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Optical solitons and conservation laws for the concatenation model with spatio-temporal dispersion (internet traffic regulation)

Ahmed H. Arnous[1], Anjan Biswas[2,3,4,5], Abdul H. Kara[6], Yakup Yıldırım[7], Luminita Moraru[8,*], Catalina Iticescu[8], Simona Moldovanu[9], and Abdulah A. Alghamdi[3]

[1] Department of Physics and Engineering Mathematics, Higher Institute of Engineering, El-Shorouk Academy, Cairo 11837, Egypt
[2] Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245, USA
[3] Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[4] Department of Applied Sciences, Cross-Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati 800201, Romania
[5] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa 0204, South Africa
[6] School of Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg, South Africa
[7] Department of Computer Engineering, Biruni University, 34010 Istanbul, Turkey
[8] Department of Chemistry, Physics and Environment, Faculty of Sciences and Environment, Dunarea de Jos University of Galati, 47 Domneasca Street, 800008 Galati, Romania
[9] Department of Computer Science and Information Technology, Faculty of Automation, Computers, Electrical Engineering and Electronics, Dunarea de Jos University of Galati, 47 Domneasca Street, 800008 Galati, Romania

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Abstract. This paper presents optical solitons with the concatenation model having spatio-temporal and chromatic dispersions. This model can advantageously curtail the Internet bottleneck effect. Two integration schemes yield these solitons. By utilizing the multipliers approach, the conservation laws are also derived.

Keywords: Solitons, Concatenation, Spatio-temporal dispersion, Conservation laws.

1 Introduction

One of the several models that is being lately addressed is the concatenation of three well known equations that are frequently visible in the field of nonlinear fiber optics. They are the Lakshmanan–Porsezian–Daniel (LPD) model, nonlinear Schrödinger's equation (NLSE) and Sasa–Satsuma equation (SSE). This concatenated model first appeared during 2014 and was studied by others to date [1–10]. There are various aspects of this model that have beenaddressed. These include rogue wave studies, the numerical study of soliton solutions by the aid of Laplace–Adomian decomposition scheme, the location of the conservation laws, the Painleve analysis, the retrieval of soliton solutions using the method of undetermined coefficients and others. Some other features that are made to be visible are the application of the Kudryashov's approaches to obtain the straddled solitons to the model, application of trial equation approach to address the model, studying the model in birefringent fibers. Later, the model was studied with nonlinear chromatic dispersion (CD) that revealed quiescent solitons. Moreover, the model was extended to birefringent fibers where a full spectrum of soliton solutions were revealed [7, 11–16].

The current paper will take these studies a bit further along. This concatenation model is being addressed with the inclusion of spatio-temporal dispersion (STD) as well as the pre-existing chromatic dispersion (CD) and the self-phase modulation (SPM) that comes with Kerr law. The advantage of the inclusion of STD is that the velocity of the soliton can be controlled. This can be advantageously used to our benefit namely to control the Internet bottleneck effect that is a growing problem with an ever-increasing demand for faster Internet. This technological marvel is being utilized for the concatenation model for the initial once. The soliton solutions are first revealed with the usage of two algorithmic approaches. Subsequently, the conservation laws are derived, and the corresponding conserved quantities are identified After providing a brief overview of the model, the results and the respective mathematical analysis are exhibited.

* Corresponding author: luminita.moraru@ugal.ro

The concatenation model with STD is formulated as [1–10]:

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + \lambda_1 [\alpha_1 q_{xxxx} + \alpha_2 (q_x)^2 q^* + \alpha_3 |q_x|^2 q + \alpha_4 |q|^2 q_{xx} + \alpha_5 q^2 q^*_{xx} + \alpha_6 |q|^4 q] + i\lambda_2 [\alpha_7 q_{xxx} + \alpha_8 |q|^2 q_x + \alpha_9 q^2 q^*_x] = 0. \quad (1)$$

The wave profile, including its spatial and temporal derivatives, can be described by the complex function $q(x, t)$. The linear temporal evolution of solitons is given by the first term, while a and b are the CD and STD coefficients and finally c represents SPM. The concatenation model is the conjoined version of three familiar, frequently visible models. For $\lambda_1 = \lambda_2 = 0$, the model collapses to NLSE, while $\lambda_1 = 0$ or $\lambda_2 = 0$ give the familiar SSE or LPD equation respectively.

2 Integration algorithms: A recapitulation

Let us examine a governing model with the structure of,

$$G(u, u_x, u_t, u_{xt}, u_{xx}, \ldots) = 0. \quad (2)$$

Here we can describe the wave profile, $u = u(x, t)$, as a function of both time and space, denoted by t and x respectively.

The wave transformation:

$$\xi = \mu(x - \upsilon t), \quad u(x, t) = U(\xi), \quad (3)$$

reduces equation (2) to,

$$P(U, -\mu\upsilon U', \mu U', \mu^2 U'', \ldots) = 0, \quad (4)$$

where the wave velocity is indicated by υ, the wave variable is denoted by ξ, and the wave width is represented by the symbol μ.

2.1 Enhanced Kudryashov's method

The fundamental principles of the methodology can be summarized as follows:

Step-1: The simplified equation (4) has a solution that can be expressed explicitly as follows:

$$U(\xi) = \lambda_0 + \sum_{l=1}^{N} \sum_{i+j=l} \lambda_{ij} Q^i(\xi) R^j(\xi), \quad (5)$$

by the aid of the auxiliary equations,

$$R'(\xi)^2 = R(\xi)^2 (1 - \chi R(\xi)^2), \quad (6)$$

and,

$$Q'(\xi) = Q(\xi)(\eta Q(\xi) - 1), \quad (7)$$

where the constants in these equations include $\lambda_{ij}(i, j = 0, 1, \ldots, N)$, η, χ, and λ_0. The value of N is determined by the balancing principle in (4).

Step-2: The solutions derived from equations (6) and (7) are presented as below,

$$R(\xi) = \frac{4d}{4d^2 e^{\xi} + \chi e^{-\xi}}, \quad (8)$$

and,

$$Q(\xi) = \frac{1}{\eta + f e^{\xi}}, \quad (9)$$

where d and f are constants.

Step-3: Inserting (5) along with equations (6) and (7) into (4), a system of algebraic equations arises. This system can then be addressed to provide the undetermined constants in (3) and (5). Finally, plugging (8) and (9) with parametric restrictions into (5) enables us bright and dark solitons.

2.2 General projective Riccati's equation method

The algorithmic approach to the general projective Riccati's equation method is listed here as follows:

Step-1: The explicit solution for the reduced equation (4) can be expressed as,

$$U(\xi) = \alpha_0 + \sum_{i=1}^{N} \phi^{i-1}(\xi)(\alpha_i \phi(\xi) + \beta_i \psi(\xi)). \quad (10)$$

The functions $\phi(\xi)$ and $\psi(\xi)$ satisfy,

$$\begin{aligned} \phi'(\xi) &= -\phi(\xi)\psi(\xi), \\ \psi'(\xi) &= \sigma - \psi^2(\xi) - \delta\phi(\xi), \end{aligned} \quad (11)$$

along with,

$$\psi^2(\xi) = \sigma - 2\delta\phi(\xi) + \frac{\delta^2 + \tau}{\sigma} \phi^2(\xi), \quad (12)$$

where β_i, $\alpha_i (i = 0, 1, \ldots, N)$, $\sigma > 0$, δ and α_0 are arbitrary constants.

Step-2: The solutions to (11) are characterized by,

$$\phi(\xi) = \frac{\sigma \mathrm{sech}[\sqrt{\sigma}\xi]}{\delta \mathrm{sech}[\sqrt{\sigma}\xi] + 1}, \quad \psi(\xi) = \frac{\sqrt{\sigma} \tanh[\sqrt{\sigma}\xi]}{\delta \mathrm{sech}[\sqrt{\sigma}\xi] + 1}, \quad (13)$$

and,

$$\phi(\xi) = \frac{\sigma \mathrm{csch}[\sqrt{\sigma}\xi]}{\delta \mathrm{csch}[\sqrt{\sigma}\xi] + 1}, \quad \psi(\xi) = \frac{\sqrt{\sigma} \coth[\sqrt{\sigma}\xi]}{\delta \mathrm{csch}[\sqrt{\sigma}\xi] + 1}, \quad (14)$$

for $\tau = -1, 1$ respectively and we obtain a positive integer number N by applying the balancing principle to the equation (4).

Step-3: Upon substituting the expressions given by equations (10)–(12) into the governing equation (4), we obtain an over-determined system of algebraic equations. By solving this system of equations, we can obtain the unknown parameters in equations (10)–(12). Finally, plugging (13) and (14) into (10) allows us hyperbolic-type wave solutions.

3 Application to the concatenation model

We can express the solution to equation (1) as a soliton wave, given by,

$$\psi(x,t) = U(\xi)e^{i\phi(x,t)}, \tag{15}$$

with,

$$\xi = k(x - vt), \tag{16}$$

where v is the speed of the soliton, and $U(\xi)$ represents the amplitude component of the soliton. The phase component $\phi(x,\, t)$ is defined as,

$$\phi(x,t) = -\kappa x + \omega t + \theta_0. \tag{17}$$

Here θ_0 is the phase constant, and κ is the frequency of the soliton. By substituting the solution form given in equation (15) into the governing equation (1) and then separating the real and imaginary parts of the resulting equation, we obtain:

$$\begin{aligned} &k^2\left(a - bv - 6\alpha_1\lambda_1\kappa^2 + 3\alpha_7\lambda_2\kappa\right)U'' + \left(-a\kappa^2 + \alpha_1\lambda_1\kappa^4 - \alpha_7\lambda_2\kappa^3\right.\\ &\left.-\omega + b\kappa\omega\right)U + \left(c - \lambda_1\kappa^2(\alpha_2 - \alpha_3 + \alpha_4 + \alpha_5)\right.\\ &\left.+(\alpha_8 - \alpha_9)\lambda_2\kappa\right)U^3 + \alpha_1\lambda_1 k^4 U'''' + (\alpha_4 + \alpha_5)\lambda_1 k^2 U^2 U''\\ &+(\alpha_2 + \alpha_3)\lambda_1 k^2 U U'^2 + \alpha_6\lambda_1 U^5 = 0, \end{aligned} \tag{18}$$

and,

$$\begin{aligned} &-k\left(2a\kappa - 4\alpha_1\lambda_1\kappa^3 + 3\alpha_7\lambda_2\kappa^2 + v - b\kappa v + b\omega\right)U'\\ &+k^3(\alpha_7\lambda_2 - 4\alpha_1\lambda_1\kappa)U''' + k((\alpha_8 + \alpha_9)\lambda_2\\ &-2(\alpha_2 + \alpha_4 - \alpha_5)\lambda_1\kappa)U^2U' = 0. \end{aligned} \tag{19}$$

The evolution of the soliton speed can be obtained from the imaginary part as follows:

$$v = \frac{2a\kappa - b\omega - 4\alpha_1\lambda_1\kappa^3 + 3\alpha_7\lambda_2\kappa^2}{b\kappa - 1}, \tag{20}$$

with parametric restrictions,

$$(\alpha_8 + \alpha_9)\lambda_2 - 2(\alpha_2 + \alpha_4 - \alpha_5)\lambda_1\kappa = 0, \tag{21}$$

and,

$$\alpha_7\lambda_2 - 4\alpha_1\lambda_1\kappa = 0. \tag{22}$$

Equation (18) can be simplified to,

$$\begin{aligned} &k^2U'''' + L_1U^2U'' + L_2U'' + L_3UU'^2 + L_4U\\ &+L_5U^5 + L_6U^3 = 0, \end{aligned} \tag{23}$$

with,

$$\begin{cases} L_1 = \dfrac{\alpha_4 + \alpha_5}{\alpha_1}, & L_2 = \dfrac{a - bv - 6\alpha_1\lambda_1\kappa^2 + 3\alpha_7\lambda_2\kappa}{\alpha_1\lambda_1},\\ L_3 = \dfrac{\alpha_2 + \alpha_3}{\alpha_1}, & L_4 = \dfrac{-a\kappa^2 + \alpha_1\lambda_1\kappa^4 - \alpha_7\lambda_2\kappa^3 - \omega + b\kappa\omega}{\alpha_1\lambda_1k^2},\\ L_5 = \dfrac{\alpha_6}{\alpha_1k^2}, & L_6 = \dfrac{c - \lambda_1(\alpha_2 - \alpha_3 + \alpha_4 + \alpha_5)\kappa^2 + (\alpha_8 - \alpha_9)\lambda_2\kappa}{\alpha_1\lambda_1k^2}. \end{cases} \tag{24}$$

The soliton velocity given by (20) is the one that carries a control parameter namely b, the coefficient of STD. This parameter can regulate the soliton flow in a triad juncture by allowing the traffic to proceed in one direction and holding off the Internet traffic in the other. Such a signaling effect can smoothen the traffic flow and mitigate the Internet bottleneck effect.

3.1 Enhanced Kudryashov's scheme

Balancing U'''' with U^5 in equation (23) yields $N = 1$, which leads to the following form of the solution:

$$U(\xi) = \lambda_0 + \lambda_{01}R(\xi) + \lambda_{10}Q(\xi). \tag{25}$$

Plugging (25) with (6) and (7) into equation (23) provides us the following results.

Result-1:

$$\begin{aligned} &\lambda_0 = \lambda_{10} = 0, \quad \lambda_{01} = \pm\sqrt{\tfrac{\chi(L_1 - L_6 + \vartheta)}{2L_5}}, \quad k = \sqrt{-(L_2 + L_4)},\\ &L_3 = \frac{3L_2(L_1 - 5L_6 + 3\vartheta) + 2L_4(3L_1 - 7L_6 + 5\vartheta)}{6L_2 + 4L_4},\\ &\vartheta = \pm\sqrt{(L_1 - L_6)^2 + 8(3L_2 + 2L_4)L_5}. \end{aligned} \tag{26}$$

As a consequence, the optoelectronic wave field comes out as,

See equation (27) bottom of the page

Selecting $\chi = \pm 4d^2$, $L_2 + L_4 < 0$ and $L_5(L_1 - L_6 + \vartheta) > 0$ allows us a bright wave profile,

$$q(x,t) = \pm\sqrt{\frac{L_1 - L_6 + \vartheta}{2L_5}}\,\mathrm{sech}\left[\sqrt{-(L_2 + L_4)}(x - vt)\right]e^{i(-\kappa x + \omega t + \theta_0)}, \tag{28}$$

while setting $\chi = \pm 4d^2$, $L_2 + L_4 < 0$ and $L_5(L_1 - L_6 + \vartheta) < 0$ enables us a singular waveform,

$$q(x,t) = \left\{\frac{\pm 4d\sqrt{\frac{\chi(L_1 - L_6 + \vartheta)}{2L_5}}}{4d^2\exp\left[\sqrt{-(L_2 + L_4)}(x - vt)\right] + \chi\exp\left[-\sqrt{-(L_2 + L_4)}(x - vt)\right]}\right\}e^{i(-\kappa x + \omega t + \theta_0)}. \tag{27}$$

$$q(x,t)=\pm\sqrt{-\frac{L_1-L_6+\vartheta}{2L_5}}\operatorname{csch}\left[\sqrt{-(L_2+L_4)}(x-vt)\right]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{29}$$

These solitons are also addressed together with the parametric restriction $(L_1-L_6)^2+8(3L_2+2L_4)L_5>0$.

Result-2:

$$\lambda_0=\sqrt{-\tfrac{L_6+\varrho}{2L_5}},\quad \lambda_{01}=0,\quad \lambda_{10}=-2\eta\lambda_0,$$

$$k=\pm\sqrt{\frac{(L_1-2L_6)(L_6+\varrho)}{2L_5}-L_2+4L_4},$$

$$L_3=2(L_6-5\varrho)+\frac{3L_2(\varrho-L_6)}{L_4}+4L_1,\quad \varrho=\pm\sqrt{L_6^2-4L_4L_5}. \tag{30}$$

Thus, the wave profile stands as,

See equation (31) bottom of the page

Choosing $\eta=\pm f$, $(L_1-2L_6)(L_6+\varrho)+2(4L_4-L_2)L_5>0$ and $L_5(L_6+\varrho)<0$ allows us the dark and singular solitons,

See equation (32) bottom of the page

and,

See equation (33) bottom of the page

respectively. These solitons are also considered with the criterion $L_6^2-4L_4L_5>0$.

3.2 General projective Riccati's equation approach

Balancing U'''' with U^5 in equation (23) leads to $N=1$, and hence, the solution can be expressed as follows:

$$U(\xi)=\alpha_0+\alpha_1\phi(\xi)+\beta_1\psi(\xi). \tag{34}$$

Plugging (34) together with (11) and (12) into equation (23) yields the following outcomes:

Result-1:

$$\tau=-1,\quad \alpha_0=0,\quad \beta_1=0,\quad \alpha_1=\pm10\sqrt{\frac{3(\delta^2-1)k^4}{L_2(3L_1+2L_3)}},$$

$$\sigma=-\frac{L_2}{5k^2},\quad L_4=\frac{4L_2^2}{25k^2},$$

$$L_6=-\frac{L_2\left((6\delta^2+3)L_1+2(\delta^2+2)L_3\right)}{30(\delta^2-1)k^2},$$

$$L_5=\frac{(4L_1+L_3)(3L_1+2L_3)}{300k^2}. \tag{35}$$

In this case, the nonlinear waveform turns out to be,

$$q(x,t)=\left\{\mp\frac{2\sqrt{\frac{3(\delta^2-1)L_2}{3L_1+2L_3}}\operatorname{sech}\left[\sqrt{-\frac{L_2}{5}}(x-vt)\right]}{\delta\operatorname{sech}\left[\sqrt{-\frac{L_2}{5}}(x-vt)\right]+1}\right\}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{36}$$

with the aid of the constraints $(\delta^2-1)(3L_1+2L_3)<0$ and $L_2<0$. For $\delta=0$, a bright soliton is recovered.

Result-2:

$$\tau=-1,\quad \alpha_0=0,\quad \alpha_1=0,\quad \beta_1=\pm\frac{2\sqrt{15}k}{\sqrt{-(3L_1+2L_3)}},$$

$$\sigma=-\frac{(\delta^2-1)L_2(3L_1+2L_3)}{5k^2\left(3(\delta^2+3)L_1+2(\delta^2-1)L_3\right)},$$

$$L_6=-\frac{L_2(3L_1+2L_3)\left((6\delta^2-9)L_1+2(\delta^2-1)L_3\right)}{30k^2\left(3(\delta^2+3)L_1+2(\delta^2-1)L_3\right)},$$

$$L_5=\frac{(4L_1+L_3)(3L_1+2L_3)}{300k^2},$$

$$L_4=\frac{2(\delta^2-1)L_2^2(3L_1+2L_3)\left(3(2\delta^2-7)L_1+4(\delta^2-1)L_3\right)}{25\left(3(\delta^2+3)kL_1+2(\delta^2-1)kL_3\right)^2}. \tag{37}$$

$$q(x,t)=\sqrt{-\frac{L_6+\varrho}{2L_5}}\left(1-\frac{2\eta}{f\exp\left[\pm\sqrt{\frac{(L_1-2L_6)(L_6+\varrho)}{2L_5}-L_2+4L_4}(x-vt)\right]+\eta}\right)e^{i(-\kappa x+\omega t+\theta_0)}. \tag{31}$$

$$q(x,t)=\pm\sqrt{-\frac{L_6+\varrho}{2L_5}}\tanh\left[\frac{1}{2}\sqrt{\frac{(L_1-2L_6)(L_6+\varrho)}{2L_5}-L_2+4L_4}(x-vt)\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{32}$$

$$q(x,t)=\pm\sqrt{-\frac{L_6+\varrho}{2L_5}}\coth\left[\frac{1}{2}\sqrt{\frac{(L_1-2L_6)(L_6+\varrho)}{2L_5}-L_2+4L_4}(x-vt)\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{33}$$

$$q(x,t)=\left\{\frac{\pm 2\sqrt{\frac{3(\delta^2-1)L_2}{3(\delta^2+3)L_1+2(\delta^2-1)L_3}}\tanh\left[\sqrt{-\frac{(\delta^2-1)L_2(3L_1+2L_3)}{5(3(\delta^2+3)L_1+2(\delta^2-1)L_3)}}(x-vt)\right]}{\delta\operatorname{sech}\left[\sqrt{-\frac{(\delta^2-1)L_2(3L_1+2L_3)}{5(3(\delta^2+3)L_1+2(\delta^2-1)L_3)}}(x-vt)\right]+1}\right\}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{38}$$

$$q(x,t)=\left\{\frac{\pm 2\sqrt{\frac{3(\delta^2+1)L_2}{3(\delta^2-3)L_1+2(\delta^2+1)L_3}}\coth\left[\sqrt{-\frac{(\delta^2+1)L_2(3L_1+2L_3)}{5(3(\delta^2-3)L_1+2(\delta^2+1)L_3)}}(x-vt)\right]}{\delta\operatorname{csch}\left[\sqrt{-\frac{(\delta^2+1)L_2(3L_1+2L_3)}{5(3(\delta^2-3)L_1+2(\delta^2+1)L_3)}}(x-vt)\right]+1}\right\}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{42}$$

Therefore, the optoelectronic wave field appears as,

See equation (38) top of the page

with the usage of the relations $(\delta^2-1)((3(\delta^2+3)L_1+2(\delta^2-1)L_3)L_2>0$ and $3L_1+2L_3<0$. For $\delta=0$, a dark soliton is retrieved.

Result-3:

$$\tau=1,\quad \alpha_0=0,\quad \beta_1=0,\quad \alpha_1=\pm 10\sqrt{\frac{3(\delta^2+1)k^4}{L_2(3L_1+2L_3)}},$$

$$\sigma=-\frac{L_2}{5k^2},\quad L_4=\frac{4L_2^2}{25k^2},$$

$$L_6=-\frac{L_2((6\delta^2-3)L_1+2(\delta^2-2)L_3)}{30(\delta^2+1)k^2},$$

$$L_5=\frac{12L_1^2+11L_3L_1+2L_3^2}{300k^2}. \tag{39}$$

As a result, the nonlinear wave profile shapes up as,

$$q(x,t)=\left\{\frac{\mp 2\sqrt{\frac{3(\delta^2+1)L_2}{3L_1+2L_3}}\operatorname{csch}\left[\sqrt{-\frac{L_2}{5}}(x-vt)\right]}{\delta\operatorname{csch}\left[\sqrt{-\frac{L_2}{5}}(x-vt)\right]+1}\right\}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{40}$$

with the help of the conditions $3L_1+2L_3<0$ and $L_2<0$. For $\delta=0$, a singular soliton is extracted.

Result-4:

$$\tau=1,\quad \alpha_0=0,\quad \alpha_1=0,\quad \beta_1=\pm\frac{2\sqrt{15}k}{\sqrt{-(3L_1+2L_3)}},$$

$$\sigma=-\frac{(\delta^2+1)L_2(3L_1+2L_3)}{5k^2(3(\delta^2-3)L_1+2(\delta^2+1)L_3)},$$

$$L_6=-\frac{L_2(3L_1+2L_3)((6\delta^2+9)L_1+2(\delta^2+1)L_3)}{30k^2(3(\delta^2-3)L_1+2(\delta^2+1)L_3)},$$

$$L_5=\frac{12L_1^2+11L_3L_1+2L_3^2}{300k^2},$$

$$L_4=\frac{2(\delta^2+1)L_2^2(3L_1+2L_3)(3(2\delta^2+7)L_1+4(\delta^2+1)L_3)}{25k^2(3(\delta^2-3)L_1+2(\delta^2+1)L_3)^2}. \tag{41}$$

Hence, the nonlinear waveform can be expressed as,

See equation (42) top of the page

with the aid of the restrictions $((3(\delta^2-3)L_1+2(\delta^2+1)L_3)L_2>0$ and $3L_1+2L_3<0$. For $\delta=0$, a singular soliton is yielded.

4 Conservation laws

Suppose $(T^t,\ T^x)$ is a conserved vector associated with the conservation law,

$$D_tT^t+D_xT^x=0, \tag{43}$$

valid along the solutions of the given differential equation. In that case, it follows that,

$$E_q[D_tT^t+D_xT^x]=0, \tag{44}$$

where E_q represents the Euler-Lagrange operator. Furthermore, assuming the existence of a non-trivial differential function Q, referred to as a "multiplier," such that,

$$QE=D_tT^t+D_xT^x, \tag{45}$$

and Q is associated with a conserved vector, then it follows that,

$$E_q(QE)=0. \tag{46}$$

This implies that each multiplier Q results in a conserved vector through a Homotopy operator. $E=0$ is the differential equation and T^t, T^x are the conserved densities and fluxes, respectively.

The concatenation model with STD, gives the following conservation laws:

1. For,

$$\alpha_2=\alpha_4=\alpha_5=0, \tag{47}$$

we get a nontrivial power (P) density as:

$$T^P = \frac{1}{2}|q|^2 + \frac{b}{2}\Im(q^* q_x). \tag{48}$$

2. If, in addition,

$$\alpha_3 = \alpha_9 = 0, \tag{49}$$

we arrive at conserved linear momentum (M) density,

$$T^M = -\frac{1}{4}b|q_x|^2 + \frac{1}{4}b\Re\left(qq^*_{xx}\right) - \frac{b}{2}\Im(q^* q_x). \tag{50}$$

3. Conserved Hamiltonian (H) density is presented as below,

$$\begin{aligned} T^H = {} & \lambda_1\left[\frac{1}{2}\alpha_1\Re\left(qq^*_{xxxx}\right) + \frac{1}{6}\alpha_6|q|^6\right] \\ & + \lambda_2\left[-\frac{1}{2}\alpha_7\Im(q^* q_{xxx}) - \frac{1}{4}\alpha_8|q|^2\Im(q^* q_x)\right] \\ & + \frac{1}{4}c|q|^4 + \frac{1}{2}a\Re\left(qq^*_{xx}\right) \\ & + \frac{1}{4}b\left[\Re\left(qq^*_{xt}\right) + \Re\left(q_x q^*_t\right)\right]. \end{aligned} \tag{51}$$

The expression for the bright 1-soliton solution, provided in equation (28), can be conveniently structured as:

$$q(x,t) = A\,\mathrm{sech}\,[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}, \tag{52}$$

where the soliton's amplitude is denoted by A, its inverse width is modeled by B, and also its velocity is formulated by v. Therefore, the following conserved quantities arise from this form of the bright soliton:

$$P = \int_{-\infty}^{\infty}\left[|q|^2 + \frac{b}{2i}\left(q^* q_x - qq^*_x\right)\right]\mathrm{d}x = \frac{2A^2}{B}(1 - b\kappa), \tag{53}$$

$$\begin{aligned} M &= \frac{1}{4}\int_{-\infty}^{\infty}\left[a|q_x|^2 - a\Re\left(qq^*_{xx}\right) + 2b\Im(q^* q_x)\right]\mathrm{d}x \\ &= \frac{A^2}{3B}\left(aB^2 + a\kappa^2 - 3b\kappa\right), \end{aligned} \tag{54}$$

and,

$$\begin{aligned} H = \int_{-\infty}^{\infty} T^H \mathrm{d}x = {} & \frac{\lambda_1\alpha_1 A^2 B}{15}\left(9B^2 + 30\kappa^2 + 5\kappa^4\right) + \frac{4\lambda_1\alpha_6 A^6}{45B} \\ & - \frac{\lambda_2\alpha_7\kappa A^2}{B}\left(\kappa^2 + 3\right) + \left(\lambda_2\alpha_8\kappa + c\right)\frac{A^4}{3B} \\ & - \frac{aA^2}{3B}\left(B^2 + 3\kappa^2\right) + \frac{bA^2}{6B}\left(vB^2 + 3\omega\kappa\right). \end{aligned} \tag{55}$$

5 Conclusion

In this paper, the concatenation model is revisited with the incorporation of STD alongside the existing CD. The SPM is with Kerr law of nonlinearity. The rational expression for the soliton velocity placed us at an advantage of controlling the Internet bottleneck effect that is responsible of slowing down the traffic flow across the globe. Such an engineering marvel is being applied to the concatenation model for the first time and this gives a true flavor of novelty to the current paper. The results of the paper are indeed encouraging and are applicable to various additional avenues. One would next need to study this technological aspect in birefringent fibers followed by dispersion-flattened fibers. This would lead to the departure from the lab to a situation where rubber meets the road. Additional effects such as stochasticity, time-dependent coefficients to the model are yet to be explored. These would lead to several novelties that would be sequentially disseminated all across the board after aligning the results with pre-existing reports [17–34].

Conflict of interest

The authors declare no conflict of interest.

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