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Effect of degree correlation on edge controllability of real networks*

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We use the controllability limit theory to study impact of correlation between in- and out-degrees (degree correlation) on edge controllability of real networks. Simulation results and analytic calculations show that the degree correlation plays an important role in the edge controllability of real networks, especially dense real networks. The upper and lower controllability limits hold for all kinds of real networks. Any edge controllability in between the limits is achievable by properly adjusting the degree correlation. In addition, we find that the edge dynamics in some real networks with positive degree correlation may be difficult to control, and explain the rationality of this anomaly based on the controllability limit theory.

Keywords: complex network, edge controllability, degree correlation, controllability limit

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1. Introduction

Complex network has been extensively studied due to its widespread use in social, biological, technological and financial systems. How to control complex networks is a challenging issue[1,2] in modern network science. According to control theory,[3,4] the dynamics in a complex network is controllable if, with a suitable choice of inputs, it can be driven from any initial state to any desired final state within finite time. Liu *et al.*[5] developed structural control theory for the nodal dynamics of complex networks and offered efficient tool based on the maximum matching to characterize the controllability of networks. A lot of work has been carried out based on the nodal dynamics and has achieved fruitful results.[6–11] However, the edge dynamics, which is suitable for modeling networks where nodes are active components with information processing capabilities, is also very important in network science. Nepusz *et al.*[12] introduced the edge dynamics and studied its structural controllability. Many interests have been stimulated toward exploring edge controllability properties of complex networks.[13–23]

A correlation between in- and out-degrees is ubiquitous in real networks.[24–26] It is reasonable to assume that such a degree correlation has influence on the edge controllability of real networks. Despite recent advances in the edge controllability, research on impact of degree correlation on the edge controllability of real networks is still missing. In this paper, we focus on this issue. Using the minimum numbers of driver nodes and driven edges to measure the edge controllability, we find that the degree correlation plays an important role in the edge controllability of real networks. Specifically, a stronger degree correlation has a greater impact on the edge controllability. The edge controllability of dense real networks is more susceptible to the degree distribution. Then we use the controllability limit theory[22] to quantify the effect of degree correlation on the edge controllability of real networks. This enables us to realize that the upper and lower controllability limits hold for all kinds of real networks. A vast range exists between the upper and lower controllability limits. Arbitrary edge controllability in between the limits can be achieved by properly adjusting the degree correlation. In addition, we find that the edge dynamics in some real networks with positive degree correlation may be difficult to control. This anomaly runs counter to the conclusion in Ref. [12]. We explain the rationality of this anomaly based on the controllability limit theory.

2. Edge controllability

The edge dynamics in a digraph $G(V,E)$ can be described by the switchboard dynamics.[12] The state vectors of the incoming and outgoing edges of a node v are denoted as $\boldsymbol{y}_v^-$ and $\boldsymbol{y}_v^+$, respectively. Factors affecting the state vector $\boldsymbol{y}_v^+$ include the state vector $\boldsymbol{y}_v^-$, the damping terms τ_v and external inputs u_v. Thus the equations for the edge dynamics can be expressed as

$$\dot{\boldsymbol{y}}_v^+ = S_v \boldsymbol{y}_v^- - \tau_v \otimes \boldsymbol{y}_v^+ + \sigma_v u_v, \tag{1}$$

where $S_v \in \mathbb{R}^{k_v^+ \times k_v^-}$ is the switching matrix. Its row number is equal to the out-degree k_v^+, and its column number is equal to the in-degree k_v^-. Here $\otimes$ denotes Hadamard product; σ_v will be 1 if node v is a driver node and, otherwise, it will be 0. In the structural edge controllability,[12] S_v must be a structural matrix, in which all nonzero elements are independent free parameters.

Let $\boldsymbol{x} = [x_1, x_2, \ldots, x_M]$ being the state vectors of all edges. Equation (1) can be written as a linear time-invariant

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system:

$$\dot{\boldsymbol{x}} = (W - T)\boldsymbol{x} + H\boldsymbol{u}, \tag{2}$$

where $W \in \mathbb{R}^{M\times M}$ is the state matrix, in which w_{ij} is nonzero if and only if the head of edge j is the tail of edge i. The damping matrix $T \in \mathbb{R}^{M\times M}$ is a diagonal matrix whose diagonal elements are the damping terms corresponding to each edge. The input matrix $H \in \mathbb{R}^{M\times M}$ will a diagonal matrix, whose ith diagonal element is σ_v, if node v is the tail of edge i.

The switchboard dynamics is relevant to some real systems with complex network topological features. It is suitable for modeling network systems where nodes are active components with information processing capabilities. For example, in the Internet with computers and routers, the edges represent physical connections such as ethernet cables, optical fiber cables and wireless connections. A node (e.g., a router) constantly processes the information received from its upstream neighbors and makes decisions which nodes to contact in the downstream neighbors. The information received and passed by a node can then be represented by the state variables on its incoming and outgoing edges. Their dynamical evolutions are governed by the switching matrix in each node. The state variables, together with the switching matrices, define the edge dynamics.

The switchboard dynamics describes the edge dynamic and gives rise to several conclusions of the structural controllability of edge dynamics that differ from nodal dynamics. The key conclusion is that the minimum set of driver node and driven edges required to control the edge dynamics are determined by the local information of nodes. Specifically, the minimum set of driver nodes is determined by selecting the divergent nodes ($k_v^+ > k_v^-$) and one arbitrary node from each balanced component ($k_v^+ = k_v^-$ for all nodes in a connected component). Here $k_v^+ - k_v^-$ of outgoing edges of a driver node denotes the driven edges. One of the outgoing edges of the selected driver node in each balanced component is the driven edge. The criterion for discerning driver nodes and driven edges is a major difference in the structural controllability between the edge dynamics and the node dynamics, and gives rise to several controllability properties of the edge dynamics that differ markedly from those associated with the nodal dynamics. One of the main conclusions is that a positive correlation between the in- and out-degrees can enhance the edge controllability.[12]

3. Effect of degree correlation on the edge controllability of real networks

We analyze the effect of the correlation between in- and out-degrees on the edge controllability of real networks. The edge controllability is measured by the minimum number N_{D} of driver nodes and the minimum number M_{D} of driven edges required to maintain full control.[12] The Pearson correlation coefficient[27] can be used to quantify the correlation between in- and out-degrees of a network. Specifically, for a network with in-degree sequence $K_{\mathrm{in}} = \{k_1^-, k_2^-, \ldots, k_N^-\}$ and out-degree sequence $K_{\mathrm{out}} = \{k_1^+, k_2^+, \ldots, k_N^+\}$, its Pearson correlation coefficient is

$$P = \frac{\sum_{i=1}^{N}(k_i^- - \overline{k^-})(k_i^+ - \overline{k^+})}{\sqrt{\sum_{i=1}^{N}(k_i^- - \overline{k^-})^2}\sqrt{\sum_{i=1}^{N}(k_i^+ - \overline{k^+})^2}}, \tag{3}$$

where $\overline{k^-} = (1/N)\sum_{i=1}^{N} k_i^-$ and $\overline{k^+} = (1/N)\sum_{i=1}^{N} k_i^+$. $P = 1$ indicates total positive linear correlation between K_{in} and K_{out}, $P = 0$ means no correlation, and $P = -1$ the total negative linear correlation. We refer to correlation between in- and out-degrees as degree correlation in the rest of this paper for simplicity.

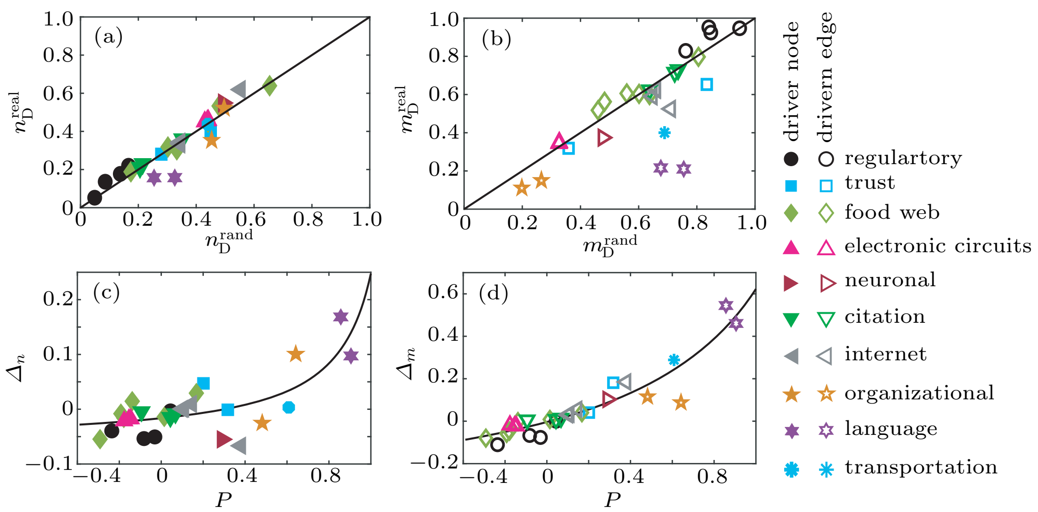

Fig. 1. Effect of degree correlation on the edge controllability of real networks. (a) The fraction of driver nodes $n_{\mathrm{D}}^{\mathrm{real}}$ obtained directly and $n_{\mathrm{D}}^{\mathrm{rand}}$ of real networks with no degree correlation. (b) The fraction of driven edges $m_{\mathrm{D}}^{\mathrm{real}}$ obtained directly and $m_{\mathrm{D}}^{\mathrm{rand}}$ of real networks with no degree correlation. (c) and (d) The differences $\Delta_n = n_{\mathrm{D}}^{\mathrm{rand}} - n_{\mathrm{D}}^{\mathrm{real}}$ and $\Delta_m = m_{\mathrm{D}}^{\mathrm{rand}} - m_{\mathrm{D}}^{\mathrm{real}}$ as the function of the Pearson correlation coefficient P of real networks. All the numerical results are obtained by averaging over 50 independent networks realizations. See Table 1 for details.

In order to study the effect of the degree correlation on the edge controllability, we apply a randomization, which keeps in-degree sequence K_{in} and out-degree sequence K_{out} of a real network unchanged but selects randomly the combination of in-degree k_i^- and out-degree k_j^+ for each node, i.e., $P \approx 0$. The minimum number of driver nodes and driven edges required to maintain full control of the edge dynamics in the randomization are defined as $N_{\mathrm{D}}^{\mathrm{rand}}$ and $M_{\mathrm{D}}^{\mathrm{rand}}$, respectively. We compare the $n_{\mathrm{D}}^{\mathrm{rand}} = N_{\mathrm{D}}^{\mathrm{rand}}/N$ ($m_{\mathrm{D}}^{\mathrm{rand}} = M_{\mathrm{D}}^{\mathrm{rand}}/M$) and $n_{\mathrm{D}}^{\mathrm{real}}$ ($m_{\mathrm{D}}^{\mathrm{real}}$) obtained directly from the real networks. As shown in Figs. 1(a) and 1(b), there is a significant deviation in the edge controllability of real networks and their randomization, especially $m_{\mathrm{D}}^{\mathrm{real}}$ and $m_{\mathrm{D}}^{\mathrm{rand}}$. This shows that the degree correlation plays an important role in the edge controllability of real networks.

To further investigate the impact of the degree correlation on the edge controllability, we show the differences $\Delta_n = n_{\mathrm{D}}^{\mathrm{rand}} - n_{\mathrm{D}}^{\mathrm{real}}$ and $\Delta_m = m_{\mathrm{D}}^{\mathrm{rand}} - m_{\mathrm{D}}^{\mathrm{real}}$ versus the Pearson correlation coefficient P of real networks. As shown in Figs. 1(c) and 1(d), a basic trend is that Δ_n and Δ_m increase with the increasing P. This shows that a stronger degree correlation has a greater impact on the edge controllability. Meanwhile, this further illustrates the huge impact of the degree correlation on the edge controllability of real networks.

4. Controllability limit theory

We use the controllability limit theory[22] to quantify the effect of the degree correlation on the edge controllability of real networks. The controllability limit is the limits of acceptable change of N_{D} and M_{D} by adjusting the degree correlation only. That is, the in-degree sequence K_{in} and out-degree sequence K_{out} remain unchanged, and in-degree k_i^- and out-degree k_j^+ of each node are reconfigured to maximize (or minimize) N_{D} and M_{D}.

We first give the calculation method of the controllability limit. In general, the balanced component is infrequent in directed networks. It has little influence on N_{D} and M_{D}.[12] Hence we neglect the possible presence of the balanced component. Then the classification of driver node and driven edge depends solely on the in- and out-degrees of each node. Specifically, a divergent node ($k_v^+ > k_v^-$) must be a driver node, and each driver node must control $k_v^+ - k_v^-$ of its outgoing edges. This allows us to calculate the controllability limit based on the maximum match and weighted maximum match.

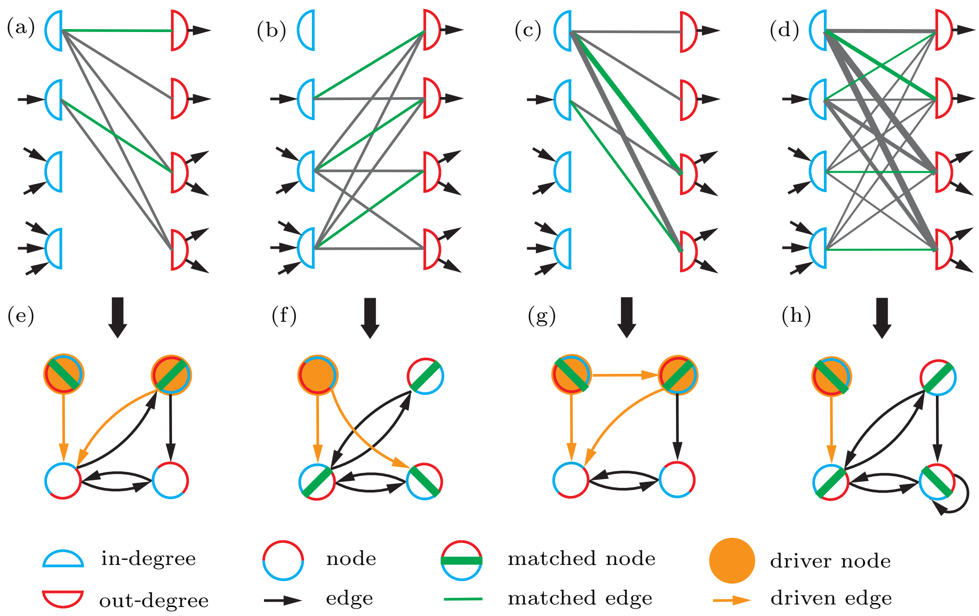

Fig. 2. Controllability limit theory. The method of calculating the controllability limits of a network with in-degree sequence $K_{\mathrm{in}} = \{0,1,2,3\}$ and out-degree sequence $K_{\mathrm{out}} = \{1,1,2,2\}$. (a) In the bipartite graph H, the node i from in-degree sequence and the node j from out-degree sequence are connected if $k_i^- < k_j^+$. Its generated network in (e) has $N_{\mathrm{D}}^{\mathrm{U}} = 2$. (b) In the bipartite graph $\overline{H}$, the node i from in-degree sequence and the node j from out-degree sequence will be connected if $k_i^- \geq k_j^+$. Its generated network in (f) has $N_{\mathrm{D}}^{\mathrm{L}} = 1$. (c) The weighted bipartite graph H^* has the same topological structure as H, and the weight of each edge is $k_j^+ - k_i^-$. Its generated network in (g) has $M_{\mathrm{D}}^{\mathrm{U}} = 3$. (d) The weighted bipartite graph $\overline{H^*}$ is generated by connecting arbitrary two nodes, and assigning the weight $k_i^- - k_j^+$ to the edges satisfying $k_i^- < k_j^+$, and 0 for other edges. Its generated network in (h) has $M_{\mathrm{D}}^{\mathrm{L}} = 1$. Note that the matching nodes of the generated networks are from the matched edges in the maximum matching, and other nodes of the generated networks are combined randomly.

As shown in Fig. 2(a), a bipartite graph H is generated by the in-degree sequence $K_{\mathrm{in}} = \{0,1,2,3\}$ and the out-degree sequence $K_{\mathrm{out}} = \{1,1,2,2\}$, where node i from in-degree sequence and node j from out-degree sequence are connected if $k_i^- < k_j^+$. Each edge in H corresponds to a potential driver node in the generated network. As shown in Fig. 2(e), its generated network has the upper limit of N_{D}, which is

$$N_{\mathrm{D}}^{\mathrm{U}} = \max(1, |M_H|), \tag{4}$$

where $|M_H|$ is the number of matching edges in the maximum matching of H. Similarly, each edge of $\overline{H}$ in Fig. 2(b) corresponds to a potential non-driver node in the generated network. As shown in Fig. 2(f), its generated network has the lower limit of N_{D}, which is

$$N_{\mathrm{D}}^{\mathrm{L}} = \max(1, N - |M_{\overline{H}}|), \tag{5}$$

where $|M_{\overline{H}}|$ is the number of matching edges in the maximum matching of $\overline{H}$.

As shown in Fig. 2(c), the weighted bipartite graph H^* has the same topological structure as H, and the weight of each edge is $k_j^+ - k_i^-$. The weight in H^* corresponds to the number of potential driven edges in the generated network. As shown in Fig. 2(g), its generated network has the upper limit of M_{D}, which is

$$M_{\mathrm{D}}^{\mathrm{U}} = \max(1, |M_{H^*}|), \tag{6}$$

where $|M_{H^*}|$ is the sum of edge weights in the weighted maximum matching of H^*. Differently, as shown in Fig. 2(d), the weighted bipartite graph $\overline{H^*}$ is generated by connecting arbitrary two nodes, and assigning the weight $k_i^- - k_j^+$ to the edges satisfying $k_i^- < k_j^+$, and 0 for other edges. The absolute value of the weight $|k_i^- - k_j^+|$ in $\overline{H^*}$ corresponds to the number of potential driven edges in the generated network. The weighted maximum matching of $\overline{H^*}$ will preferentially select edges with weight 0 and smaller weight $|k_i^- - k_j^+|$. Thus its generated network has the lower limit of M_{D}, which is

$$M_{\mathrm{D}}^{\mathrm{L}} = \max(1, |M_{\overline{H^*}}|), \tag{7}$$

where $|M_{\overline{H^*}}|$ is the absolute values of the sum of edge weights in the weighted maximum matching of $\overline{H^*}$.

5. Controllability limit of real networks

We use the tools developed above to determine the controllability limit of real networks. The upper and lower limits of n_{D} (m_{D}) of different types of real networks are displayed in Figs. 3(a) and 3(b). An important observation is that the upper and lower limits of n_{D} (m_{D}) hold for all kinds of real networks. Any values of n_{D} (m_{D}) in between the limits are achievable by properly adjusting the degree correlation. This demonstrates the significant effect of the degree correlation on the edge controllability of real networks.

As shown in Figs. 3(c) and 3(d), another notable result is that the differences $n_{\mathrm{D}}^{\mathrm{U}} - n_{\mathrm{D}}^{\mathrm{L}}$ ($m_{\mathrm{D}}^{\mathrm{U}} - m_{\mathrm{D}}^{\mathrm{L}}$) increase as the average degree $\langle k \rangle$ increases. This shows that degree correlation has a bigger effect on n_{D} (m_{D}) in dense real networks, in which the range between the upper and lower limits is larger. In other words, the edge controllability of dense real networks is more susceptible to the degree correlation.

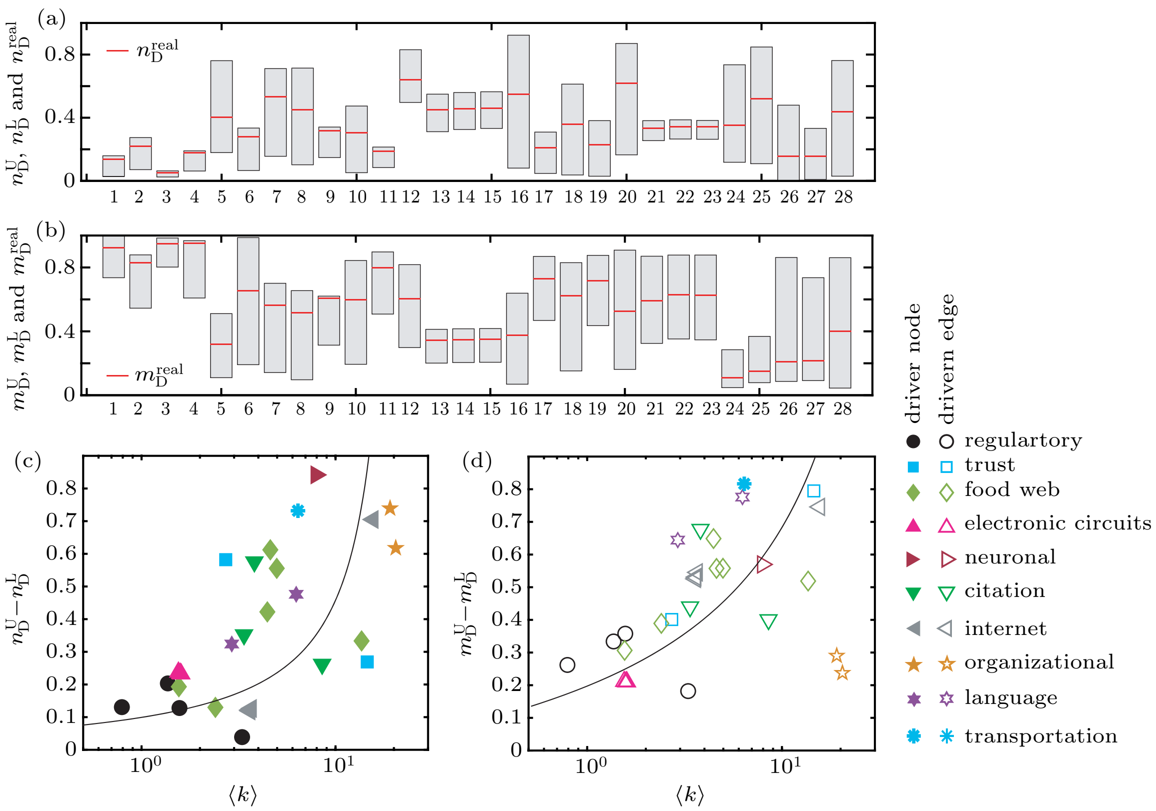

Fig. 3. Controllability limit of real networks. (a) The fraction of driver nodes $n_{\mathrm{D}}^{\mathrm{real}}$ obtained directly and the controllability limit ($n_{\mathrm{D}}^{\mathrm{U}}$ and $n_{\mathrm{D}}^{\mathrm{L}}$) of real networks. (b) The fraction of driven edges $m_{\mathrm{D}}^{\mathrm{real}}$ obtained directly and the controllability limit ($m_{\mathrm{D}}^{\mathrm{U}}$ and $m_{\mathrm{D}}^{\mathrm{L}}$) of real networks. (c) and (d) The differences $n_{\mathrm{D}}^{\mathrm{U}} - n_{\mathrm{D}}^{\mathrm{L}}$ and $m_{\mathrm{D}}^{\mathrm{U}} - m_{\mathrm{D}}^{\mathrm{L}}$ versus the average degree $\langle k \rangle$ of real networks. The numbers in (a) and (b) refer to the network indices in Table 1.

6. Anomaly in edge controllability of real networks

One of the main conclusions in Ref. [12] is that the positive correlation between the in- and out-degrees can enhance the edge controllability. That is, for a real network with positive degree correlation $P > 0$, its differences read $\Delta_n > 0$ and $\Delta_m > 0$. Figure 4 shows the Pearson correlation coefficient P, the differences Δ_n and Δ_m of each real network. As show in Fig. 4(a), we find an anomaly that the edge dynamics in some real networks with positive degree correlation may be difficult to be controlled. A typical example is that $\Delta_n = -0.066$ and $P = 0.379$ in the Political blogs network (No. 20 in Table 1). However, as shown in Fig. 4(b), a similar anomaly does not appear from the point of m_{D}.

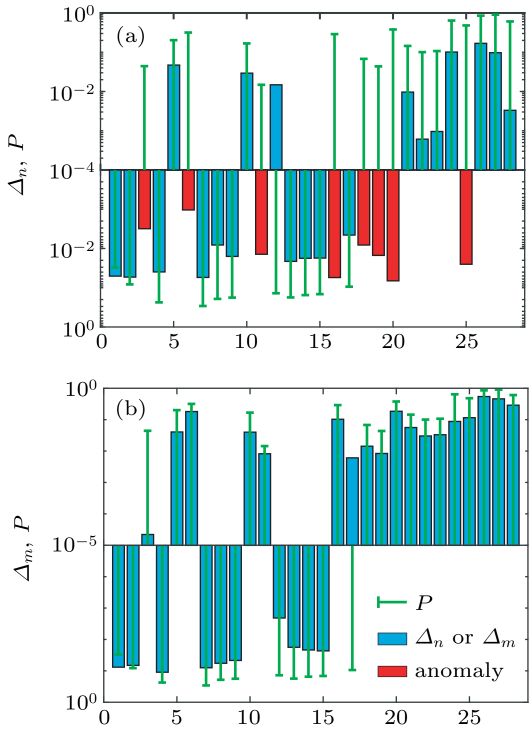

Fig. 4. Anomaly in edge controllability of real networks. (a) The Pearson correlation coefficient P and the differences Δ_n of real networks. (b) The Pearson correlation coefficient P and the differences Δ_m of real networks. All the numerical results are obtained by averaging over 50 independent networks in realizations. The numbers refer to the network indices in Table 1.

The reason for the anomaly is that there are some nodes in real networks whose out-degree is slightly larger than in-degree. These nodes not only maintain a positive correlation between the in- and out-degrees, but also generate a large number of driver nodes. For example, for a network generated by $K_{\mathrm{in}} = K_{\mathrm{out}} = \{1, 2, \ldots, 100\}$, the case of all nodes with the same in- and out-degrees causes the network to reach the lower limit $n_{\mathrm{D}}^{\mathrm{L}} = 0.01$. Its Pearson correlation coefficient is $P_{\mathrm{ND}}^{\mathrm{L}} = 1$. Conversely, the case of 99 nodes with $k_v^- = k_v^+ - 1$ causes the network to reach the upper limit $n_{\mathrm{D}}^{\mathrm{U}} = 0.99$. Its Pearson correlation coefficient is $P_{\mathrm{ND}}^{\mathrm{U}} = 0.94$. One can see that the positive degree correlation applies to both upper and lower limits. This shows that the positive degree correlation may reduce the edge controllability from the point of n_{D}. However, a similar anomaly does not appear from the point of m_{D}. Specifically, in the above example, the case of all nodes with the same in- and out-degrees leads to $m_{\mathrm{D}}^{\mathrm{L}} = 0.01$ and $P_{\mathrm{MD}}^{\mathrm{L}} = 1$; conversely, the case of each node with in-degree $k_v^- = \mathrm{i}$ and out-degree $k_v^+ = 101 - \mathrm{i}$ ($\mathrm{i} = 1, 2, \ldots, 100$) leads to $m_{\mathrm{D}}^{\mathrm{U}} = 0.495$ and $P_{\mathrm{MD}}^{\mathrm{U}} = -1$.

We further study the anomaly based on the controllability limit theory, and conduct the following simulations. Firstly, we calculate the range of P of model networks and real networks. Specifically, the in-degree and out-degree sequences of a network are sorted from small to large, and are denoted as $K_{\mathrm{in}} = \{k_1^-, k_2^-, \ldots, k_N^-\}$ and $K_{\mathrm{out}} = \{k_1^+, k_2^+, \ldots, k_N^+\}$, respectively. When we assign individual node with in-degree k_i^- and out-degree k_i^+ ($i = 1, 2, \ldots, N$), its generated network has the strongest positive degree correlation and the largest p-value. On the contrary, when we assign individual node with in-degree k_i^- and out-degree k_{N-i+1}^+ ($i = 1, 2, \ldots, N$), its generated network has the strongest negative degree correlation and the smallest p-value. Note that P may not reach 1 or -1 for some networks.

Secondly, for a generated network with the upper limit $N_{\mathrm{D}}^{\mathrm{U}}$, we calculate its Pearson correlation coefficient $P_{\mathrm{ND}}^{\mathrm{U}}$. Note that we use the maximum matching to determine $N_{\mathrm{D}}^{\mathrm{U}}$. The in- and out-degrees of the nodes, which correspond to the matched edges in H, are fixed in the generated network. However, the in- and out-degrees of the rest nodes are adjustable. This leads to the fact that $P_{\mathrm{ND}}^{\mathrm{U}}$ is allowed to fluctuate in some extent, which is $P_{\mathrm{ND}}^{\mathrm{U}} \in [P', P'']$. We assign the minimum unmatched in-degree to the maximum unmatched out-degree in turn, and calculate P'. Then we assign the minimum unmatched in-degree to the minimum unmatched out-degree in turn, and calculate P''. In a similar way, we calculate $P_{\mathrm{MD}}^{\mathrm{U}}$, $P_{\mathrm{ND}}^{\mathrm{L}}$ and $P_{\mathrm{MD}}^{\mathrm{L}}$ of model and real networks.

In Figs. 5(a)–5(d), we give the ranges of P, $P_{\mathrm{ND}}^{\mathrm{U}}$, $P_{\mathrm{ND}}^{\mathrm{L}}$, $P_{\mathrm{MD}}^{\mathrm{U}}$ and $P_{\mathrm{MD}}^{\mathrm{L}}$ of model networks. As shown in Figs. 5(a) and 5(c), the positive degree correlation ($P \approx 0.5$) applies to both $n_{\mathrm{D}}^{\mathrm{U}}$ and $n_{\mathrm{D}}^{\mathrm{L}}$ in the model networks with large $\langle k \rangle$. This shows that the positive degree correlation may reduce the edge controllability from the point of n_{D}. Conversely, as shown in Figs. 5(b) and 5(d), if we consider the edge controllability from the point of m_{D}, the positive degree correlation indeed enhances the edge controllability. The same result also appeared on real networks. As shown in Fig. 5(e), except one of the food web (No. 7 in Table 1) and the electronic circuits (Nos. 13, 14 and 15 in Table 1), the positive degree correlation applies to both $n_{\mathrm{D}}^{\mathrm{U}}$ and $n_{\mathrm{D}}^{\mathrm{L}}$ in real networks. Conversely, as shown in Fig. 5(f), the anomaly does not appear from the point of m_{D} in real networks.

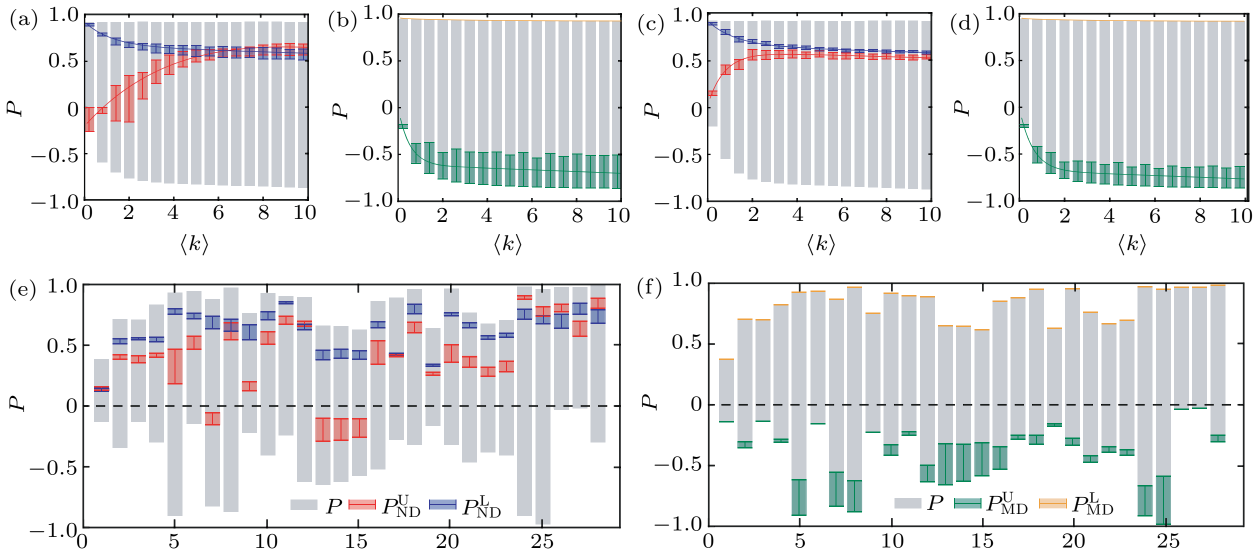

Fig. 5. Anomaly in edge controllability. The range of Pearson correlation coefficient P in (a)–(d) model networks and (e)–(f) real networks. The Pearson correlation coefficient $P_{\rm ND}^{\rm U}$ (red) and $P_{\rm ND}^{\rm L}$ (blue) in [(a), (c)] model networks and (e) real networks. The Pearson correlation coefficient $P_{\rm MD}^{\rm U}$ (green) and $P_{\rm MD}^{\rm L}$ (orange) in [(b), (d)] model networks and (f) real networks. The model network is generated by given degree distribution, where in-degree follows exponent distribution and out-degree follows Poisson distribution in (a) and (b), and in-degree follows Poisson distribution and out-degree follows exponent distribution in (c) and (d). See Appendix for how to construct a model network. All the numerical results are obtained by averaging over 50 independent networks in realizations. The numbers in (e)–(f) refer to the network indices in Table 1.

7. Conclusions

We have studied the effect of degree correlation on edge controllability of real networks. Simulation results show that a stronger degree correlation has a greater impact on the edge controllability. Meanwhile, degree correlation plays a more important role in edge controllability of dense networks. Then we use the controllability limit theory to quantify the effect of degree correlation on edge controllability. Evaluation of real networks indicates that the upper and lower controllability limits hold for all kinds of real networks. Any edge controllability in between the limits are achievable by properly adjusting the degree correlation. In addition, we find that the edge dynamics in the networks with positive degree correlation may be difficult to control, and explain this anomaly based on the controllability limit theory. The results not only deepen our understanding of edge controllability of real networks, but also raise several new problems. Future research directions include the effect of the degree correlation on the controllable subspace, target control and control energy of real networks.

Appendix A: Real networks

The details of the real networks we have studied are presented in Table 1.

Table 1. Simulation results of real networks. For each real network, we show its type, name, nodes' number N, edges' number M, the Pearson correlation coefficient P, the fraction of driver nodes and driven edges calculated in the real network ($n_{\rm D}^{\rm real}$ and $m_{\rm D}^{\rm real}$), after randomization ($n_{\rm D}^{\rm rand}$ and $m_{\rm D}^{\rm rand}$), and the controllability limits ($n_{\rm D}^{\rm U}$, $n_{\rm D}^{\rm L}$, $m_{\rm D}^{\rm U}$ and $m_{\rm D}^{\rm L}$).

Type	No.	Name	N	M	P	$n_{\rm D}^{\rm real}$	$m_{\rm D}^{\rm real}$	$n_{\rm D}^{\rm rand}$	$m_{\rm D}^{\rm rand}$	$n_{\rm D}^{\rm U}$	$n_{\rm D}^{\rm L}$	$m_{\rm D}^{\rm U}$	$m_{\rm D}^{\rm L}$
Regulatory	1	Ownership-USCorp[28]	8497	6726	−0.031	0.136	0.924	0.086	0.848	0.159	0.028	1.000	0.738
	2	TRN-EC-2[29]	423	578	−0.082	0.220	0.829	0.166	0.762	0.274	0.071	0.879	0.545
	3	TRN-Yeast-1[30]	4684	15451	0.044	0.052	0.947	0.049	0.947	0.064	0.025	0.984	0.803
	4	TRN-Yeast-2[29]	688	1079	−0.236	0.177	0.952	0.138	0.841	0.190	0.063	0.968	0.610
Trust	5	Prison inmate[31]	67	182	0.201	0.403	0.319	0.450	0.359	0.761	0.179	0.511	0.110
	6	Wiki Vote[32]	7115	103689	0.318	0.281	0.653	0.279	0.834	0.335	0.066	0.987	0.192
Food web	7	St.Marks[33]	45	224	−0.292	0.533	0.563	0.479	0.483	0.711	0.156	0.701	0.143
	8	Seagrass[34]	49	226	−0.192	0.449	0.518	0.441	0.46	0.714	0.102	0.655	0.097
	9	Grassland[35]	88	137	−0.179	0.318	0.606	0.302	0.559	0.341	0.148	0.620	0.314
	10	Ythan[35]	135	601	0.168	0.304	0.597	0.333	0.637	0.474	0.052	0.844	0.195
	11	Silwood[36]	154	370	0.014	0.188	0.797	0.174	0.806	0.214	0.084	0.897	0.508
	12	Little Rock[37]	183	2494	−0.138	0.639	0.603	0.654	0.601	0.831	0.497	0.818	0.299
Electronic	13	S208a[29]	122	189	−0.177	0.451	0.344	0.430	0.326	0.549	0.311	0.413	0.201
circuits	14	s420a[29]	252	399	−0.154	0.456	0.348	0.439	0.327	0.560	0.325	0.416	0.206
	15	s838a[29]	512	819	−0.146	0.459	0.350	0.441	0.327	0.564	0.332	0.418	0.208

Table 1. (*Continued*).

Type	No.	Name	N	M	P	n_D^{real}	m_D^{real}	n_D^{rand}	m_D^{rand}	n_D^U	n_D^L	m_D^U	m_D^L
Neuronal	16	C. elegans[38]	297	2359	0.291	0.549	0.374	0.494	0.477	0.923	0.081	0.639	0.069
Citation	17	Small World[39]	233	1988	−0.094	0.210	0.729	0.206	0.735	0.309	0.047	0.869	0.469
	18	SciMet[39]	2729	10416	0.068	0.360	0.623	0.352	0.638	0.613	0.037	0.830	0.153
	19	Kohonen[40]	3772	12731	0.044	0.230	0.715	0.215	0.724	0.381	0.029	0.876	0.436
Internet	20	Political blogs[41]	1224	19090	0.379	0.619	0.525	0.553	0.710	0.870	0.165	0.908	0.162
	21	p2p-1[42]	10876	39994	0.145	0.334	0.591	0.344	0.647	0.381	0.255	0.870	0.325
	22	p2p-2[42]	8846	31839	0.101	0.344	0.628	0.344	0.659	0.387	0.265	0.878	0.352
	23	p2p-3[42]	8717	31525	0.107	0.343	0.625	0.344	0.658	0.383	0.264	0.878	0.347
Organizational	24	Freeman-1[43]	34	695	0.642	0.353	0.111	0.454	0.199	0.735	0.118	0.285	0.047
	25	Consulting[44]	46	879	0.482	0.522	0.150	0.497	0.266	0.848	0.109	0.369	0.078
Language	26	English words[31]	7381	46281	0.857	0.158	0.210	0.326	0.755	0.479	0.003	0.862	0.087
	27	French words[31]	8325	24295	0.905	0.157	0.216	0.254	0.676	0.333	0.009	0.736	0.092
Transportation	28	USair97[45]	332	2126	0.608	0.437	0.400	0.440	0.689	0.762	0.030	0.861	0.045

Appendix B: Model networks

A model network with N nodes is structured by giving in- and out-degree distributions, including Poisson distribution and exponential distribution. According to a given degree distribution, a degree sequence can be obtained, where the in- and out-degree sequences are denoted by $K_{\text{in}} = \{k_1^-, k_2^-, \ldots, k_N^-\}$ and $K_{\text{out}} = \{k_1^+, k_2^+, \ldots, k_N^+\}$, respectively. Note that N must be large enough to ensure that the degree sequence is completely encoded by the degree distribution. Meanwhile, we can finely tune the degree sequence to ensure $\sum_i k_i^- = \sum_i k_i^+$, which will not change the intrinsic degree distribution.

A directed network starts from N isolated nodes. Each node is assigned in-degree k_i^- and out-degree k_j^+ from in- and out-degree sequences, respectively. Each time, two nodes v with $k_v^- > 0$ and node u with $k_u^+ > 0$ are randomly selected and connected from u to v. Then the in-degree of the node v turns into $k_v^- - 1$ and the out-degree of the node u turns into $k_u^+ - 1$. Repeat this process until all nodes meet the given in- and out-degrees. Note that the multiple edges in the generated network will be processed by edge swapping, i.e., turning edges e_{uv} and e_{kl} to edges e_{ul} and e_{kv} if there exist multiple edges e_{uv}, where $k \neq u$ and $l \neq v$.

We can build different networks by the in- and out-degree sequences. For example, as shown in Fig. 2, the in- and out-degrees sequences are $K_{\text{in}} = \{0,1,2,3\}$ and $K_{\text{out}} = \{1,1,2,2\}$, respectively. As shown in Figs. 2(e)–2(h), by matching the in- and out-degree sequences based on different strategies, different networks are generated by the same in- and out-degree sequences.

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