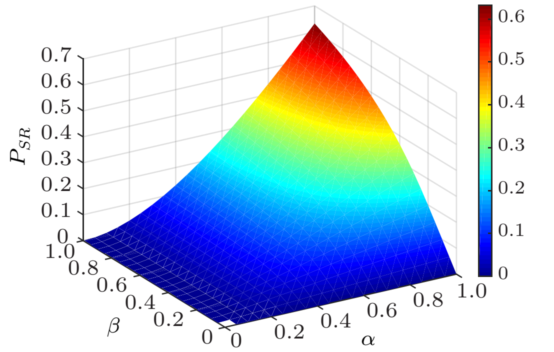

Fig. 3. Direct immunity probability P_{SR} changes with the interventional force α, and the real information credibility β. When $\alpha=0.2859$ and $\beta=1$, P_{SR} takes a maximum value of 0.7620.

3. Stability analysis

In the study of prevention and control measures for infectious diseases, people tend to be most concerned about the conditions under which the disease originates and dies out. The desire is to take corresponding measures to rapidly achieve or approach extinction and to reduce the number of infections. The basic reproduction number is one of the most important indicators used to measure the outbreak and extinction of infectious diseases in the propagation dynamics model. It is usually expressed by R_0. This is defined as the number of patients who can be infected during the average period of illness, which manifests via two situations:

(1) $R_0\leqslant 1$ means that the maximum number of patients that can be transmitted in an average infection period is less than one. Thus, the disease will gradually disappear. At this time, the model only has a disease-free equilibrium, which is globally asymptotically stable.

(2) $R_0>1$ means that a patient can infect more than one new patient in the average period of infection. Thus, the disease will always exist and will form an endemic. At this time, in addition to the disease-free equilibrium point, there is an endemic-disease equilibrium point. The disease-free equilibrium point of the model is unstable, and the endemic disease equilibrium point is globally asymptotically stable.

The next-generation matrix proposed by Van Den Driessche[29] is an effective method for calculating the basic reproduction number. The main steps are as follows:

(1) Select the infectious-state node, record the new infected item as a vector function, $\mathcal{F}(x)$, and the removal item as a vector function, $\mathcal{V}(x)$.

(2) Substitute the disease-free equilibrium point, obtain the Jacobian matrix, and obtain the appearance-rate matrix of the new communicator, $\boldsymbol{F}$, and the individual migration rate matrix, $\boldsymbol{V}$.

(3) Calculate $\boldsymbol{F}\boldsymbol{V}^{-1}$; its spectral radius is the basic reproduction number, R_0.

According to the characteristic that newly added nodes in a complex network are more easily connected to nodes having a higher degree, the influence of node degree should be considered during the process of analyzing public-opinion propagation.

Assuming that $\rho_k(t)$ represents the density of diseased nodes in the node group having scale k at time t, the following differential equation is satisfied:[30,31]

$$\frac{\mathrm{d}\rho_k(t)}{\mathrm{d}t}=\lambda(k)(1-\rho_k(t))\Theta_k(t)-\rho_k(t),\quad(k=1,2,\ldots).\tag{8}$$

Considering that $\lambda(k)=\widetilde{\lambda(k)}k$, $\widetilde{\lambda(k)}$ represents the propagation rate related to degree, and $\Theta_k(t)$ represents the probability that an edge from a node of degree k will connect to the propagation node I.[32] Thus, there is

$$\Theta_k(t)=\sum_{k'}\frac{\vartheta(k')}{k'}P(k'|k)\rho_{k'}(t),\tag{9}$$

where $1/k'$ represents the probability that a neighbor node of a certain node in the propagation state, and degree k contacts the node in unit time. $\vartheta(k')$ represents the number of times the node having degree k' has effectively contacted other nodes per unit time. $P(k'|k)$ represents the conditional probability that a node of degree k will randomly touch a node of degree k' through an edge.

For a degree-independent network, the degrees of different nodes are uncorrelated. That is, $P(k'|k)$ is independent of k but proportional to $k'P(k')$.[33] Thus,

$$P(k'|k)=\frac{k'P(k')}{\langle k\rangle},\tag{10}$$

where $\langle k\rangle$ is the average degree of the network. For the general function $f(x)$, there is

$$\langle f(x)\rangle:=\sum_k f(k)P(k),\tag{11}$$

where $P(k)$ is the degree distribution or the probability of any node whose degree is k. Because this article is studied on degree-independent networks, equation (9) can be simplified to

$$\Theta(t)=\frac{1}{\langle k\rangle}\sum_{k'}^{n}\vartheta(k')P(k')I_{k'}(t),\tag{12}$$

where $\Theta(t)$ represents the probability that any edge in the network points to a node in the propagation state. Therefore, the probability of changing from an unknown state to a hesitant state can be defined as

$$P_{SC}=\lambda\langle k\rangle S_k(t)\Theta(t).\tag{13}$$

Assuming the left side to be zero in Eq. (4), and when $I_k=0$, $k=1,2,\ldots,n$, the model is in the disease-free equilibrium state, and there is a disease-free equilibrium point at this time

$$E_0=(S^0,C^0,I^0)=(A/\mu,0,0).\tag{14}$$

When $I>0$, the endemic disease equilibrium point of the model is $E^*=(S^*,C^*,I^*)$, where

$$\begin{aligned}S^*&=\frac{(P_{CI}+P_{CR}+\mu)(P_{IR}+\mu)}{P_{SC}P_{CI}},\\ C^*&=\frac{AP_{SC}P_{SI}-(P_{SR}+\mu)(P_{CI}+P_{CR}+\mu)(P_{IR}+\mu)}{P_{SC}P_{CI}(P_{CI}+P_{CR}+\mu)},\\ I^*&=\frac{AP_{SC}P_{SI}-(P_{SR}+\mu)(P_{CI}+P_{CR}+\mu)(P_{IR}+\mu)}{P_{SC}(P_{CI}+P_{CR}+\mu)(P_{IR}+\mu)}.\end{aligned}\tag{15}$$

Owing to $S_k(t)+C_k(t)+I_k(t)+R_k(t)=1$, equation (4) can be simplified to

$$\begin{cases}\dfrac{\mathrm{d}C_k}{\mathrm{d}t}=P_{SC}(1-C_k-I_k-R_k)I_k-(P_{CI}+P_{CR}+\mu)C_k,\\ \dfrac{\mathrm{d}I_k}{\mathrm{d}t}=P_{CI}C_k-(P_{IR}+\mu)I_k,\\ \dfrac{\mathrm{d}R_k}{\mathrm{d}t}=P_{SR}(1-C_k-I_k-R_k)I_k+P_{CR}C_k+P_{IR}I_k-\mu R_k.\end{cases}\tag{16}$$

Divide vector $\boldsymbol{x}$ and vector $\boldsymbol{y}$ into

$$\dot{\boldsymbol{x}}=\begin{bmatrix}C(t)\\ I(t)\end{bmatrix},\quad\dot{\boldsymbol{y}}=\begin{bmatrix}S(t)\\ R(t)\end{bmatrix}.\tag{17}$$

Suppose that

$$\dot{\boldsymbol{x}}=\mathcal{F}(\boldsymbol{x})-\mathcal{V}(\boldsymbol{x}).\tag{18}$$

The rate of new infected is

$$\mathcal{F}(\boldsymbol{x})=(\mathcal{F}_1(\boldsymbol{x}),\ldots,\mathcal{F}_n(\boldsymbol{x}),0,\ldots,0,0,\ldots,0)^{\mathrm{T}}.\tag{19}$$

The individual migration rate is

$$\begin{aligned}\mathcal{V}(\boldsymbol{x})=\big(&\mathcal{V}_1(\boldsymbol{x}),\ldots,\mathcal{V}_n(\boldsymbol{x}),\mathcal{V}_{n+1}(\boldsymbol{x}),\ldots,\mathcal{V}_{2n}(\boldsymbol{x}),\\ &\mathcal{V}_{2n+1}(\boldsymbol{x}),\ldots,\mathcal{V}_{3n}(\boldsymbol{x})\big)^{\mathrm{T}}.\end{aligned}\tag{20}$$

Then, there are

$$\dot{\boldsymbol{x}}=\mathcal{F}(\boldsymbol{x})-\mathcal{V}(\boldsymbol{x})=\begin{bmatrix}P_{SC}S_kI_k\\ 0\end{bmatrix}-\begin{bmatrix}(P_{CR}+P_{CI}+\mu)C_k\\ -P_{CI}C_k+(P_{IR}+\mu)I_k\end{bmatrix}.\tag{21}$$

Calculating the Jacobian matrix for $\mathcal{F}(\boldsymbol{x})$ and $\mathcal{V}(\boldsymbol{x})$, there are

$$\boldsymbol{F}=\mathrm{Jacobian}(\mathcal{F}(\boldsymbol{x}))=\begin{bmatrix}0 & P_{SC}S_k\\ 0 & 0\end{bmatrix},\tag{22}$$

$$\boldsymbol{V}=\mathrm{Jacobian}(\mathcal{V}(x))=\begin{bmatrix}P_{CI}+P_{CR}+\mu & 0\\ -P_{CI} & P_{IR}+\mu\end{bmatrix}.\tag{23}$$

The basic reproduction number from Ref. [34] is

$$R_0=\tilde{\rho}(\boldsymbol{F}\boldsymbol{V}^{-1}),\tag{24}$$

where $\tilde{\rho}$ represents the spectral radius of the matrix. Then, the basic reproduction number of the model is

$$R_0=\frac{P_{CI}P_{SC}S_k}{(P_{IR}+\mu)(P_{CI}+P_{CR}+\mu)}.\tag{25}$$

Bringing the disease-free balance point in Eq. (14), we obtain

$$R_0=\frac{P_{CI}P_{SC}A}{\mu(P_{IR}+\mu)(P_{CI}+P_{CR}+\mu)}.\tag{26}$$

When $R_0 \leqslant 1$, there are fewer users who obtain public-opinion information in the network. Thus, the information will disappear in the network eventually, and the model will be locally progressively stable at the disease-free equilibrium point E_0. When $R_0 > 1$, the number of infected nodes (i.e., users who contact and spread the public opinion information) in the network will gradually increase until it reaches a stable state. Then, this number fluctuates around the endemic disease balance point E^*.

4. Simulation analysis

4.1. Dataset

The characteristics of social networks are consistent with Barabási–Albert (BA) scale-free networks, including the possession of typical power-law distribution characteristics.[35,36] Thus, the dataset in this paper adopts a BA scale-free network having 1000 nodes. The schematic of the topology is shown in Fig. 4, and the corresponding characteristic parameters are shown in Table 1. To visually display the nature of BA scale-free networks, the network's degree distribution is shown in Fig. 5.

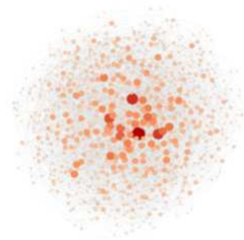

Fig. 4. BA scale-free network topology. Nodes having higher degrees are darker in color and larger in area, and the relationship between the nodes is indicated by a solid gray line.

Table 1. Characteristic parameters of BA scale-free network.

Node	Edge	Average degree	Max degree	Min degree	Average path length	Average clustering coefficient
1000	7981	7.981	230	8	3.233	0.029

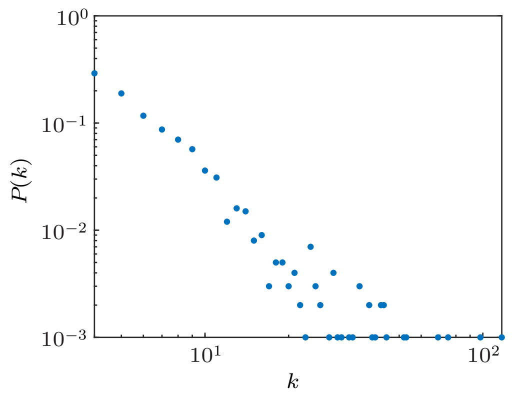

Fig. 5. BA scale-free network degree distribution logarithmic coordinate graph. The x-axis, k, represents the degree of nodes in the network, and the y-axis, $P(k)$, represents the distribution of correspondence degrees.

As can be seen from Fig. 5 and Table 1, the degree distribution of BA scale-free network nodes approximately follows the power-law distribution, which reflects the scale-free characteristic, and the maximum and minimum degrees have a large gap.[37] However, the average degree and clustering coefficient are small. All of these reflect the characteristics of a small world, which reflect characteristics similar to real social networks. Thus, a BA scale-free network is selected for the simulation data.

4.2. The influence of basic reproduction number on the propagation of public opinion

To analyze the impact of the basic reproduction number on the spread of public opinion, considering the fact that the total number of nodes in the network has limited fluctuations around N, that is, $A = \mu$, the two cases, $R_0 > 1$ and $R_0 \leqslant 1$, are simulated and analyzed. The results are shown in Fig. 6.

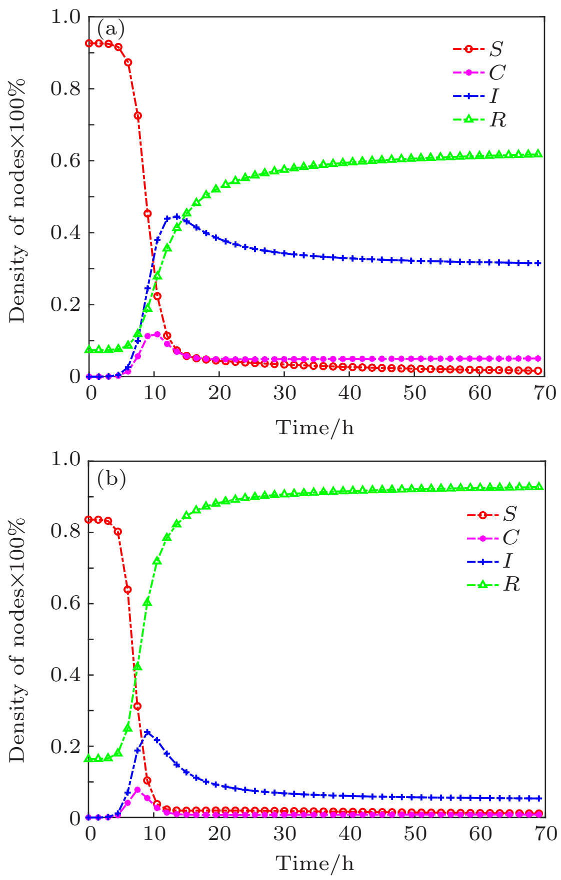

Fig. 6. Effect of basic reproduction numbers on the spread of public opinion: (a) random chosen $R_0 = 1.3914 > 1$; (b) random chosen $R_0 = 0.9635 < 1$.

It can be seen from Fig. 6(a) that, when $R_0 > 1$, after 60-h propagation, the network reaches a stable state with $(S(t), C(t), I(t), R(t)) \rightarrow (0.0203, 0.0582, 0.3162, 0.6053)$. The node density in the immune state R is about 0.6, and the node density in the propagation state I is about 0.31, indicating that although there are a certain proportion of users in the immune state R in the stable state, public-opinion propagators always exist in the network and that public opinion will continue to spread, similar to the spread of epidemics, showing a spreading trend. It can be seen from Fig. 6(b) that, when $R_0 \leqslant 1$, there is a peak in the number of propagators. When the peak is reached, the density of the I state node will show a downward trend when the public opinion reaches a stable state with $(S(t), C(t), I(t), R(t)) \to (0, 0, 0.0467, 0.9533)$. At this time, the node density of the I state is almost zero, and most of the nodes are in the immune state, indicating that the vast majority of internet users are not interested in the public opinion, so the public opinion is difficult to continue to influence. This is similar to the spread of an endemic disease that will eventually disappear from the network.

4.3. The influence of interventional force on the propagation of public opinion

To analyze the influence of interventional force α on the spread of public opinion, assuming that other variables are the same, $\alpha = 0$, 0.1, 0.3, 0.5, 0.7 are used to apply DI analysis to the propagation model. The result is shown in Fig. 7.

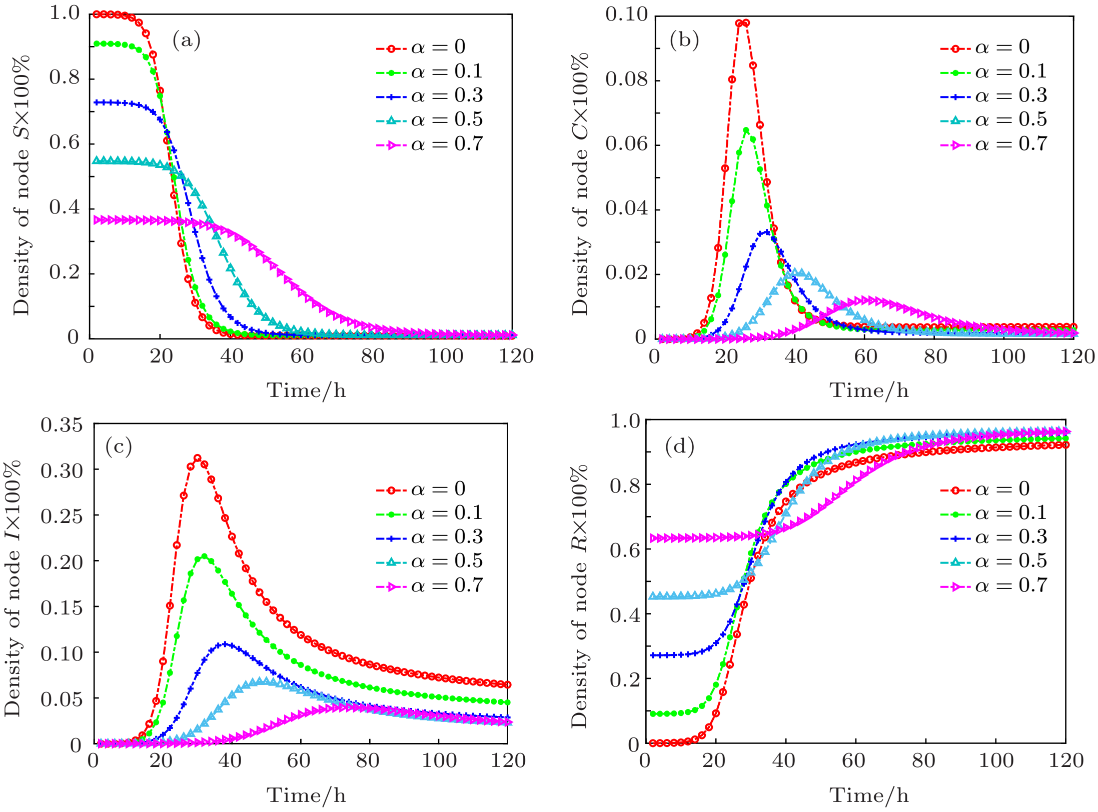

Fig. 7. Impact of interventional force α on the propagation of public opinion. α takes 0,0.1,0.3,0.55,0.7: density changes of (a) S state node, (b) C state node, (c) I state node, and (d) R state node.

It can be seen from Figs. 7(a) and 7(d), that the magnitude of the interventional force is reflected in the numbers of nodes in the unknown state S and the immune state R at the initial state of public-opinion propagation, where the magnitude of the interventional force, α, is inversely related to the node density of the unknown state S, while positively correlated with the node density of the immune state R. Thus, the greater the interventional force α, the fewer S-state users and more R-state users during the initial propagation. Simultaneously, the greater the intervention force α, the slower the density of the S-state node density and the R-state node density are reduced, indicating that the interventional force α can suppress the fluctuation of the number of nodes in different states in the network and maintain network stability. As can be seen from Fig. 7(b), the greater the interventional force α, the slower the growth rate of the hesitant state C in the process of public-opinion propagation, which delays the time for the hesitant state C node density to reach the peak. The time required for the number of C-state nodes to reach the peak is shortened from 61 hours with interventional force $\alpha = 0.7$ to 22 hours without interventional force. This also reduces the corresponding peak value and its proportion in the spread of public opinion. It can be seen from Fig. 7(c) that the greater the interventional force α, the slower the growth rate of the propagation state I in the early stage of public-opinion propagation. This indicates that increasing the interventional force α can slow the outbreak of public opinion in the network. The time is increased, and the number of corresponding peaks is reduced as well. In the steady state, the node density of the I state when the interventional force $\alpha = 0.7$ is lower than that when the interventional force $\alpha = 0$ by 65.97%. The above conditions indicate that interventional force α can change the proportion of the number of nodes in each state of the initial stage of public-opinion propagation, increasing the interventional force α can increase the number of users who are immune in the initial stage of public-opinion propagation, and reduce the time and scope of public-opinion propagation in the network, which effectively suppresses the spread of malignant public opinion.

4.4. The influence of real information credibility on the propagation of public opinion

To analyze the influence of real information credibility β on the spread of public opinion, based on the premise that other variables are the same, $\beta = 0$, 0.1, 0.3, 0.5, 0.7 are used to apply DI analysis to the propagation model. The result is shown in Fig. 8.

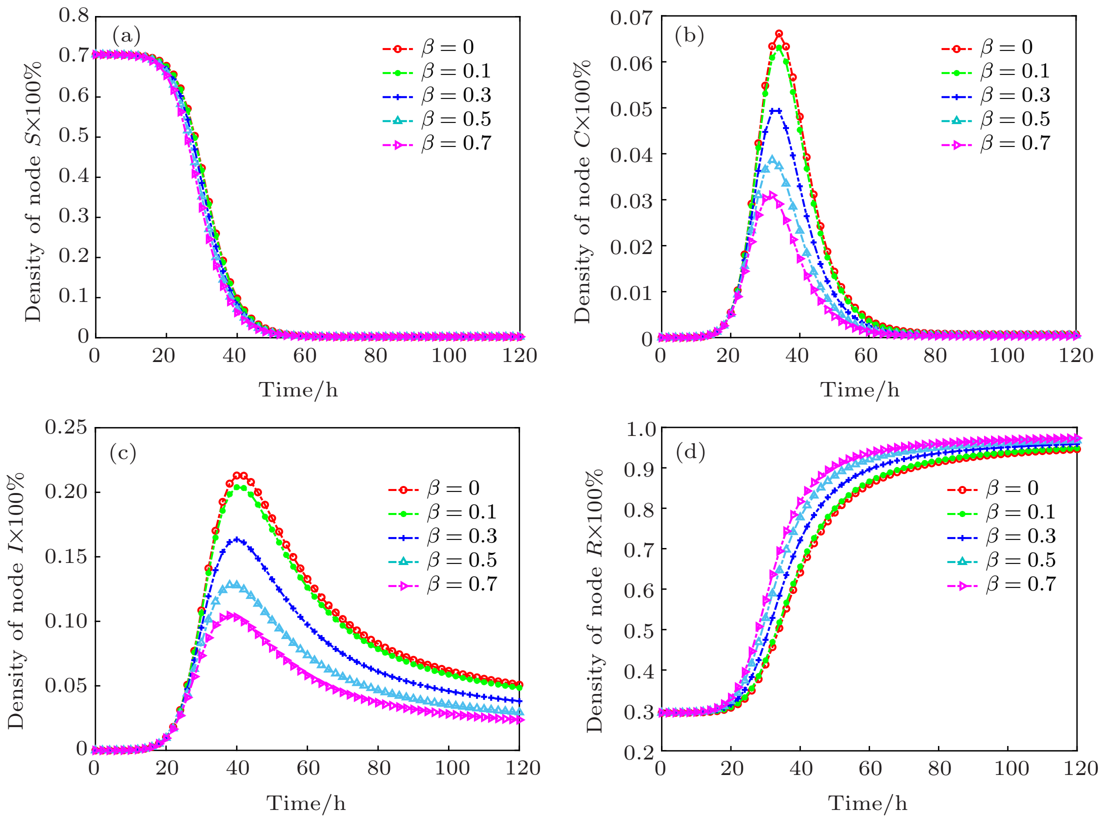

Fig. 8. Impact of real information credibility β on the propagation of public opinion β takes 0,0.1,0.3,0.5,0.7, respectively: density changes of (a) S state node, (b) C state node, (c) I state node, and (d) R state node.

It can be seen from Fig. 8(b) that, with the increase of real information credibility β, the peak of node density in hesitant-state C decreases significantly. When the real information credibility is $\beta = 0.7$, the node density in hesitant state C decreases 49.23%, compared with $\beta = 0$, indicating that increasing the credibility of real information β can reduce the number of hesitant nodes in the network and further reduce the spread of nodes to the propagation state I. Thus, the spread of public opinion is suppressed. It can be seen from Fig. 8(c) that, with the increase of the real information credibility β, the peak of the node density in propagation state I decreases significantly. When the real information credibility is $\beta = 0.7$, the node density in the propagation state I decreases 38.74% compared with $\beta = 0$, indicating that increasing the real information credibility β can reduce the node density of the propagation state I, and inhibit the further spread of public opinion. As can be seen from Fig. 8(d), increasing the credibility of real information β increases the growth rate of the density of R state nodes. Simultaneously, the number of immune-state nodes also increases in the stable state, indicating that increasing the credibility of real information β can reduce the spread of public opinion. All of the above shows that real information credibility β can significantly affect the spread of public opinion on social networks. The larger real information credibility β is, the less people trend to believe public opinion. So relevant departments can affect the spread of public opinion by adjusting real information credibility β to avoid the spread of negative public opinion.

4.5. The influence of interventional time on the propagation of public opinion

To analyze the influence of interventional time T on the spread of public opinion, assuming that other variables are the same, we consider the time, $T = 3$, 10, 20, 30, 40, to apply DI analysis to the propagation model. The result is shown in Fig. 9.

It can be observed from Fig. 9(b) that, before direct immunity is applied, the node density of the hesitant state C will reach its peak 9 h after the start of propagation. When direct immunity is applied at $T = 3$, the propagation of hesitant state C will be blocked, so that the peak corresponding to the hesitant C in the state of the interventional time, $T = 3$, is much smaller than the peak value when no direct immunization is applied. This effectively reduces the number of hesitation states C to the propagation state I, and inhibits the spread of public opinion. It can be seen from Fig. 9(c) that, when direct immunization is involved at $T = 3$, the peak value of the node density of the propagation state I is reduced by 45.82% compared with other states, which effectively suppresses the spread of public opinion. When direct immunization is involved at $T = 10$, the corresponding node density of the propagation state I is just at the peak of the theoretical condition that no direct immunity is applied. At this time, when direct immunity is applied, the node density of the propagation state I suddenly decreases, inhibiting the large-scale spread of public opinion. When the interventional time $T = 20$, 30, 40, it also inhibits the spread of public opinion. However, the intensity of the effect decreases in turn. As can be seen from Fig. 9(d), the earlier the interventional time, the earlier the node density of the immune state R enters a stable state and the shorter the propagation time of public opinion, the worse the propagation effect. The above results indicate that the earlier the interventional time, the shorter the time and scope of public opinion spread on the network. This means that relevant departments can suppress the spread of negative public opinion by taking timely and effective measures, thereby reducing the impact of negative public opinion.

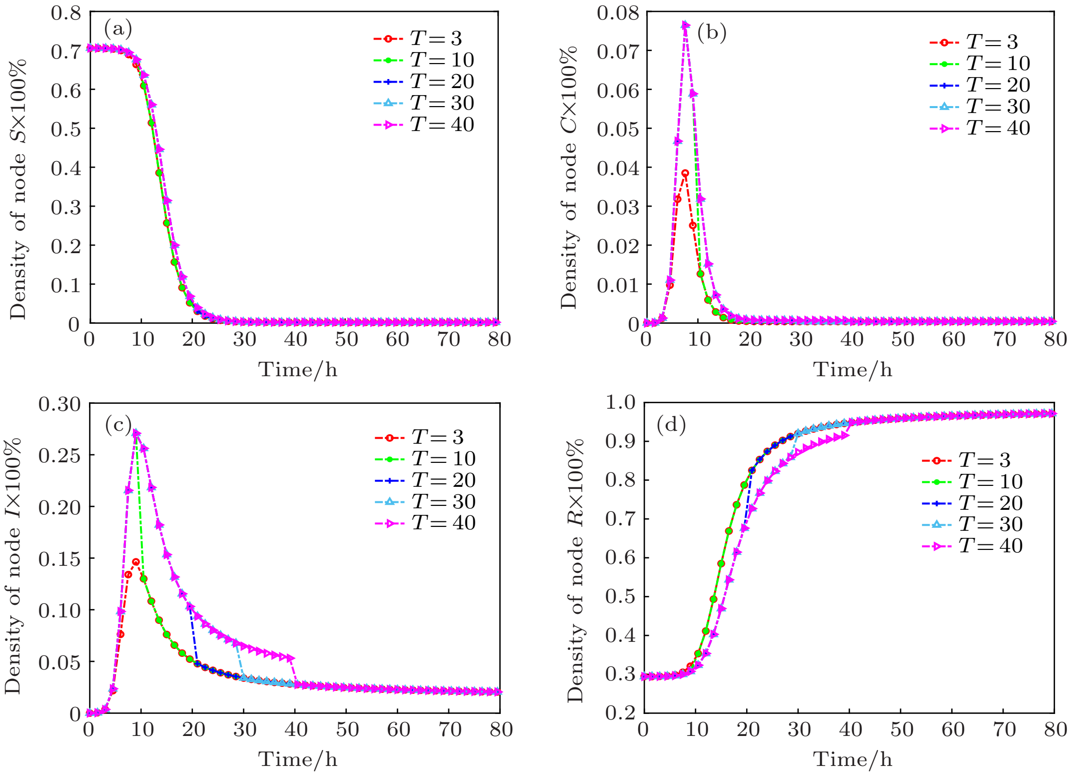

Fig. 9. Impact of interventional time T on the propagation of public opinion. T takes 3, 10, 20, 30, 40, respectively: density changes (a) of S state node, (b) C state node, (c) I state node, and (d) R state node.

4.6. Comparative analysis of different models

To analyze the different effects of the DI-SCIR model, it is compared with SIR and SCIR. The results are shown in Fig. 10.

It can be seen from Figs. 10(a) and 10(d) that, owing to the availability of interventional force, the node density of the unknown state S, and the immune state R, is not 0 at the beginning, and a proportion of immunized users in the initial stage of propagation exists. This prompts the unknown state S, and the immune state R, to enter a stable state sooner, reducing the time for public opinion to spread in the network. It can be seen from Fig. 10(b) that the existence of the hesitant state C is not considered in the SIR model. The model in this paper significantly reduces the peak of the node density of the hesitant state

C, thereby effectively reducing the number of hesitant-state nodes transferred to the propagation state I. This suppresses the spread of public opinion. It can be seen from Fig. 10(c) that the peak value of propagation state I is the largest in the SIR model, followed by the SCIR model, in which the peak value drops by 25.74%. The smallest is obtained in the DI-SCIR model, whose peak value drops by 59.12%. Meanwhile, the time required for the propagation state I to reach the peak also decreases in turn, indicating that the DI-SCIR model can significantly reduce the number of propagators and the propagation time in the network. This effectively suppresses the spread of public opinion in the network.

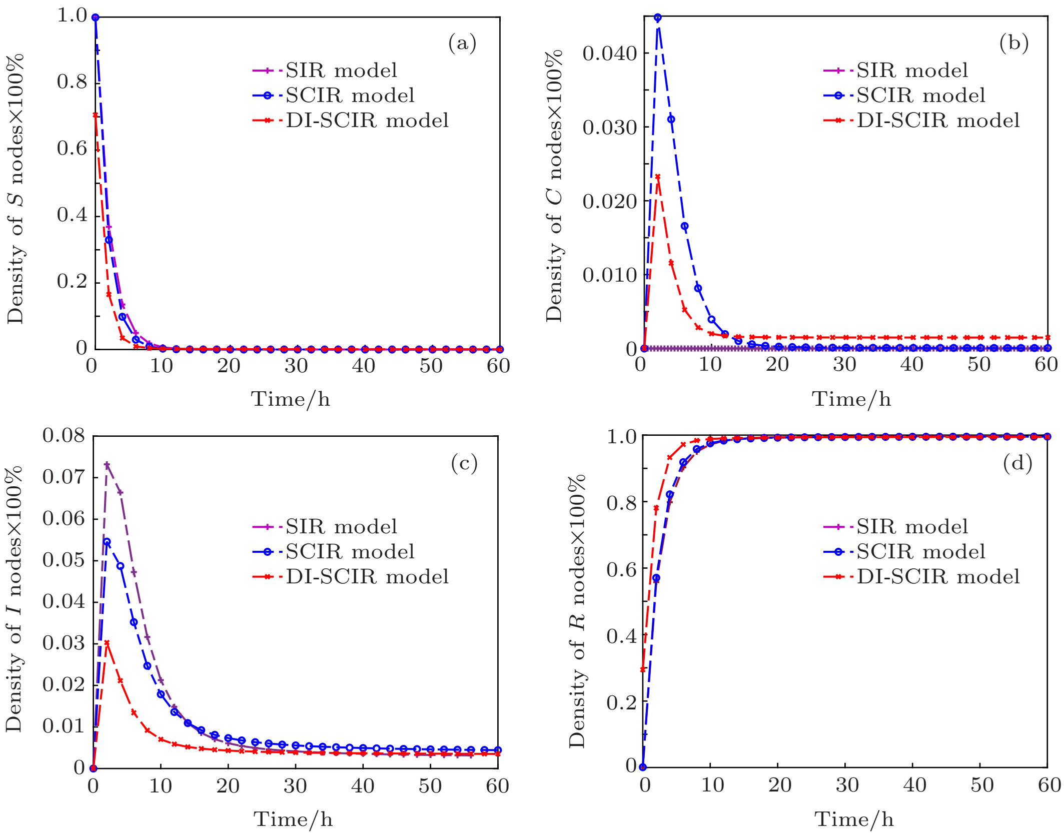

Fig. 10. Impact of different models on the spread of public opinion. The models are SIR, SCIR, and the DI-SCIR public-opinion propagation models based on real-time online users: (a) density changes of(a) of S state node, (b) C state node, (c) I state node, and (d) R state node.

5. Conclusions

Extant public-opinion propagation models do not consider the fluctuation of the number of network users and the influence of social factors on the effect of public-opinion immunity. Thus, it is difficult to accurately describe the rules of public-opinion propagation in social networks. This article comprehensively considered the activity of social-network users and the regulatory role of relevant government departments in the control of propagation of public opinion. As a result, the DI-SCIR public-opinion propagation model based on real-time online users is presented. The model analyzes the rules of social-network public-opinion spread. Theoretical and simulation verification showed that the model can more accurately reflect the spread of public opinion. When the basic reproduction number $R_0 > 1$, the public opinion continues to spread in the network, similar to the spread of epidemics. When $R_0 \leqslant 1$, the public opinion automatically disappears after a period of time in the network, similar to the spread of endemic diseases. Social factors can effectively control the spread of public opinion to increase the interventional force α, and the credibility of real information, β, can increase the number of R-state users who are immune at the initial moment with a probability of direct immunization P_{SR}, thereby inhibiting the spread of network public opinion. The smaller the interventional time T, the sooner the spread of the public opinion can be controlled. The results of this article provide a good reference for the relevant departments to develop a control capability for the spread of public opinion so that they can actively resolve crises of malignant public sentiment, which is of great importance when maintaining social security and stability.

As a limitation, this article did not consider the upper limits of the increase in the amount of online public-opinion information when establishing social-network users' additional and offline rates. In future research, we plan to draw on the logistics population growth model to formulate the additional and offline rates to establish a more accurate model. At the same time, the model designed in this paper defines each state transition probability as a constant, without considering the influence of factors such as network structure (e.g., node degree) and user interest (e.g., social reinforcement effect) on the spread of public opinion, these can be future research directions to improve the DI-SCIR public-opinion propagation model.

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