


# Chirped gap solitons with Kudryashov's law of self-phase modulation having dispersive reflectivity

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**Abstract.** The present study is devoted to investigate the chirped gap solitons with Kudryashov's law of self-phase modulation having dispersive reflectivity. Thus, the mathematical model consists of coupled nonlinear Schrödinger equation (NLSE) that describes pulse propagation in a medium of fiber Bragg gratings (BGs). To reach an integrable form for this intricate model, the phase-matching condition is applied to derive equivalent equations that are handled analytically. By means of auxiliary equation method which possesses Jacobi elliptic function (JEF) solutions, various forms of soliton solutions are extracted when the modulus of JEF approaches 1. The generated chirped gap solitons have different types of structures such as bright, dark, singular, W-shaped, kink, anti-kink and Kink-dark solitons. Further to this, two soliton waves namely chirped bright quasi-soliton and chirped dark quasi-soliton are also created. The dynamic behaviors of chirped gap solitons are illustrated in addition to their corresponding chirp. It is noticed that self-phase modulation and dispersive reflectivity have remarkable influences on the pulse propagation. These detailed results may enhance the engineering applications related to the field of fiber BGs.

**Keywords:** Chirped gap solitons, Bragg gratings, Kudryashov's law.

## 1 Introduction

The new technology in information industry depends broadly on optical fibers since its presence as a prominent mechanism that transmits light and signals over long distances and local area networks or computer networks [1–5]. The field of optical fibers can lead to further developments in the engineering and industrial applications that serve wide ranges of sectors [6, 7]. In particular, the essential applications of optical fibers include telecommunications, sensors, bio-medicals, and fibre lasers [8–13]. The process of transmitting data is made by the soliton propagation due to the balance between chromatic dispersion (CD) and fiber nonlinearity. Unfortunately, the low contribution from CD may causes a restriction in the transmission scenario. This crisis can be effectively manipulated by making use of Bragg gratings (BGs) technology which compensates for low CD. In the last decade, many experts around the world have extensively studied the dynamic of solitons in fiber BGs with different forms of nonlinear refractive index such as Kerr law, quadratic-cubic law, parabolic law, polynomial law, parabolic-nonlocal combo law and many others [14–20]. Additionally, the characteristic of soliton propagation associated with the frequency chirp influence has been being studied continuously through the years as the chirp has significant advantages including pulse compression and amplification in optical fiber [21–27].

Rece, a significant model known as Kudryashov's equation (KE) [28] was developed to study the soliton pulse propagation in the field of nonlinear optics. The KE is considered as a part of the family of nonlinear Schrödinger equation

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and it is generated from a law of refractive index. Since its appearance in 2019, the KE has been discussed by many scholars to deal with some physical features like conservation laws and optical soliton behaviors by means of various integration schemes and techniques such as Lie symmetry analysis, extended sinh-Gordon equation expansion method, complete discriminant system for a polynomial, new mapping method, unified auxiliary equation scheme, improved modified extended tanh-function approach, extended trial function method and unified ansätze framework. Different soliton structures were derived such as bright, dark, singular, bright-dark, singular-dark solitons and others. For more details about obtained results, reader is referred to the references [29–36]. The governing KE is given by

$$iq_t + aq_{xx} + \left( \frac{b_1}{|q|^{2n}} + \frac{b_2}{|q|^n} + b_3|q|^n + b_4|q|^{2n} \right) q = 0, \quad (1)$$

where the first term represents the time evolution and  $i = \sqrt{-1}$ . The term with the coefficient  $a$  stands for the group velocity dispersion while the terms with the coefficients  $b_1, b_2, b_3, b_4$  describe the effect of self-phase modulation (SPM). In the literature, some generalized models of equation (1) are discussed to investigate optical solitons by applying distinct strategies, see as example references [37–39].

The model of KE can be also implemented to fiber BGs to examine its influence on the pulse propagation. For example, Zayed et al. [16] detected the applicability of KE to fiber BGs with dispersive reflectivity and Kerr law of nonlinear refractive index when  $n = 1, n = 2, n = 3$ . Using the extended Kudryashov's scheme, both chirped and chirp-free optical solitons are retrieved and they are found to have the forms of dark and singular soliton profiles. It is necessary to be mentioned that the chirping associated to these solitons is expressed in terms of constant. Our current work aims to investigate the chirped gap solitons with Kudryashov's law of self-phase modulation having dispersive reflectivity when  $n = 2$ . Herein, we assume that the chirp has a form of nonlinear function.

As stated above, this study focuses on the model of Kudryashov's equation (KE) in fiber medium having BGs effect. The vector-coupled KE reads [16]

$$iq_t + a_1 r_{xx} + \frac{f_1 q}{b_1 |q|^4 + c_1 |q|^2 |r|^2 + d_1 |r|^4} + \frac{g_1 q}{h_1 |q|^2 + k_1 |r|^2} + (l_1 |q|^2 + m_1 |r|^2) q + (n_1 |q|^4 + p_1 |q|^2 |r|^2 + s_1 |r|^4) q + i\alpha_1 q_x + \beta_1 r = 0, \quad (2)$$

$$ir_t + a_2 q_{xx} + \frac{f_2 r}{b_2 |r|^4 + c_2 |r|^2 |q|^2 + d_2 |q|^4} + \frac{g_2 r}{h_2 |r|^2 + k_2 |q|^2} + (l_2 |r|^2 + m_2 |q|^2) r + (n_2 |r|^4 + p_2 |r|^2 |q|^2 + s_2 |q|^4) r + i\alpha_2 r_x + \beta_2 q = 0, \quad (3)$$

where the functions  $q(x, t)$  and  $r(x, t)$  stand for forward and backward propagating waves, respectively, whereas  $a_j$  for  $j = 1, 2$  represent the coefficients of dispersive reflectivity. In the coupled equations above,  $b_j, h_j, l_j$  and  $n_j$  indicate the coefficients of self-phase modulation (SPM) and  $c_j, d_j, k_j, m_j, p_j$  and  $s_j$  denote the cross-phase modulation XPM, respectively. The coefficients  $f_j$  and  $g_j$  represent the combination of SPM and XPM. Finally,  $\alpha_j$  account for inter-modal dispersion and  $\beta_j$  define detuning parameters. All of the coefficients are real valued constants and  $i = \sqrt{-1}$ .

The following sections of paper are formatted as follows. In Section 2, the governing model is analyzed and reduced to an integrable form. Section 3 displays the derivation of chirped gap solitons with the aid of auxiliary equation method. The structures and behaviors of created solitons are discussed and described in Section 4. Finally, the summary of obtained results is given in Section 5.

## 2 Mathematical analysis of governing model

In order to reduce the coupled-KE give by (2) and (3) to an integrable form, its complex structure is analyzed using the transformation given by

$$q(x, t) = \psi_1(\xi) e^{i(\phi(\xi) - \omega t)}, \quad (4)$$

$$r(x, t) = \psi_2(\xi) e^{i(\phi(\xi) - \omega t)}, \quad (5)$$

where  $\xi = x - vt$  while  $\omega$  and  $v$  are real constants indicating the wave number and the soliton velocity. The two functions  $\psi_1(\xi)$  and  $\psi_2(\xi)$  account for the amplitudes of the solitons whereas the function  $\phi(\xi)$  represents nonlinear phase shift.

The corresponding chirp is identified as  $\delta\omega(x, t) = -\frac{\partial}{\partial x}[\phi(\xi) - \omega t] = -\frac{d\phi(\xi)}{d\xi}$ .

Inserting (4) and (5) into the coupled system (2) and (3) and breaking down into the imaginary and real components, we reach

$$a_1\psi''_2 + \omega\psi_1 + \beta_1\psi_2 + (v - \alpha_1)\psi_1\phi' - a_1\psi_2\phi'^2 + \frac{f_1\psi_1}{b_1\psi_1^4 + c_1\psi_1^2\psi_2^2 + d_1\psi_1^4} + \frac{g_1\psi_1}{h_1\psi_1^2 + k_1\psi_2^2} + (l_1\psi_1^2 + m_1\psi_2^2)\psi_1 + (n_1\psi_1^4 + p_1\psi_1^2\psi_2^2 + s_1\psi_2^4)\psi_1 = 0, \tag{6}$$

$$a_2\psi''_1 + \omega\psi_2 + \beta_2\psi_1 + (v - \alpha_2)\psi_2\phi' - a_2\psi_1\phi'^2 + \frac{f_2\psi_2}{b_2\psi_2^4 + c_2\psi_2^2\psi_1^2 + d_2\psi_1^4} + \frac{g_2\psi_2}{h_2\psi_2^2 + k_2\psi_1^2} + (l_2\psi_2^2 + m_2\psi_1^2)\psi_2 + (n_2\psi_2^4 + p_2\psi_2^2\psi_1^2 + s_2\psi_1^4)\psi_2 = 0, \tag{7}$$

and

$$(\alpha_1 - v)\psi'_1 + a_1(\psi_2\phi'' + 2\psi'_2\phi') = 0, \tag{8}$$

$$(\alpha_2 - v)\psi'_2 + a_2(\psi_1\phi'' + 2\psi'_1\phi') = 0. \tag{9}$$

To handle this complexity, we assume

$$\psi_2 = \gamma\psi_1, \tag{10}$$

where  $\gamma \neq 1$  is a real constant. Accordingly, the set of equations (6)–(9) are converted into

$$a_1\gamma\psi_1^3\psi''_1 + \frac{f_1}{b_1 + c_1\gamma^2 + d_1\gamma^4} + \frac{g_1\psi_1^2}{h_1 + k_1\gamma^2} + \left[\omega + \beta_1\gamma + (v - \alpha_1)\phi' - a_1\gamma\phi'^2\right]\psi_1^4 + (l_1 + m_1\gamma^2)\psi_1^6 + (n_1 + p_1\gamma^2 + s_1\gamma^4)\psi_1^8 = 0, \tag{11}$$

$$a_2\psi_1^3\psi''_1 + \frac{f_2\gamma}{b_2\gamma^4 + c_2\gamma^2 + d_2} + \frac{g_2\gamma\psi_1^2}{h_2\gamma^2 + k_2} + \left[\omega\gamma + \beta_2 + (v - \alpha_2)\gamma\phi' - a_2\phi'^2\right]\psi_1^4 + (l_2\gamma^2 + m_2)\gamma\psi_1^6 + (n_2\gamma^4 + p_2\gamma^2 + s_2)\gamma\psi_1^8 = 0, \tag{12}$$

and

$$(\alpha_1 - v)\psi'_1 + a_1\gamma(\psi_1\phi'' + 2\psi'_1\phi') = 0, \tag{13}$$

$$(\alpha_2 - v)\gamma\psi'_1 + a_2(\psi_1\phi'' + 2\psi'_1\phi') = 0. \tag{14}$$

The system of equations (13) and (14) can be integrated to yield

$$\phi' = \frac{v - \alpha_1}{2a_1\gamma} + \frac{\rho_1\psi_1^{-2}}{a_1\gamma}, \tag{15}$$

$$\phi' = \frac{(v - \alpha_2)\gamma}{2a_2} + \frac{\rho_2\psi_1^{-2}}{a_2}, \tag{16}$$

where  $\rho_1$  and  $\rho_2$  are the integration constants. Due to the equivalency between equations (15) and (16), one arrives at the constraint conditions given by

$$a_2\rho_1 - a_1\gamma\rho_2 = 0, \tag{17}$$

$$(a_2 - a_1\gamma^2)v - (a_2\alpha_1 - a_1\alpha_2\gamma^2) = 0. \tag{18}$$

From equation (18) we come by the velocity of the soliton in the form

$$v = \frac{a_2\alpha_1 - a_1\alpha_2\gamma^2}{a_2 - a_1\gamma^2}. \quad (19)$$

Then, the chirp expression can be addressed as

$$\delta\omega(x, t) = -\left[\frac{v - \alpha_1}{2a_1\gamma} + \frac{\rho_1\psi_1^{-2}}{\gamma a_1}\right]. \quad (20)$$

Using (15) and (16), the coupling equations (11) and (12) are changed into

$$a_1\gamma\psi_1^3\psi_1'' - \frac{\rho_1^2}{a_1\gamma} + \frac{f_1}{b_1 + c_1\gamma^2 + d_1\gamma^4} + \frac{g_1\psi_1^2}{h_1 + k_1\gamma^2} + \left[\omega + \beta_1\gamma + \frac{(v - \alpha_1)^2}{4a_1\gamma}\right]\psi_1^4 + (l_1 + m_1\gamma^2)\psi_1^6 + (n_1 + p_1\gamma^2 + s_1\gamma^4)\psi_1^8 = 0, \quad (21)$$

$$a_2\psi_1^3\psi_1'' - \frac{a_2\rho_1^2}{a_1^2\gamma^2} + \frac{f_2\gamma}{b_2\gamma^4 + c_2\gamma^2 + d_2} + \frac{g_2\gamma\psi_1^2}{h_2\gamma^2 + k_2} + \left[\omega\gamma + \beta_2 + \frac{(v - \alpha_2)^2\gamma^2}{4a_2}\right]\psi_1^4 + (l_2\gamma^2 + m_2)\gamma\psi_1^6 + (n_2\gamma^4 + p_2\gamma^2 + s_2)\gamma\psi_1^8 = 0. \quad (22)$$

These coupled equations are equivalent based on the conditions given by

$$a_2 = a_1\gamma, \quad (23)$$

$$f_2\gamma(b_1 + c_1\gamma^2 + d_1\gamma^4) = f_1(b_2\gamma^4 + c_2\gamma^2 + d_2), \quad (24)$$

$$g_2\gamma(h_1 + k_1\gamma^2) = g_1(h_2\gamma^2 + k_2), \quad (25)$$

$$4a_1\gamma(\omega\gamma + \beta_2) + (v - \alpha_2)^2\gamma^2 = 4a_1\gamma(\omega + \beta_1\gamma) + (v - \alpha_1)^2, \quad (26)$$

$$(l_2\gamma^2 + m_2)\gamma = l_1 + m_1\gamma^2, \quad (27)$$

$$(n_2\gamma^4 + p_2\gamma^2 + s_2)\gamma = n_1 + p_1\gamma^2 + s_1\gamma^4. \quad (28)$$

Performing the balance between the terms  $\psi_1^3\psi_1''$  and  $\psi_1^8$  in equation (21) brings about the relation

$$4N + 2 = 8N, \quad (29)$$

which leads to  $N = 1/2$ . To ensure closed form solutions, we put forward the transformation of the form

$$\psi_1(\xi) = P^{\frac{1}{2}}(\xi). \quad (30)$$

Upon implementing (30), equation (21) collapses into

$$P''^2 - 2PP'' + \sigma_0 + \sigma_1P + \sigma_2P^2 + \sigma_3P^3 + \sigma_4P^4 = 0, \quad (31)$$

where the constants  $\sigma_j$ , ( $j = 0, 1, 2, 3, 4$ ) are defined as

$$\begin{aligned} \sigma_0 &= \frac{4\rho_1^2}{a_1^2\gamma^2} - \frac{4f_1}{a_1\gamma(b_1 + c_1\gamma^2 + d_1\gamma^4)}, & \sigma_1 &= \frac{-4g_1}{a_1\gamma(h_1 + k_1\gamma^2)}, & \sigma_2 &= -\frac{4a_1\gamma(\omega + \beta_1\gamma) + (v - \alpha_1)^2}{a_1^2\gamma^2}, \\ \sigma_3 &= -\frac{4(l_1 + m_1\gamma^2)}{a_1\gamma}, & \sigma_4 &= -\frac{4(n_1 + p_1\gamma^2 + s_1\gamma^4)}{a_1\gamma}. \end{aligned} \quad (32)$$

### 3 Chirped gap solitons

Our target now is to derive the chirped gap solitons to the coupled-KE by finding the solutions of equation (31) using a new extended auxiliary equation method [40]. This strategy provides various forms of Jacobi elliptic function solutions. To start with, we assume that equation (31) has a solution in the form

$$P(\xi) = \frac{\eta_1 + \eta_2 F^2(\xi)}{\eta_3 + \eta_4 F^2(\xi)}, \tag{33}$$

where  $\eta_j$ , ( $j = 1, 2, 3, 4$ ) are constants to be identified and the function  $F(\xi)$  satisfies the first order ordinary differential equation given by

$$(F'(\xi))^2 = A_0 + A_2 F(\xi)^2 + A_4 F(\xi)^4 + A_6 F(\xi)^6, \tag{34}$$

where  $A_j$ , ( $j = 0, 2, 4, 6$ ) are constants to be determined. Equation (34) admits solutions having the form

$$F(\xi) = \frac{1}{2} \left[ -\frac{A_4}{A_6} (1 \pm f(\xi)) \right]^{\frac{1}{2}}, \tag{35}$$

where the function  $f(\xi)$  can be expressed in terms of the Jacobi elliptic functions (JEFs)  $\text{sn}(\xi, m)$ ,  $\text{cn}(\xi, m)$ ,  $\text{dn}(\xi, m)$  and others, where  $0 < m < 1$  is the modulus of JEFs that degenerate to hyperbolic functions and trigonometric functions as  $m$  approaches 1 or 0, respectively. Substituting (33) into equation (31) and using equation (34), we find a polynomial in terms of  $F(\xi)^j F(\xi)^l$ , ( $j = 0, 1; l = 0, 1, \dots$ ). Collecting the coefficients of terms with the same powers and equating them to zero yields a system of algebraic equations for  $\eta_j$ , ( $j = 1, 2, 3, 4$ ),  $A_j$ , ( $j = 0, 2, 4, 6$ ) and  $\sigma_j$ , ( $j = 0, 1, 2, 3, 4$ ). Solving this system gives us the following cases of solutions.

**Case 1.**

$$\begin{aligned} \eta_4 = 0, \quad \eta_1 = \frac{\eta_2(8A_4\eta_3 - \sigma_3\eta_2)}{32A_6\eta_3}, \quad \eta_2 = \pm\eta_3\sqrt{\frac{12A_6}{\sigma_4}}, \quad \sigma_0 = \frac{\eta_1(8A_0\eta_2^3\eta_3 - 8A_2\eta_1\eta_2^2\eta_3 + 24A_6\eta_1^3\eta_3 + \sigma_3\eta_1^2\eta_2^2)}{2\eta_2^2\eta_3^3}, \\ \sigma_1 = 0, \quad \sigma_2 = \frac{8A_2\eta_2^2\eta_3 - 48A_6\eta_1^2\eta_3 - 3\sigma_3\eta_2^2\eta_1}{2\eta_2^2\eta_3}. \end{aligned} \tag{36}$$

*Family 1.* If  $A_0 = \frac{A_4^3(m^2 - 1)}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(5m^2 - 1)}{16A_6m^2}$ , then we arrive at the Jacobi elliptic function solutions of the coupled equations (2) and (3) as

$$q(x, t) = \left\{ \frac{1}{8A_6\sigma_4} \left[ -3A_6\sigma_3 \pm 4A_4\sqrt{3A_6\sigma_4}\text{sn}\left(\frac{A_4}{2m}\sqrt{\frac{1}{A_6}}(x - vt)\right) \right] \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{37}$$

$$r(x, t) = \gamma q(x, t), \tag{38}$$

and

$$q(x, t) = \left\{ \frac{1}{8mA_6\sigma_4} \left[ -3mA_6\sigma_3 \pm 4A_4\sqrt{3A_6\sigma_4}\text{ns}\left(\frac{A_4}{2m}\sqrt{\frac{1}{A_6}}(x - vt)\right) \right] \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{39}$$

$$r(x, t) = \gamma q(x, t), \tag{40}$$

where  $\sigma_4 > 0$ ,  $A_6 > 0$ . As  $m \rightarrow 1$ , solutions (37) and (38) change to the soliton solutions given by

$$q(x, t) = \left\{ \frac{1}{8A_6\sigma_4} \left[ -3A_6\sigma_3 \pm 4A_4\sqrt{3A_6\sigma_4}\tanh\left(\frac{A_4}{2}\sqrt{\frac{1}{A_6}}(x - vt)\right) \right] \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{41}$$

$$r(x, t) = \gamma q(x, t), \tag{42}$$

while solutions (39) and (40) fall into the singular soliton solutions as

$$q(x, t) = \left\{ \frac{1}{8A_6\sigma_4} \left[ -3A_6\sigma_3 \pm 4A_4\sqrt{3A_6\sigma_4} \coth \left( \frac{A_4}{2} \sqrt{\frac{1}{A_6}}(x - vt) \right) \right] \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \quad (43)$$

$$r(x, t) = \gamma q(x, t), \quad (44)$$

*Family 2.* If  $A_0 = \frac{A_4^3}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(4m^2 + 1)}{16A_6m^2}$ , then one can obtain the Jacobi elliptic function solutions of the coupled equations (2) and (3) as

$$q(x, t) = \left\{ \frac{1}{8A_6\sigma_4} \left[ -3A_6\sigma_3 \pm 4A_4\sqrt{3A_6\sigma_4} \operatorname{cn} \left( \frac{A_4}{2m} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right] \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \quad (45)$$

$$r(x, t) = \gamma q(x, t), \quad (46)$$

where  $\sigma_4 < 0$ ,  $A_6 < 0$ . As  $m \rightarrow 1$ , solutions (45) and (46) reduce to the soliton solutions of the form

$$q(x, t) = \left\{ \frac{1}{8A_6\sigma_4} \left[ -3A_6\sigma_3 \pm 4A_4\sqrt{3A_6\sigma_4} \operatorname{sech} \left( \frac{A_4}{2} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right] \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \quad (47)$$

$$r(x, t) = \gamma q(x, t). \quad (48)$$

## Case 2.

$$\eta_3 = 0, \quad \eta_2 = \frac{(4A_2 - \sigma_2)\eta_1}{12A_0}, \quad \eta_4 = \frac{\sigma_3\eta_1}{8A_0}, \quad \sigma_0 = \frac{4(A_0\eta_2^3 - A_2\eta_1\eta_2^2 + A_4\eta_1^2\eta_2 - A_6\eta_1^3)}{\eta_1\eta_4^2}, \quad \sigma_1 = 0, \quad \sigma_4 = 0. \quad (49)$$

If  $A_0 = \frac{A_4^3}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(4m^2 + 1)}{16A_6m^2}$ , the Jacobi elliptic function solutions of the coupled equations (2) and (3) are secured as

$$q(x, t) = \left\{ \frac{(A_4^2 + 4m^2[A_4^2 - A_6\sigma_2]) \left[ 1 + \operatorname{cn} \left( \frac{A_4}{2m} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right] - 6A_4^2}{6m^2A_6\sigma_3 \left[ 1 + \operatorname{cn} \left( \frac{A_4}{2m} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \quad (50)$$

$$r(x, t) = \gamma q(x, t), \quad (51)$$

where  $A_6 < 0$ . As  $m \rightarrow 1$ , solutions (50) and (51) reduce to the soliton solutions of the form

$$q(x, t) = \left\{ \frac{(5A_4^2 - 4A_6\sigma_2) \left[ 1 + \operatorname{sech} \left( \frac{A_4}{2} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right] - 6A_4^2}{6A_6\sigma_3 \left[ 1 + \operatorname{sech} \left( \frac{A_4}{2} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \quad (52)$$

$$r(x, t) = \gamma q(x, t). \quad (53)$$

**Case 3.**

$$\begin{aligned} \eta_2 = 0, \quad \eta_4 = \pm \frac{2\eta_1}{\sigma_0} \sqrt{-A_6\sigma_0}, \quad \sigma_1 = 0, \quad \sigma_2 = \frac{12A_0\eta_3\eta_4^3 - 8A_2\eta_3^2\eta_4^2 + 12A_6\eta_3^4 + \sigma_4\eta_1^2\eta_4^2}{\eta_3^2\eta_4^2}, \\ \sigma_3 = \frac{-(16A_0\eta_3\eta_4^3 - 8A_2\eta_3^2\eta_4^2 + 8A_6\eta_3^4 + 2\sigma_4\eta_1^2\eta_4^2)}{\eta_1\eta_4^2\eta_3}, \quad \sigma_4 = \frac{12\eta_3(-A_0\eta_4^3 + A_2\eta_3\eta_4^2 - A_4\eta_3^2\eta_4 + A_6\eta_3^3)}{\eta_4^2\eta_1^2}. \end{aligned} \tag{54}$$

*Family 1.* If  $A_0 = \frac{A_4^3(m^2 - 1)}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(5m^2 - 1)}{16A_6m^2}$ , then we arrive at the Jacobi elliptic function solutions of the coupled equations (2) and (3) as

$$q(x, t) = \left\{ \frac{2A_6\sigma_0\eta_1}{2A_6\sigma_0\eta_3 \mp A_4\eta_1\sqrt{-A_6\sigma_0} \left[ 1 + \operatorname{sn} \left( \frac{A_4}{2m} \sqrt{\frac{1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{55}$$

$$r(x, t) = \gamma q(x, t), \tag{56}$$

and

$$q(x, t) = \left\{ \frac{2mA_6\sigma_0\eta_1}{2mA_6\sigma_0\eta_3 \mp A_4\eta_1\sqrt{-A_6\sigma_0} \left[ m + \operatorname{ns} \left( \frac{A_4}{2m} \sqrt{\frac{1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{57}$$

$$r(x, t) = \gamma q(x, t), \tag{58}$$

where  $\sigma_0 < 0$ ,  $A_6 > 0$ . When  $m \rightarrow 1$ , solutions (55) and (56) become the soliton solutions given by

$$q(x, t) = \left\{ \frac{2A_6\sigma_0\eta_1}{2A_6\sigma_0\eta_3 \mp A_4\eta_1\sqrt{-A_6\sigma_0} \left[ 1 + \tanh \left( \frac{A_4}{2} \sqrt{\frac{1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{59}$$

$$r(x, t) = \gamma q(x, t), \tag{60}$$

while solutions (57) and (58) result in the singular soliton solutions as

$$q(x, t) = \left\{ \frac{2A_6\sigma_0\eta_1}{2A_6\sigma_0\eta_3 \mp A_4\eta_1\sqrt{-A_6\sigma_0} \left[ 1 + \operatorname{coth} \left( \frac{A_4}{2} \sqrt{\frac{1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{61}$$

$$r(x, t) = \gamma q(x, t). \tag{62}$$

*Family 2.* If  $A_0 = \frac{A_4^3}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(4m^2 + 1)}{16A_6m^2}$ , we reach the Jacobi elliptic function solutions of the coupled equations (2) and (3) as

$$q(x, t) = \left\{ \frac{2A_6\sigma_0\eta_1}{2A_6\sigma_0\eta_3 \mp A_4\eta_1\sqrt{-A_6\sigma_0} \left[ 1 + \operatorname{cn} \left( \frac{A_4}{2m} \sqrt{\frac{1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{63}$$

$$r(x, t) = \gamma q(x, t), \tag{64}$$

where  $\sigma_0 > 0$ ,  $A_6 < 0$ . As  $m \rightarrow 1$ , solutions (45) and (46) convert to the soliton solutions of the form

$$q(x, t) = \left\{ \frac{2A_6\sigma_0\eta_1}{2A_6\sigma_0\eta_3 \mp A_4\eta_1\sqrt{-A_6\sigma_0} \left[ 1 + \operatorname{sech}\left(\frac{A_4}{2}\sqrt{\frac{-1}{A_6}}(x-vt)\right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi)-\omega t)}, \quad (65)$$

$$r(x, t) = \gamma q(x, t). \quad (66)$$

**Case 4.**

$$\begin{aligned} \sigma_1 = 0, \quad \sigma_0 &= \frac{4\eta_1(A_0\eta_2^3 - A_2\eta_1\eta_2^2 + A_4\eta_1^2\eta_2 - A_6\eta_1^3)}{(\eta_1\eta_4 - \eta_2\eta_3)^2}, \quad \sigma_4 = \frac{-12\eta_3(A_0\eta_4^3 - A_2\eta_3\eta_4^2 + A_4\eta_3^2\eta_4 - A_6\eta_3^3)}{(\eta_1\eta_4 - \eta_2\eta_3)^2}, \\ \sigma_2 &= \frac{-(24A_0\eta_3\eta_4 - 8A_2\eta_3^2 + 4\eta_1^2\sigma_4 + 3\eta_1\eta_3\sigma_3)}{2\eta_3^2}, \quad \sigma_3 = \frac{-8(9A_0\eta_3\eta_4^2 - 6A_2\eta_3^2\eta_4 + 3A_4\eta_3^3 + \eta_1^2\eta_4\sigma_4 - \eta_1\eta_2\eta_3\sigma_4)}{3\eta_3(\eta_1\eta_4 - \eta_2\eta_3)}. \end{aligned} \quad (67)$$

*Family 1.* If  $A_0 = \frac{A_4^3(m^2 - 1)}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(5m^2 - 1)}{16A_6m^2}$ , the Jacobi elliptic function solutions of the coupled equations (2) and (3) are retrieved as

$$q(x, t) = \left\{ \frac{4A_6\eta_1 - A_4\eta_2 \left[ 1 + \operatorname{sn}\left(\frac{A_4}{2m}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]}{4A_6\eta_3 - A_4\eta_4 \left[ 1 + \operatorname{sn}\left(\frac{A_4}{2m}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi)-\omega t)}, \quad (68)$$

$$r(x, t) = \gamma q(x, t), \quad (69)$$

and

$$q(x, t) = \left\{ \frac{4mA_6\eta_1 - A_4\eta_2 \left[ m + \operatorname{ns}\left(\frac{A_4}{2m}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]}{4mA_6\eta_3 - A_4\eta_4 \left[ m + \operatorname{ns}\left(\frac{A_4}{2m}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi)-\omega t)}, \quad (70)$$

$$r(x, t) = \gamma q(x, t), \quad (71)$$

where  $A_6 > 0$ . When  $m \rightarrow 1$ , solutions (68) and (69) become the soliton solutions given by

$$q(x, t) = \left\{ \frac{4A_6\eta_1 - A_4\eta_2 \left[ 1 + \tanh\left(\frac{A_4}{2}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]}{4A_6\eta_3 - A_4\eta_4 \left[ 1 + \tanh\left(\frac{A_4}{2}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi)-\omega t)}, \quad (72)$$

$$r(x, t) = \gamma q(x, t), \quad (73)$$

while solutions (70) and (71) result in the singular soliton solutions as

$$q(x, t) = \left\{ \frac{4A_6\eta_1 - A_4\eta_2 \left[ 1 + \operatorname{coth}\left(\frac{A_4}{2}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]}{4A_6\eta_3 - A_4\eta_4 \left[ 1 + \operatorname{coth}\left(\frac{A_4}{2}\sqrt{\frac{1}{A_6}}(x-vt)\right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi)-\omega t)}, \quad (74)$$

$$r(x, t) = \gamma q(x, t). \quad (75)$$



Family 2. If  $A_0 = \frac{A_4^3}{32A_6^2m^2}$ ,  $A_2 = \frac{A_4^2(4m^2 + 1)}{16A_6m^2}$ , we reach the Jacobi elliptic function solutions of the coupled equations (2) and (3) as

$$q(x, t) = \left\{ \frac{4A_6\eta_1 - A_4\eta_2 \left[ 1 + \operatorname{cn} \left( \frac{A_4}{2m} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right]}{4A_6\eta_3 - A_4\eta_4 \left[ 1 + \operatorname{cn} \left( \frac{A_4}{2m} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{76}$$

$$r(x, t) = \gamma q(x, t), \tag{77}$$

where  $A_6 < 0$ . When  $m \rightarrow 1$ , solutions (76) and (77) turn into the soliton solutions of the form

$$q(x, t) = \left\{ \frac{4A_6\eta_1 - A_4\eta_2 \left[ 1 + \operatorname{sech} \left( \frac{A_4}{2} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right]}{4A_6\eta_3 - A_4\eta_4 \left[ 1 + \operatorname{sech} \left( \frac{A_4}{2} \sqrt{\frac{-1}{A_6}}(x - vt) \right) \right]} \right\}^{\frac{1}{2}} e^{i(\phi(\xi) - \omega t)}, \tag{78}$$

$$r(x, t) = \gamma q(x, t). \tag{79}$$

Based upon the results obtained above and its counterpart in [16], the term with the parameter  $g_1$  has to be neglected so as to reach closed form solutions for the coupled KE, meaning that  $g_1 = 0$ . Accordingly, equation (21) collapses to an elliptic-type differential equation having the form

$$\psi_1'' - \frac{\sigma_2}{4}\psi_1 - \frac{\sigma_3}{4}\psi_1^3 - \frac{\sigma_4}{4}\psi_1^5 = 0, \tag{80}$$

under the restriction condition

$$f_1 a_1 \gamma = \rho_1^2 (b_1 + c_1 \gamma^2 + d_1 \gamma^4), \tag{81}$$

where  $\sigma_2$ ,  $\sigma_3$  and  $\sigma_4$  are as defined in (32). Equation (80) is known to have various types of soliton solutions. One can find, for instance, a quasi-soliton solution given as

$$\psi_1(\xi) = \frac{\kappa_1 \operatorname{sech}(\Omega \xi)}{\sqrt{1 + \kappa_2 \operatorname{sech}^2(\Omega \xi)}}, \tag{82}$$

where

$$\Omega = \frac{1}{2} \sqrt{\sigma_2}, \quad \kappa_1^4 = \left( \frac{12\sigma_2^2}{3\sigma_3^2 - 16\sigma_2\sigma_4} \right), \quad \kappa_2 = -\frac{1}{2} \left( 1 + \sigma_3 \sqrt{\frac{3}{3\sigma_3^2 - 16\sigma_2\sigma_4}} \right), \tag{83}$$

provided that  $\sigma_2 > 0$  and  $3\sigma_3^2 - 16\sigma_2\sigma_4 > 0$  to guarantee real values for the pulse width and amplitude. From this finding, the coupled equations (2) and (3) possess chirped bright quasi-soliton solution in the form

$$q(x, t) = \frac{\kappa_1 \operatorname{sech}[\Omega(x - vt)]}{\sqrt{1 + \kappa_2 \operatorname{sech}^2[\Omega(x - vt)]}} e^{i(\phi(\xi) - \omega t)}, \tag{84}$$

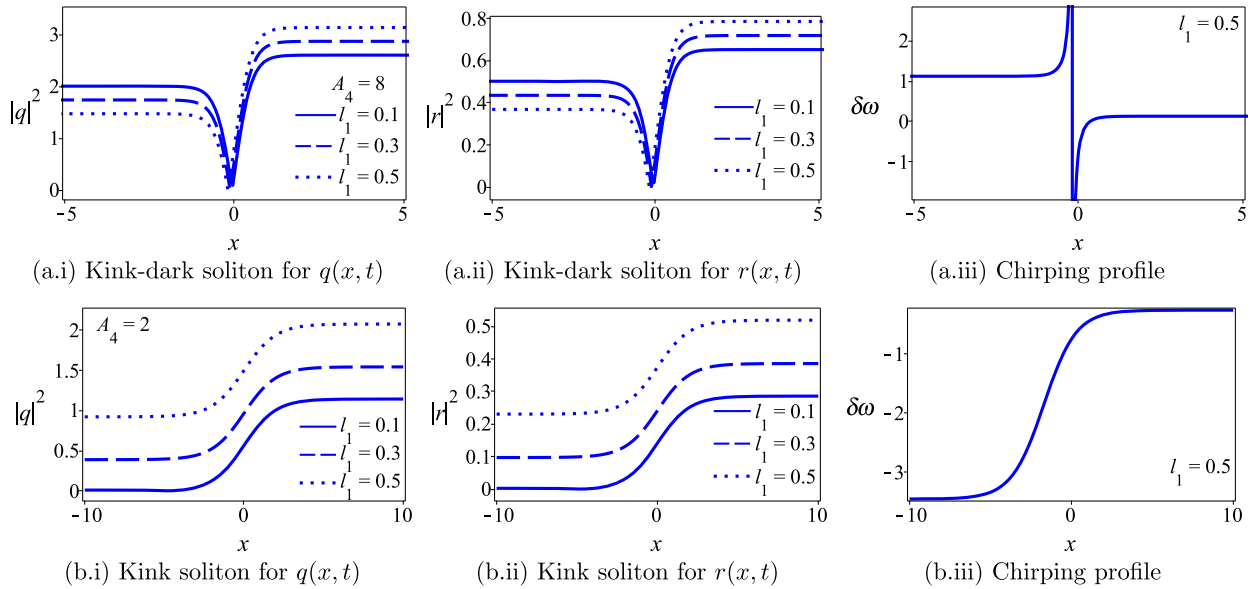
$$r(x, t) = \gamma q(x, t). \tag{85}$$

Additionally, we can secure another form of quasi-soliton solution for equation (80) as

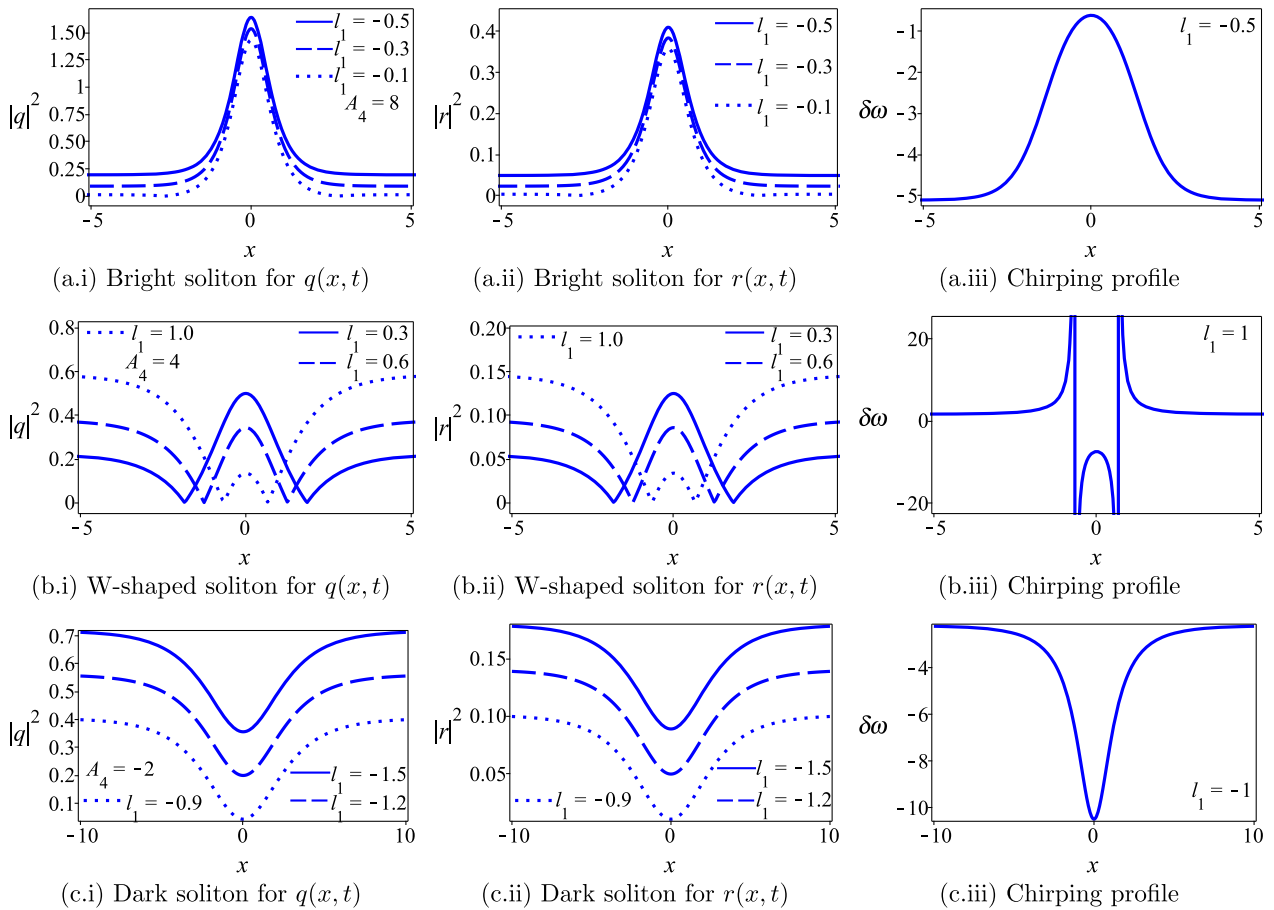
$$\psi_1(\xi) = \frac{\mu_1 \tanh(\Lambda \xi)}{\sqrt{1 + \mu_2 \operatorname{sech}^2(\Lambda \xi)}}, \tag{86}$$

where

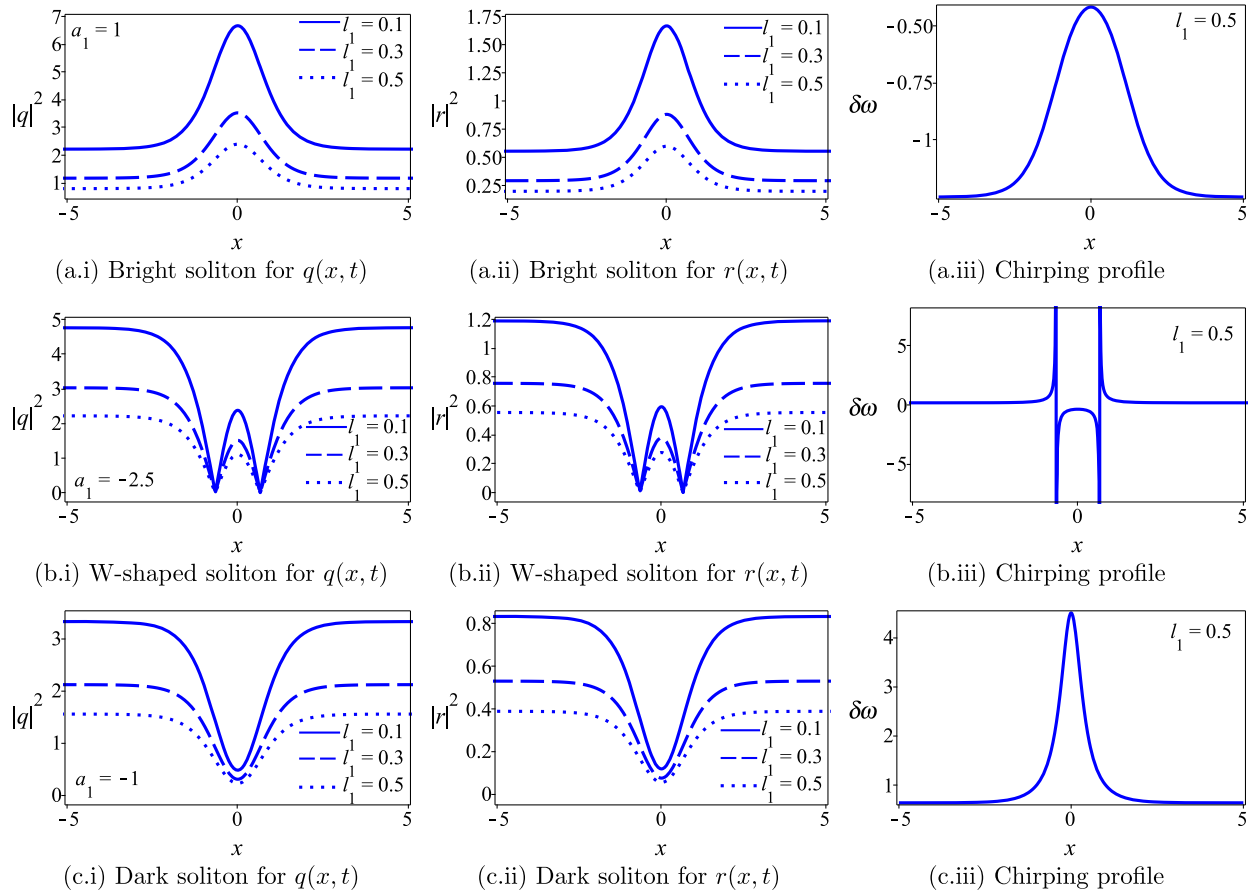
$$\Lambda = \frac{\mu_1}{4} \sqrt{2\sigma_3 + 4\sigma_4\mu_1^2}, \quad \mu_2 = \frac{2\sigma_4\mu_1^2}{3\sigma_3 + 4\sigma_4\mu_1^2}, \tag{87}$$



**Figure 1.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (41) and (42) along with chirping profile.



**Figure 2.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (47) and (48) along with chirping profile.



**Figure 3.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (52) and (53) along with chirping profile.

under the constraint conditions

$$\sigma_2 + \sigma_3\mu_1^2 + \sigma_4\mu_1^4 = 0, \tag{88}$$

provided that  $2\sigma_3 + 4\sigma_4\mu_1^2 > 0$  to ensure the validity of constructing quasi-soliton wave. Making use of these results, the coupled equations (2) and (3) has chirped dark quasi-soliton solution presented as

$$q(x, t) = \frac{\mu_1 \tanh[\Lambda(x - vt)]}{\sqrt{1 + \mu_2 \text{sech}^2[\Lambda(x - vt)]}} e^{i(\phi(\xi) - \omega t)}, \tag{89}$$

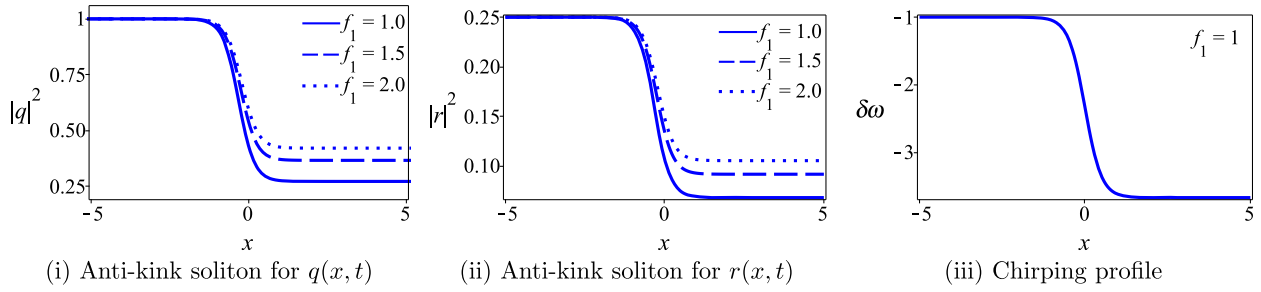
$$r(x, t) = \gamma q(x, t). \tag{90}$$

In all solutions obtained above, the wave number  $\omega$  is an arbitrary constant, the soliton velocity  $v$  is identified in (19) and the nonlinear phase shift  $\phi(\xi)$  can be found from (15). The chirping associated to each soliton is extracted by (20).

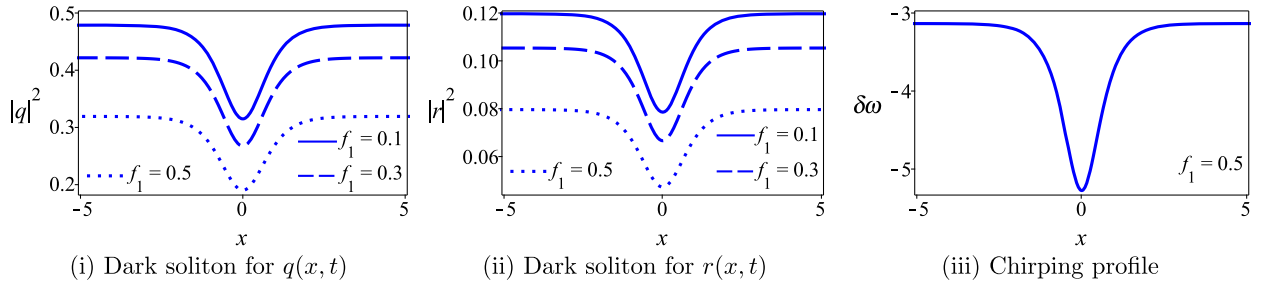
### 4 Results and discussion

As done analytically above, the implemented mathematical approach has yielded a variety of exact solutions to the coupled-KE given by (2) and (3). These solutions describe distinct soliton structures for which the corresponding nonlinear chirp is expressed in terms of the reciprocal of soliton intensity. The dynamical behaviors of derived soliton waves are represented graphically to understand their physical meaning in fiber Bragg gratings medium. Thus, we illustrate the intensity profiles of gap solitons using the model parameters. The chirping associated to these solitons is also plotted.

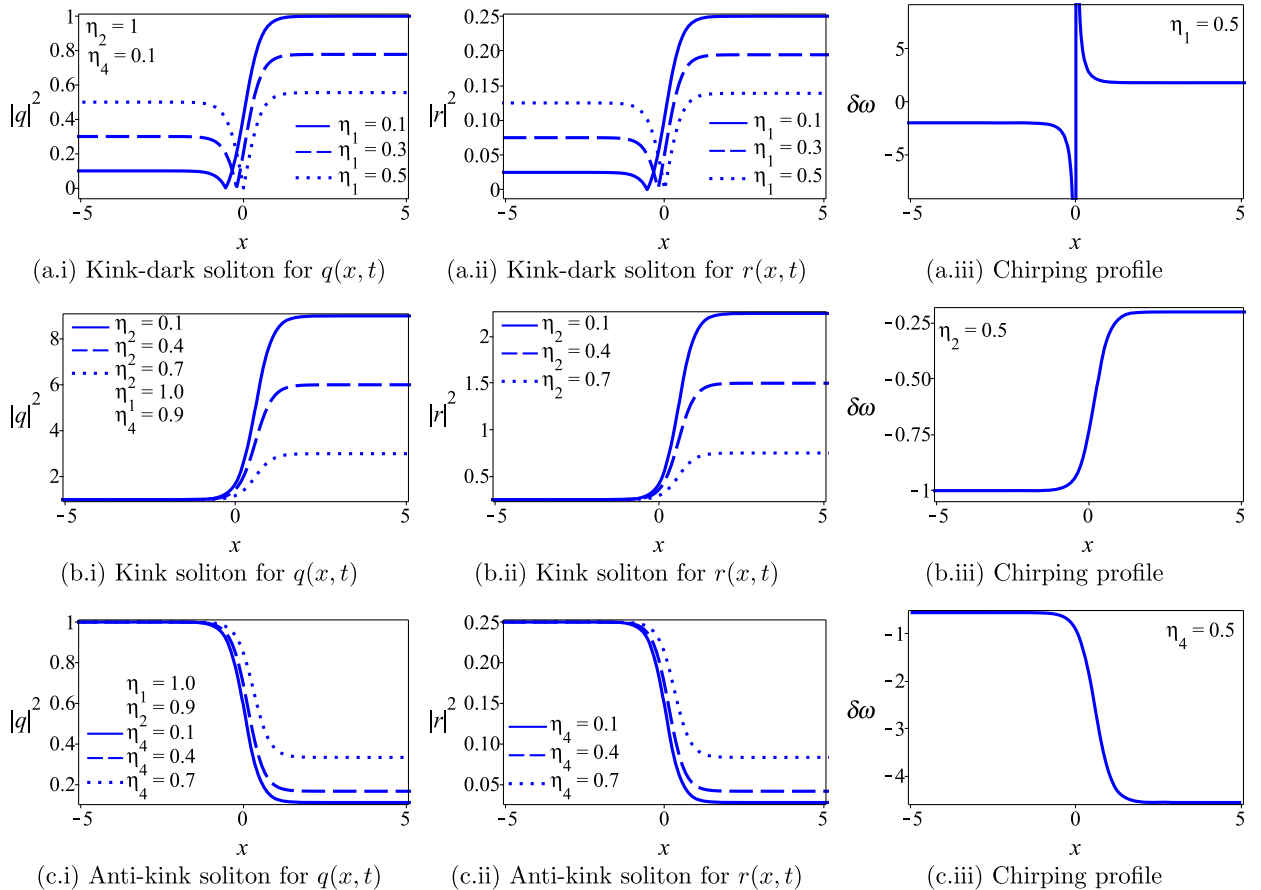
Figure 1 displays the behaviors of solutions (41) and (42) with the model parameters given by  $a_1 = a_2 = 1$ ,  $\gamma = \alpha_1 = \alpha_2 = \rho_1 = m_1 = p_1 = 0.5$ ,  $n_1 = -0.5$ ,  $s_1 = 1.5$ ,  $A_6 = 4$ . Based on the change in the value of  $A_4$ , it can be observed that these solutions describe two soliton structures in addition to their corresponding chirp. As it can be seen from Figure 1a,



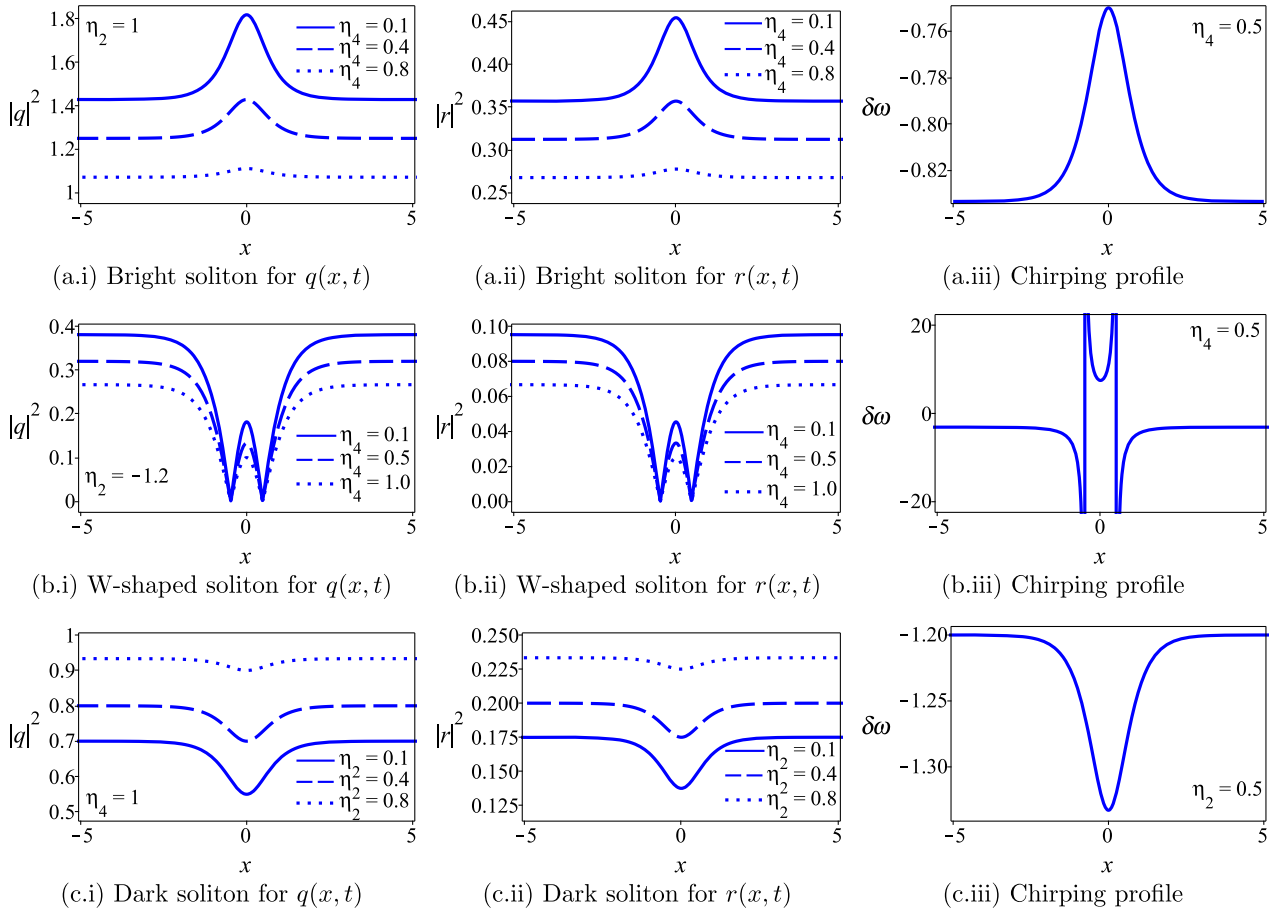
**Figure 4.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (59) and (60) along with chirping profile.



**Figure 5.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (65) and (66) along with chirping profile.



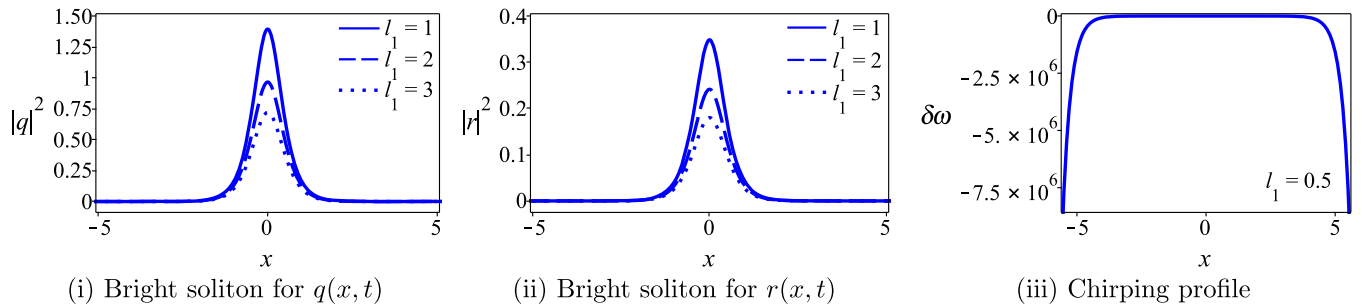
**Figure 6.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (72) and (73) along with chirping profile.



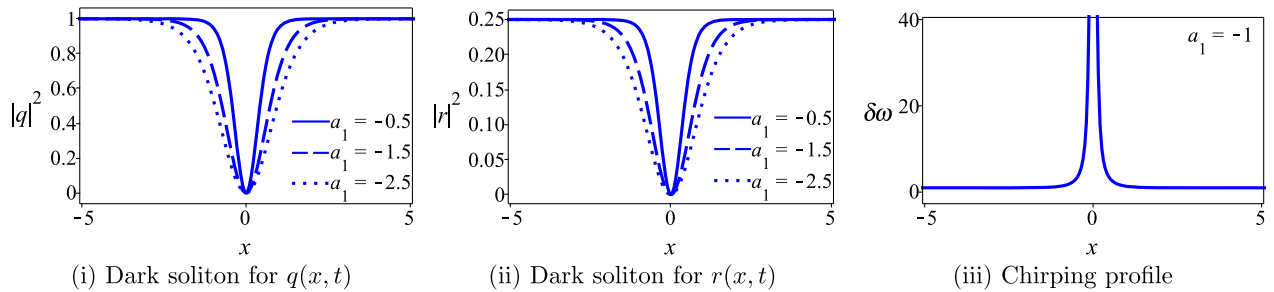
**Figure 7.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (78) and (79) along with chirping profile.

the graph shows kink-dark soliton with  $A_4 = 8$  while Figure 1b exhibits kink wave with  $A_4 = 2$ . We can clearly notice that Figure 2 demonstrates three forms of solitons that are deduced from solutions (47) and (48) which are plotted with same values of parameters as in Figure 1 except  $A_6 = -4$  and with different values of  $A_4$  and  $l_1$ . The first soliton form represents bright soliton wave as depicted in Figure 2a when  $A_4 = 2$ ;  $l_1 = -0.5, -0.3, -0.1$ , the second soliton form describes soliton wave having W shape as shown in Figure 2b when  $A_4 = 4$ ;  $l_1 = 0.3, 0.6, 1$  and the third wave form is dark soliton as presented in Figure 2c when  $A_4 = -2$ ;  $l_1 = -1.5, -1.2, -0.9$ . We have also found that solutions (52) and (53) describe three types of solitons having the former structures as shown in Figure 3 with same values of parameters as in Figure 2 and  $A_4 = 8, \omega = 1, \beta = 0.5$ . The bright soliton in Figure 3a is plotted with  $a_1 = 1$ , the W-shaped soliton in 3b is plotted with  $a_1 = -2.5$  and the dark soliton in 3c is plotted with  $a_1 = -1$ . In Figure 4, the graph illustrates anti-kink soliton characterizing solutions (59) and (60) for the values of parameters  $a_1 = a_2 = b_1 = c_1 = 1, \gamma = \alpha_1 = \alpha_2 = \rho_1 = \eta_1 = \eta_3 = d_1 = 0.5, A_6 = 4, A_4 = 8$  while Figure 5 depicts dark soliton profile that represents solutions (65) and (66) where  $A_6 = -4$ . Moreover, we observe that Figure 6 presents three solitonic structures describing solutions (72) and (73) for the values of parameters  $a_1 = a_2 = \eta_3 = 1, \gamma = \alpha_1 = \alpha_2 = \rho_1 = 0.5, A_4 = 8, A_6 = 4$ . The first structure is kink-dark soliton as displayed in Figure 6a with  $\eta_2 = 1, \eta_4 = 0.1$  and  $\eta_1 = 0.1, 0.3, 0.5$ . The second structure is kink soliton as plotted in Figure 6b with  $\eta_1 = 1, \eta_4 = 0.9$  and  $\eta_2 = 0.1, 0.4, 0.7$ . The third structure is anti-kink soliton as presented in Figure 2c with  $\eta_1 = 1, \eta_2 = 0.9$  and  $\eta_4 = 0.1, 0.4, 0.7$ . Obviously, one can see that Figure 7 demonstrates three wave forms which are bright, W-shaped and dark solitons describing solutions (78) and (79) with same values of parameters as in Figure 6 besides  $\eta_1 = 1, A_6 = -4$ . The bright soliton is shown in Figure 7a with  $\eta_2 = 1, \eta_4 = 0.1, 0.4, 0.8$ ; the W-shaped soliton is shown in Figure 7b with  $\eta_2 = -1.2, \eta_4 = 0.1, 0.5, 1$  and the dark soliton is shown in Figure 7c with  $\eta_4 = 1, \eta_2 = 0.1, 0.4, 0.8$ . The special case of chirped bright quasi-soliton solution (84) and (85) is depicted in Figure 8 with same values of parameters as in Figure 2 and  $\omega = 1, \beta = \eta_1 = 0.5$ . Further to this, the chirped dark quasi-soliton solution (89) and (90) is delineated in Figure 9 with same values of parameters as in Figure 8 and  $\mu = 1, a_1 = -0.5, -1.5, -2.5$ .

From the dynamical behaviors of solitons presented in Figures 1–8, it can be clearly seen that SPM causes remarkable variations in the amplitude of chirped gab solitons. On the other hand, one can notice from Figure 9 that the width of chirped dark quasi-soliton is severely affected by the changes in dispersive reflectivity.



**Figure 8.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (84) and (85) along with chirping profile.



**Figure 9.** Soliton intensity for  $q(x, t)$  and  $r(x, t)$  given in (89) and (90) along with chirping profile.

## 5 Conclusion

The current work concentrated on the chirped gap solitons with Kudryashov's law of self-phase modulation having dispersive reflectivity. The medium of fiber BGs is dominated by a coupled NLSE which is reduced to an integrable form by introducing specific conditions. The extended auxiliary equation method which has solutions in terms of JEFs is applied to extract soliton solutions when the modulus of JEFs tends to 1. Due to manipulating the values of model parameters, it is found that some of solutions construct several chirped soliton structures with their corresponding chirp. The derived chirped soliton waves include bright, dark, singular, W-shaped, kink, anti-kink and Kink-dark solitons. In addition to this, the behaviors of solitons point out that SPM enhances the amplitude of waves. Besides, it is noticed that the width of dark quasi-soliton is obviously affected by dispersive reflectivity. The results in this work could reveal important details about the dynamics of chirped gap solitons that might lead to improvements in the industrial sector related to the field of fiber BGs.

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