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Chirped gap solitons in fiber Bragg gratings with polynomial law of nonlinear refractive index

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Abstract. The objective of the present study is to examine the behaviors of chirped optical solitons in fiber Bragg gratings (BGs) with dispersive reflectivity. The form of nonlinear refractive index represents polynomial law nonlinearity. By virtue of phase-matching condition, the discussed model of coupled nonlinear Schrödinger equation is reduced to an integrable form. Consequently, chirped optical solitons having various profiles such as W-shaped, bright, dark, kink and anti-kink solitons are derived. Further to this, the chirp associated with these soliton structures are extracted. The impact of dispersive reflectivity, self-phase modulation and cross-phase modulation on the pulse propagation is investigated and it is induced that the changes of self-phase modulation and cross-phase modulation cause a marked rise in soliton amplitude which is subject to minor variations by dispersive reflectivity. The physical evolutions of chirped optical solitons are described along with the corresponding chirp to pave the way for possible applications in the field of fiber BGs.

Keywords: Chirped solitons, Bragg gratings, Polynomial law.

1 Introduction

The existence of soliton in the filed of optical fibers has made a wide improvement in various sectors of our life such as communications, medical, aerospace and many others [1–3]. In communications, for example, soliton operates as one of the effective carriers that transmit huge information through optical fibers over transcontinental and transoceanic distances [4–8]. Basically, soliton in fiber-optic communication systems is known as optical soliton which denotes a light pulse propagating through a nonlinear optical medium without any change in its shape and velocity [9–11]. The emergence of optical soliton is based on controllable interaction of dispersion and nonlinearity of the pulse propagation. The investigation of optical soliton behaviors can be done through discussing a model of the nonlinear Schrödinger equation (NLSE) or its generalized forms [12–16].

During propagation scenario within an optical fiber, soliton is found to be badly affected by the low count of chromatic dispersion (CD) and hence the data transmission is obfuscated. This problem can be handled via applying a new technology, namely, dispersion compensating fiber. One of the perfect candidates for this role is Bragg gratings (BGs) because of their low loss, small footprint, and low optical nonlinearity. Practically, BGs provides induced dispersion to replenish the low count of CD and sufficiently ensure the smooth formation of solitons which are transmitted along fibers for intercontinental distances. Based on the physical nature of fiber BGs medium, the self-phase modulation arises from the nonlinear refractive index structures which has different types such as Kerr law, quadratic-cubic law, parabolic law, polynomial law, parabolic-nonlocal

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combo law and many others. This diversity of forms of nonlinear refractive index leads to create distinct forms of NLSE.

Various studies in literature are implemented in the area of fiber BGs under the influences of different types of nonlinearity to examine the behaviors of optical solitons by making use of powerful integration schemes. For instance, Biswas et al. [17] scrutinized coupled NLSE in fiber BGs with parabolic form of nonlinearity. They extracted three forms of solitons called bright, dark and singular solitons. With the aid of modified simple equation, dark and singular optical solitons to fiber Bragg gratings with Kerr law are obtained by a group of authors [18]. The latter model is studied as well in presence of the four-wave mixing terms [19]. As a result, chirped and chirp-free bright, dark and singular solitons are revealed. Moreover, solitons in fiber BGs with five forms of nonlinear refractive index are investigated using the sine-Gordon equation method [20]. Distinct structures of solitons including bright, dark, singular and combo singular solitons are retrieved. For more details about the previous studies that discussed optical solitons in fiber BGs, the reader is referred to the references [21–35].

The recent experimental studies declare that optical solitons with nonlinear chirping have effective role in engineering applications such as the design of fiber-optic amplifiers, spread spectrum communications, photonic and optoelectronic devices. Hence, a lot of intensive theoretical works are directed to the investigation of chirped solitons in fiber-optic media. Many experts have dealt with various mathematical models to derive miscellaneous types of chirped solitons by means of several analytical techniques. Some of obtained soliton structures include kink, dark and bright solitons [36–39]; chirped self-similar bright and kink solitons [40]; dark-dark and bright-bright soliton pairs [41]; chirped self-similar gray and kink waves [42], see also [43–46]. Our present work focuses on deriving chirped optical soliton solutions in fiber BGs. The nonlinear chirping associated to each soliton solution is also created.

The most substantial purpose of this study is to inquire into the dimensionless form of the coupled NLSE in fiber BGs having polynomial law of nonlinearity given by [47, 48]

$$iq_t + a_1 r_{xx} + (b_1|q|^2 + c_1|r|^2)q + (d_1|q|^4 + f_1|q|^2|r|^2 + g_1|r|^4)q + (l_1|q|^6 + m_1|q|^4|r|^2 + n_1|q|^2|r|^4 + p_1|r|^6)q + ih_1 q_x + k_1 r = 0, \tag{1}$$

$$ir_t + a_2 q_{xx} + (b_2|r|^2 + c_2|q|^2)r + (d_2|r|^4 + f_2|r|^2|q|^2 + g_2|q|^4)r + (l_2|r|^6 + m_2|r|^4|q|^2 + n_2|r|^2|q|^4 + p_2|q|^6)r + ih_2 r_x + k_2 q = 0, \tag{2}$$

where the functions $q(x, t)$ and $r(x, t)$ indicate forward and backward propagating waves, respectively, while a_j for $j = 1, 2$ denote the coefficients of dispersive reflectivity. The terms having b_j account for the coefficients of self-phase modulation (SPM) whereas the terms with c_j represent the cross-phase modulation (XPM) for cubic nonlinearity portion. Regarding quintic nonlinear part, d_j are the coefficients of SPM while f_j and g_j are the coefficients of XPM. For septic nonlinearity, l_j are the coefficients of SPM while m_j, n_j and p_j are the coefficients of XPM. Finally, h_j stand for inter-modal dispersion and k_j represent detuning parameters. All of the coefficients are real valued constants and $i = \sqrt{-1}$.

This work is essentially devoted to investigating chirped optical solitons in fiber BG with polynomial law of nonlinear refractive index. The coupled NLSE (1) and (2) is dealt with analytic strategy and reduced to an integrable form under the phase-matching condition. Then, soliton solutions are created by implementing two forms for the method of undetermined coefficients. The corresponding chirp to each soliton solution is also derived as a nonlinear function in terms of the reciprocal of intensity. The physical interpretations of constructed optical solitons are displayed.

2 Mathematical analysis and reduction of governing model

Our aim now is to analyze the system of coupled NLSE (1) and (2) and to reduce it to an integrable form. To achieve this process, a complexvariable transformation is assumed as

$$q(x, t) = u_1(\tau)e^{i(\Upsilon(\tau)-\omega t)}, \tag{3}$$

$$r(x, t) = u_2(\tau)e^{i(\Upsilon(\tau)-\omega t)}, \tag{4}$$

where $\tau = x - vt$ while ω and v are real constants indicating the wave number and the soliton velocity. The functions $u_1(\tau)$ and $u_2(\tau)$ stand for the amplitudes of the solitons whereas $\Upsilon(\tau)$ represents the nonlinear phase shift. The corresponding chirp is identified as $\delta\omega(x, t) = -\frac{\partial}{\partial x}[\Upsilon(\tau) - \omega t] = -\Upsilon'(\tau)$. Substituting (3) and (4) into the coupled system (1) and (2) and then separating the imaginary and real components, one can arrive at

$$a_1 u_2'' - a_1 u_2 \Upsilon'^2 + (v - h_1)u_1\Upsilon' + \omega u_1 + k_1 u_2 + b_1 u_1^3 + c_1 u_1 u_2^2 + d_1 u_1^5 + f_1 u_1^3 u_2^2 + g_1 u_1 u_2^4 + l_1 u_1^7 + m_1 u_1^5 u_2^2 + n_1 u_1^3 u_2^4 + p_1 u_1 u_2^6 = 0, \tag{5}$$

$$a_2 u_1'' - a_2 u_1 \Upsilon'^2 + (v - h_2)u_2\Upsilon' + \omega u_2 + k_2 u_1 + b_2 u_2^3 + c_2 u_2 u_1^2 + d_2 u_2^5 + f_2 u_2^3 u_1^2 + g_2 u_2 u_1^4 + l_2 u_2^7 + m_2 u_2^5 u_1^2 + n_2 u_2^3 u_1^4 + p_2 u_2 u_1^6 = 0, \tag{6}$$

and

$$(h_1 - v)u_1' + a_1(u_2\Upsilon'' + 2u_2\Upsilon') = 0, \tag{7}$$

$$(h_2 - v)u_2' + a_2(u_1\Upsilon'' + 2u_1\Upsilon') = 0. \tag{8}$$

To manipulate the equations obtained above, the following relation is proposed as

$$u_2 = \rho u_1, \tag{9}$$

where $\rho \neq 1$ is a real constant. Subsequently, the set of equations (5)–(8) change into

$$\rho a_1 u_1'' + [\omega + \rho k_1 + (v - h_1)\Upsilon' - \rho a_1 \Upsilon'^2]u_1 + (b_1 + \rho^2 c_1)u_1^3$$
$$+(d_1 + \rho^2 f_1 + \rho^4 g_1)u_1^5 + (l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1)u_1^7 = 0, \tag{10}$$

$$a_2 u_1'' + [\rho\omega + k_2 + \rho(v - h_2)\Upsilon' - a_2 \Upsilon'^2]u_1 + (\rho^3 b_2 + \rho c_2)u_1^3$$
$$+(\rho^5 d_2 + \rho^3 f_2 + \rho g_2)u_1^5 + (\rho^7 l_2 + \rho^5 m_2 + \rho^3 n_2 + \rho p_2)u_1^7 = 0, \tag{11}$$

and

$$(h_1 - v)u_1' + \rho a_1(u_1 \Upsilon'' + 2u_1 \Upsilon') = 0, \tag{12}$$

$$\rho(h_2 - v)u_1' + a_2(u_1 \Upsilon'' + 2u_1 \Upsilon') = 0. \tag{13}$$

From the integrability of equations (12) and (13), one can obtain

$$\Upsilon' = \frac{v - h_1}{2\rho a_1} + \frac{\delta_1 u_1^{-2}}{\rho a_1}, \tag{14}$$

$$\Upsilon' = \frac{\rho(v - h_2)}{2a_2} + \frac{\delta_2 u_1^{-2}}{a_2}, \tag{15}$$

where δ_1 and δ_2 are the integration constants. Since equations (14) and (15) are equivalent, they give rise the constraint condition of the form

$$(a_2 - \rho^2 a_1)vu_1^2 - (h_1 a_2 - h_2 \rho^2 a_1)u_1^2 + 2(\delta_1 a_2 - \delta_2 \rho a_1) = 0. \tag{16}$$

To reduce the level of complexity in the system of equations (14) and (15), we assume that

$$\delta_2 \rho a_1 = \delta_1 a_2, \tag{17}$$

and, as a consequence, equation (16) collapses to the expression for the velocity of the soliton as

$$v = \frac{h_1 a_2 - h_2 \rho^2 a_1}{a_2 - \rho^2 a_1}. \tag{18}$$

As a result, the chirp can be written as

$$\delta\omega(x, t) = -\left[\frac{v - h_1}{2\rho a_1} + \frac{\delta_1 u_1^{-2}}{\rho a_1}\right]. \tag{19}$$

Plugging (14) and (15) into equations (10) and (11), respectively, we find

$$\rho a_1 u_1'' - \frac{\delta_1^2}{\rho a_1} u_1^{-3} + \left[\omega + \rho k_1 + \frac{(v - h_1)^2}{4\rho a_1}\right]u_1 + (b_1 + \rho^2 c_1)u_1^3$$
$$+(d_1 + \rho^2 f_1 + \rho^4 g_1)u_1^5 + (l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1)u_1^7 = 0, \tag{20}$$

$$a_2 u_1'' - \frac{\delta_1^2 a_2}{\rho^2 a_1^2} u_1^{-3} + \left[\rho\omega + k_2 + \frac{\rho^2 (v - h_2)^2}{4a_2}\right]u_1 + (\rho^3 b_2 + \rho c_2)u_1^3$$
$$+(\rho^5 d_2 + \rho^3 f_2 + \rho g_2)u_1^5 + (\rho^7 l_2 + \rho^5 m_2 + \rho^3 n_2 + \rho p_2)u_1^7 = 0. \tag{21}$$

Integrating the last equations, this leads to

$$\rho a_1 u_1'^2 - \frac{\delta_1^2}{\rho a_1} u_1^{-2} + \left[\omega + \rho k_1 + \frac{(v - h_1)^2}{4\rho a_1}\right]u_1^2 + \frac{1}{2}(b_1 + \rho^2 c_1)u_1^4$$
$$+\frac{1}{3}(d_1 + \rho^2 f_1 + \rho^4 g_1)u_1^6 + \frac{1}{4}(l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1)u_1^8 + 2\mu_1 = 0, \tag{22}$$

$$a_2 u_1'^2 - \frac{\delta_1^2 a_2}{\rho^2 a_1^2} u_1^{-2} + \left[\rho\omega + k_2 + \frac{\rho^2 (v - h_2)^2}{4a_2}\right]u_1^2 + \frac{1}{2}(\rho^3 b_2 + \rho c_2)u_1^4$$
$$+\frac{1}{3}(\rho^5 d_2 + \rho^3 f_2 + \rho g_2)u_1^6 + \frac{1}{4}(\rho^7 l_2 + \rho^5 m_2 + \rho^3 n_2 + \rho p_2)u_1^8 + 2\mu_2 = 0, \tag{23}$$

where μ_1 and μ_2 are the constants of integration. Equations (22) and (23) are equivalent under the following conditions presented as

$$a_2 = \rho a_1, \tag{24}$$

$$\rho\omega + k_2 + \frac{\rho^2 (v - h_2)^2}{4a_2} = \omega + \rho k_1 + \frac{(v - h_1)^2}{4\rho a_1}, \tag{25}$$

$$\rho^3 b_2 + \rho c_2 = b_1 + \rho^2 c_1, \tag{26}$$

$$\rho^5 d_2 + \rho^3 f_2 + \rho g_2 = d_1 + \rho^2 f_1 + \rho^4 g_1, \tag{27}$$

$$\rho^7 l_2 + \rho^5 m_2 + \rho^3 n_2 + \rho p_2 = l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1, \tag{28}$$

$$\mu_2 = \mu_1. \tag{29}$$

As both equations (22) and (23) are equivalent, the rest of procedure in this work are performed through tackling equation (22). Let us introduce the variable transformation given by

$$u_1^2 = F, \tag{30}$$

which enables us to convert equation (22) into

$$\rho a_1 F'^2 - \frac{4\delta_1^2}{\rho a_1} + 8\mu_1 F + 4\left[\omega + \rho k_1 + \frac{(v - h_1)^2}{4\rho a_1}\right]F^2 + 2(b_1 + \rho^2 c_1)F^3$$
$$+\frac{4}{3}(d_1 + \rho^2 f_1 + \rho^4 g_1)F^4 + (l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1)F^5 = 0. \tag{31}$$

Equation (31) can be rearranged to have the form

$$\rho a_1 F'' + 4\mu_1 + 4\left[\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right]F + 3(b_1 + \rho^2 c_1)F^2$$
$$+\frac{8}{3}(d_1 + \rho^2 f_1 + \rho^4 g_1)F^3 + \frac{5}{2}(l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1)F^4 = 0. \tag{32}$$

3 Chirped soliton solutions

To derive the chirped soliton solutions, the method of undetermined coefficients with two forms having the hyperbolic secant and tangent functions is employed to equation (32). Then, the obtained solutions are inserted into the relation (30) together with (3) to arrive at the chirped optical solitons for the coupled system (1) and (2).

3.1 First expression with hyperbolic tangent function

We express that the solution of equation (32) has the form

$$F(\tau) = \alpha_0 + \alpha_1 \tanh(\alpha_3\tau) + \alpha_2 \tanh^2(\alpha_3\tau), \tag{33}$$

where α_0, α_1, α_2 and α_3 are constants to be identified. Substituting the ansatz (33) into equation (32) and then equating all coefficients of all powers of $\tanh(\alpha_3\tau)$ to zero, one can obtain a system of algebraic equations that determines the value of constants α_0, α_1, α_2 and α_3. Solving this system brings about the following sets of solutions.

Set 1

$$\alpha_0 = \frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1 + \rho^2 c_1)}, \tag{34}$$

$$\alpha_1 = 0, \tag{35}$$

$$\alpha_2 = -\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{36}$$

$$\alpha_3 = \sqrt{\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}, \tag{37}$$

under the constraint conditions

$$d_1 + \rho^2 f_1 + \rho^4 g_1 = 0, \tag{38}$$

$$l_1 + \rho^2 m_1 + \rho^4 n_1 + \rho^6 p_1 = 0. \tag{39}$$

Substituting (34)–(37) into (33) and using the relations (30) and (9) along with (3) and (4), we arrive at an exact solution in the form of chirped bright soliton for the coupled system (2) and (3) as

$$q(x,t) = \left[\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2 c_1)}\left\{1 - 3\tan\text{h}^2\left(\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)},$$
$$r(x,t) = \rho\left[\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2 c_1)}\left\{1 - 3\tan\text{h}^2\left(\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)}, \tag{40}$$

where $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1) > 0$. The associated chirp is presented as

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} + \frac{\delta_1}{\rho a_1}\left\{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2 c_1)}\right.\right.$$
$$\left.\left.\times\left[1 - 3\tan\text{h}^2\left(\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right]\right\}^{-1}\right]. \tag{41}$$

Set 2

$$\alpha_0 = -\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{42}$$

$$\alpha_1 = 0, \tag{43}$$

$$\alpha_2 = \frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{44}$$

$$\alpha_3 = \sqrt{-\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}, \tag{45}$$

under the constraint conditions (38) and (39). Substituting these outcomes into (33) and using the relations (30) and (9) together with (3) and (4), we obtain the chirped bright soliton for the coupled system (2) and (3) given by

$$q(x,t) = \sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}}\sec\text{h}\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)e^{i(\varphi(\tau)-\omega t)},$$
$$r(x,t) = \rho\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}}\sec\text{h}\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)e^{i(\varphi(\tau)-\omega t)}, \tag{46}$$

where $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1) < 0$ and $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)$ $(b_1 + \rho^2 c_1) < 0$. The chirp expression is addressed as

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} - \frac{\delta_1}{\rho a_1}\left\{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}\right.\right.$$
$$\left.\left.\sec\text{h}^2\left(\sqrt{-\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}^{-1}\right]. \tag{47}$$

Set 3

$$\alpha_0 = -\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{48}$$

$$\alpha_1 = -\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{49}$$

$$\alpha_2 = 0, \tag{50}$$

$$\alpha_3 = \sqrt{-\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}, \tag{51}$$

under the constraint conditions (39) and

$$16\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(d_1 + \rho^2 f_1 + \rho^4 g_1) = 3(b_1 + \rho^2 c_1)^2. \tag{52}$$

Exploiting (33)–(51) with (33) and using the relations (30) and (9) together with (3) and (4), we present the chirped dark soliton pair for the coupled system (2) and (3) as

$$q(x,t) = \left[-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}\left\{1 + \tan\,\mathrm{h}\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)},$$
$$r(x,t) = \rho\left[-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}\left\{1 + \tan\,\mathrm{h}\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)}, \tag{53}$$

where $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1) < 0$ and $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)$ $(b_1 + \rho^2 c_1) < 0$. The corresponding chirping is introduced as

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} - \frac{\delta_1}{\rho a_1}\left\{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}\left[1 + \tan\,\mathrm{h}\left(\sqrt{-\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right]\right\}^{-1}\right]. \tag{54}$$

Set 4

$$\alpha_0 = -\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{55}$$

$$\alpha_1 = \frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}, \tag{56}$$

$$\alpha_2 = 0, \tag{57}$$

$$\alpha_3 = \sqrt{-\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}, \tag{58}$$

under the constraint conditions (39) and (52). Plugging these findings into (33) and using the relations (30) and (9) together with (3) and (4), we secure the chirped dark soliton pair for the coupled system (2) and (3) as

$$q(x,t) = \left[-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}\left\{1 - \tan\,\mathrm{h}\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)},$$
$$r(x,t) = \rho\left[-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}\left\{1 - \tan\,\mathrm{h}\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)}, \tag{59}$$

where $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1) < 0$ and $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)$ $(b_1 + \rho^2 c_1) < 0$. The associated chirp is caught in the form

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} - \frac{\delta_1}{\rho a_1}\left\{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1 + \rho^2 c_1)}\left[1 - \tan\,\mathrm{h}\left(\sqrt{-\frac{\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)\right]\right\}^{-1}\right]. \tag{60}$$

Set 5

$$\alpha_0 = 0, \tag{61}$$

$$\alpha_1 = \sqrt{-\frac{3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{2(d_1 + \rho^2 f_1 + \rho^4 g_1)}}, \tag{62}$$

$$\alpha_2 = 0, \tag{63}$$

$$\alpha_3 = \sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{\rho a_1}}, \tag{64}$$

under the constraint conditions (39) and

$$b_1 + \rho^2 c_1 = 0. \tag{65}$$

Employing (61)–(64) with (33) and using the relations (30) and (9) together with (3) and (4), we acquire the chirped dark soliton solution for the coupled system (2) and (3) as

$$q(x,t) = \left[\sqrt{-\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{2(d_1+\rho^2 f_1+\rho^4 g_1)}}\tan\,\mathrm{h}\left(\sqrt{\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{\rho a_1}}\tau\right)\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)},$$
$$r(x,t) = \rho\left[\sqrt{-\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{2(d_1+\rho^2 f_1+\rho^4 g_1)}}\tan\,\mathrm{h}\left(\sqrt{\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{\rho a_1}}\tau\right)\right]^{\frac{1}{2}} e^{i(\varphi(\tau)-\omega t)}, \tag{66}$$

where $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1) > 0$ and $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(d_1 + \rho^2 f_1 + \rho^4 g_1) < 0$. The chirping is procured as

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} + \frac{\delta_1}{\rho a_1}\left\{\sqrt{-\frac{3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{2(d_1 + \rho^2 f_1 + \rho^4 g_1)}}\ \tan\text{h}\left(\sqrt{\frac{2\left(\omega + \rho \mathrm{k}_1 + \frac{(\mathrm{v}-\mathrm{h}_1)^2}{4\rho \mathrm{a}_1}\right)}{\rho \mathrm{a}_1}}\tau\right)\right\}^{-1}\right]. \quad (67)$$

3.2 Second expression with hyperbolic secant function

We consider that equation (32) has an exact soliton solution identified as

$$F(\tau) = \beta_0 + \frac{\beta_1 \sec\text{h}(\beta_3\tau)}{1 + \sec\text{h}(\beta_3\tau)} + \frac{\beta_2 \sec\text{h}^2(\beta_3\tau)}{1 + \sec\text{h}^2(\beta_3\tau)}, \quad (68)$$

where β_0, β_1, β_2 and β_3 are constants to be determined. Applyiny ansatz (68) to equation (32) gives rise to a polynomial in $\text{sech}(\beta_3\tau)$ of various powers. Equating each coefficient in this polynomial to zero yields a system of algebraic equations that induces the following sets of solutions.

Set 1

$$\beta_0 = \left(-\frac{9}{2} + \frac{\sqrt{17}}{2}\right)\sqrt{\frac{-3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17 + 9\sqrt{17})(d_1 + \rho^2 f_1 + \rho^4 g_1)}}, \quad (69)$$

$$\beta_1 = 0, \quad (70)$$

$$\beta_2 = 8\sqrt{\frac{-3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17 + 9\sqrt{17})(d_1 + \rho^2 f_1 + \rho^4 g_1)}}, \quad (71)$$

$$\beta_3 = 4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17 + 9\sqrt{17})\rho a_1}}, \quad (72)$$

under the constraint conditions (39) and

$$b_1 + \rho^2 c_1 = 4(-3 + \sqrt{17}) \times \sqrt{\frac{\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(d_1 + \rho^2 f_1 + \rho^4 g_1)}{-3(-17 + 9\sqrt{17})}}. \quad (73)$$

Substituting (69)–(72) into (68) and using the relations (30) and (9) along with (3) and (4), we extract the chirped bright soliton pair for the coupled system (2) and (3) in the form:

See Equation (74) bottom of the page

where $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1) > 0$ and $\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)(d_1 + \rho^2 f_1 + \rho^4 g_1) < 0$. The corresponding chirping is obtained as

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} + \frac{\delta_1}{\rho a_1}\left\{\frac{1}{2}\sqrt{\frac{-3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})(d_1 + \rho^2 f_1 + \rho^4 g_1)}} \times \left[(-9 + \sqrt{17}) + \frac{16\sec\text{h}^2\left(4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}{1 + \sec\text{h}^2\left(4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}\right]\right\}^{-1}\right]. \quad (75)$$

$$q(x,t) = \left[\frac{1}{2}\sqrt{\frac{-3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17 + 9\sqrt{17})(d_1 + \rho^2 f_1 + \rho^4 g_1)}}\left\{\left(-9 + \sqrt{17}\right) + \frac{16\sec\text{h}^2\left(4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}{1 + \sec\text{h}^2\left(4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau) - \omega t)},$$

$$r(x,t) = \rho\left[\frac{1}{2}\sqrt{\frac{-3\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17 + 9\sqrt{17})(d_1 + \rho^2 f_1 + \rho^4 g_1)}}\left\{\left(-9 + \sqrt{17}\right) + \frac{16\sec\text{h}^2\left(4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}{1 + \sec\text{h}^2\left(4\sqrt{\frac{2\left(\omega + \rho k_1 + \frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}\right\}\right]^{\frac{1}{2}} e^{i(\varphi(\tau) - \omega t)}, \quad (74)$$

Set 2

$$\beta_0=\left(\frac{9}{2}+\frac{\sqrt{17}}{2}\right)\sqrt{\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})(d_1+\rho^2 f_1+\rho^4 g_1)}}, \quad (76)$$

$$\beta_1=0, \quad (77)$$

$$\beta_2=-8\sqrt{\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})(d_1+\rho^2 f_1+\rho^4 g_1)}}, \quad (78)$$

$$\beta_3=4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})\rho a_1}}, \quad (79)$$

under the constraint conditions (39) and

$$b_1+\rho^2 c_1=-4(3+\sqrt{17})\times\sqrt{\frac{\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(d_1+\rho^2 f_1+\rho^4 g_1)}{3(17+9\sqrt{17})}}. \quad (80)$$

By virtue of these results with (68) and using the relations (30) and (9) in company with (3) and (4), we come by the chirped dark soliton for the coupled system (2) and (3) of the form

See Equation (81) bottom of the page

where $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1)<0$ and $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(d_1+\rho^2 f_1+\rho^4 g_1)>0$. The corresponding chirping reads as

$$\delta\omega(x,t)=-\left[\frac{v-h_1}{2\rho a_1}+\frac{\delta_1}{\rho a_1}\left\{\frac{1}{2}\sqrt{\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})(d_1+\rho^2 f_1+\rho^4 g_1)}}\right.\right.$$
$$\left.\left.\times\left[(9+\sqrt{17})-\frac{16\sec\mathrm{h}^2\left(4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(-17+9\sqrt{17})\rho a_1}}\tau\right)}{1+\sec\mathrm{h}^2\left(4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})\rho a_1}}\tau\right)}\right]\right\}^{-1}\right]. \quad (82)$$

Set 3

$$\beta_0=0, \quad (83)$$

$$\beta_1=0, \quad (84)$$

$$\beta_2=-\frac{6\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)}, \quad (85)$$

$$\beta_3=\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}, \quad (86)$$

under the constraint conditions (39) and

$$6\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(d_1+\rho^2 f_1+\rho^4 g_1)=(b_1+\rho^2 c_1)^2. \quad (87)$$

Inserting (83)–(86) into (68) and using the relations (30) and (9) together with (3) and (4), one can derive the chirped bright soliton solution for the coupled system (2) and (3) given by

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$$q(x,t)=\left[\frac{1}{2}\sqrt{\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})(d_1+\rho^2 f_1+\rho^4 g_1)}}\left\{\left(9+\sqrt{17}\right)-\frac{16\sec\mathrm{h}^2\left(4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})\rho a_1}}\tau\right)}{1+\sec\mathrm{h}^2\left(4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})\rho a_1}}\tau\right)}\right\}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},$$

$$r(x,t)=\rho\left[\frac{1}{2}\sqrt{\frac{3\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})(d_1+\rho^2 f_1+\rho^4 g_1)}}\left\{\left(9+\sqrt{17}\right)-\frac{16\sec\mathrm{h}^2\left(4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})\rho a_1}}\tau\right)}{1+\sec\mathrm{h}^2\left(4\sqrt{-\frac{2\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(17+9\sqrt{17})\rho a_1}}\tau\right)}\right\}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)}, \quad (81)$$

$$q(x,t)=\left[-\frac{6\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)}\frac{\operatorname{sec h}^2\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}^2\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},$$

$$r(x,t)=\rho\left[-\frac{6\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)}\frac{\operatorname{sec h}^2\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}^2\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},\tag{88}$$

where $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1)<0$ and $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(b_1+\rho^2c_1)<0$. The chirping can be found as

See Equation (89) bottom of the page

Set 4

$$\beta_0=-\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2c_1)},\tag{90}$$

$$\beta_1=\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)},\tag{91}$$

$$\beta_2=0,\tag{92}$$

$$\beta_3=2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}},\tag{93}$$

under the constraint conditions (38) and (39). Implementing these outcomes into (68) and using the relations (30) and (9) along with (3) and (4), we reach the chirped bright soliton for the coupled system (2) and (3) in the form

$$q(x,t)=\left[-\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2c_1)}\left\{1-\frac{3\operatorname{sec h}\left(2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}\left(2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right\}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},$$

$$r(x,t)=\rho\left[-\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2c_1)}\left\{1-\frac{3\operatorname{sec h}\left(2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}\left(2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right\}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},\tag{94}$$

where $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1)>0$ and $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(b_1+\rho^2c_1)<0$. The corresponding chirping is attained as

$$\delta\omega(x,t)=-\left[\frac{v-h_1}{2\rho a_1}-\frac{\delta_1}{\rho a_1}\left\{\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{3(b_1+\rho^2c_1)}\times\left[1-\frac{3\operatorname{sec h}\left(2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}\left(2\sqrt{\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right]\right\}^{-1}\right].\tag{95}$$

Set 5

$$\beta_0=0,\tag{96}$$

$$\beta_1=-\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)},\tag{97}$$

$$\beta_2=0,\tag{98}$$

$$\beta_3=2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}},\tag{99}$$

under the constraint conditions (38) and (39). Substituting (96)–(99) into (68) and using the relations (30) and (9) as well as (3) and (4), a form of nonlinearly chirped bright soliton for the coupled system (2) and (3) is constructed as

$$q(x,t)=\left[-\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)}\frac{\operatorname{sec h}\left(2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}\left(2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},$$

$$r(x,t)=\rho\left[-\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)}\frac{\operatorname{sec h}\left(2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}\left(2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right]^{\frac{1}{2}}e^{i(\varphi(\tau)-\omega t)},\tag{100}$$

where $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(\rho a_1)<0$ and $\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)(b_1+\rho^2c_1)<0$. The chirping associated to this soliton solution is presented as

$$\delta\omega(x,t)=-\left[\frac{v-h_1}{2\rho a_1}-\frac{\delta_1}{\rho a_1}\left\{\frac{6\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2c_1)}\frac{\operatorname{sec h}^2\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\operatorname{sec h}^2\left(\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right\}^{-1}\right].\tag{89}$$

$$\delta\omega(x,t) = -\left[\frac{v-h_1}{2\rho a_1} - \frac{\delta_1}{\rho a_1}\left\{\frac{4\left(\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}\right)}{(b_1+\rho^2 c_1)} \frac{\sec\text{h}\left(2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}{1+\sec\text{h}\left(2\sqrt{-\frac{\omega+\rho k_1+\frac{(v-h_1)^2}{4\rho a_1}}{\rho a_1}}\tau\right)}\right\}^{-1}\right]. \tag{101}$$

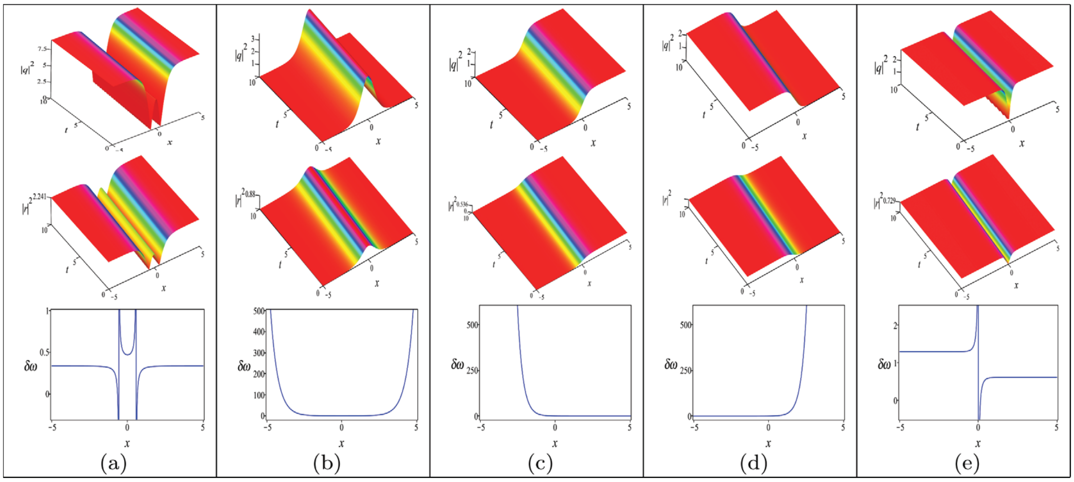

Figure 1. The intensity profiles of W-shaped, bright, kink, anti-kink and dark solitons given by solutions (40), (46), (53), (59) and (66).

See Equation (101) at the top of the page

4 Results and remarks

The chirped optical solitons obtained above for the coupled system (1) and (2) differ entirely from that obtained in the literature. In addition to that, the chirping is found to have a nonlinear function in terms of the reciprocal of intensity. The derived optical solitons consist of various structures which are W-shaped, bright, dark, kink and anti-kink solitons. To provide clear insight on the dynamics of solitons, the graphical representations of the analytical results are represented. The intensity profiles of optical solitons are depicted by selecting suitable values of parameters.

In Figure 1, we exhibit the behaviors of extracted soliton solutions by the first form of undetermined coefficients method along with the corresponding chirping. The plots in Figure 1a display a profile of W-shaped wave that describes the solution (40). The graphs shown in Figure 1b demonstrate a structure of bright soliton wave which characterizes the solution (46). Furthermore, the behavior of solution (53) illustrates kink-wave soliton as shown in Figure 1c while the plots in Figure 1d present the profile of anti-kink wave that describes the soliton solution (59). Additionally, the graphs of solution (66) are revealed in Figure 1e which describe a structure of dark soliton.

Likewise, soliton solutions obtained via the second form of undetermined coefficients method are displayed in Figure 2 besides the associated chirp. The graphs in Figure 2a characterize the soliton pulse with the shape of W for solution (74). Moreover, the evolution of solution (81) is given in Figure 2b that represents a structure of dark soliton wave. Obviously, it can be noted that the plots in Figure 2c demonstrate a profile of bright soliton pulse describing solution (88). Further to this, one can see that the graphs in Figure 2d show the propagation of W-shaped solitons which represents solution (94). Finally, the behavior of solution (100) is plotted in Figure 2e that delineates a profile of bright soliton wave.

Interestingly, the influence level of dispersive reflectivity, self-phase modulation and cross-phase modulation on the pulse propagation is shown in Figure 1. These effects are examined especially for the bright soliton given by solution (46), as example, with the parameter values $\omega = k_1 = h_1 = c_1 = 1$, $\rho_1 = 0.5$, $v = 0.05$, $a_1 = -2.5$, $b_1 = -0.5$, $t = 0$. One can see from Figure 3a that the soliton amplitude is weakly affected because of the variations of dispersive reflectivity a_1 in comparison to the noticeable impacts caused by self-phase modulation b_1 and cross-phase modulation c_1. However, the most negative value of a_1 enhances the amplitude more than the least negative value. Additionally, it is obviously in Figure 3b that self-phase modulation increases the pulse amplitude remarkably compared to cross-phase modulation strength exhibited in Figure 3c.

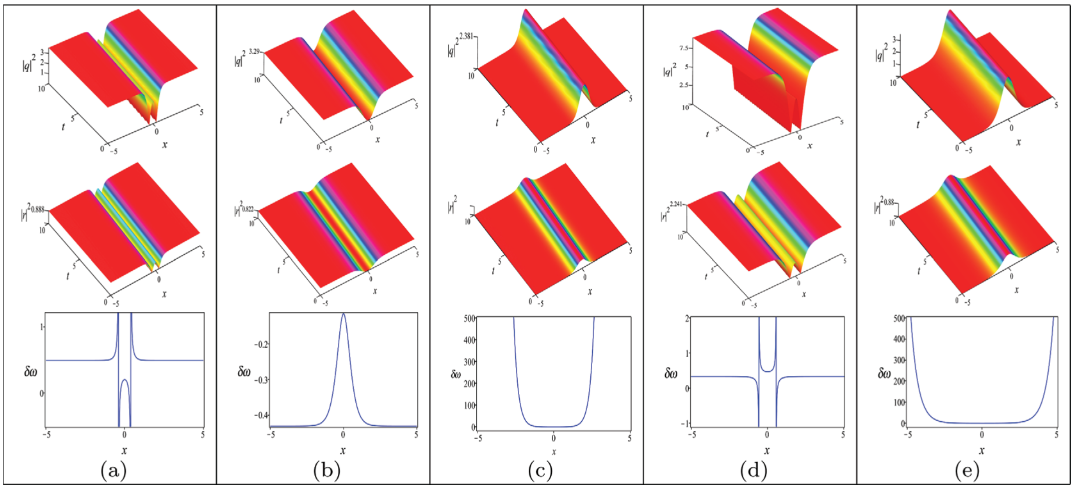

Figure 2. The intensity profiles of W-shaped, bright and dark solitons presented by solutions (74), (81), (88), (94) and (100).

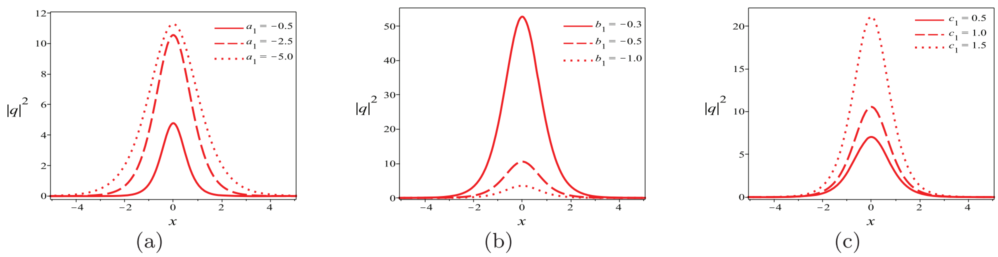

Figure 3. Effects of dispersive reflectivity, self-phase modulation and cross-phase modulation on the pulse propagation for solution (46).

Conclusion

This study discussed essentially the chirped optical solitons in fiber BGs with dispersive reflectivity having polynomial law of nonlinearity. The model of the coupled NLSE is analyzed under specific conditions in order to be straightforwardly integrable. Then, the soliton solutions are extracted by means of the undetermined coefficients approach which was given in two forms. The created optical solitons have several structures that included W-shaped, bright, dark, kink and anti-kink solitons. The chirping expressions associated with solitons were derived for all obtained solutions as well. The intensities of optical solitons are illustrated in addition to the chirping profiles. Besides, it is found that both of self-phase modulation and cross-phase modulation can highly amplify the soliton amplitude while there is a weak growth of amplitude by reason of dispersive reflectivity. The results obtained are expected to serve the field of optical fibers with BGs.

In the forthcoming work, the current model is studied via the technique of complete discrimination system for polynomial. Due to the intricate form of the coupled NLSE, various implicit solutions are revealed under specific restrictions. Furthermore, an exotic form of the soliton ansatz method having combination of hyperbolic secant and tangent functions is applied to equation (32). Consequently, different types of optical solitons are retrieved based on the existence conditions. Later, we intend to further expand our studies to incorporate Bragg gratings with cubic-quartic form of CD or highly dispersive form of CD applied to Bragg gratings and other structures so as to examine the dynamics of optical solitons.

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