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Review Article

Multiplication Processes in High-Density H-^{11}B Fusion Fuel

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Proton-boron fusion would offer considerable advantages for the purpose of energy production as the reaction is aneutronic and does not involve radioactive species. Its exploitation, however, appears to be particularly challenging due to the low reactivity of the H-^{11}B fuel at temperatures up to 100 keV. Fusion chain-reaction concepts have been proposed as possible means to overcome this limitation. Relevant findings are reviewed in this article. Energy-amplification processes are also presented, which are of interest for beam-fusion experiments and fast ignition of H-^{11}B fuel. Directions for further work are outlined as well.

1. Introduction

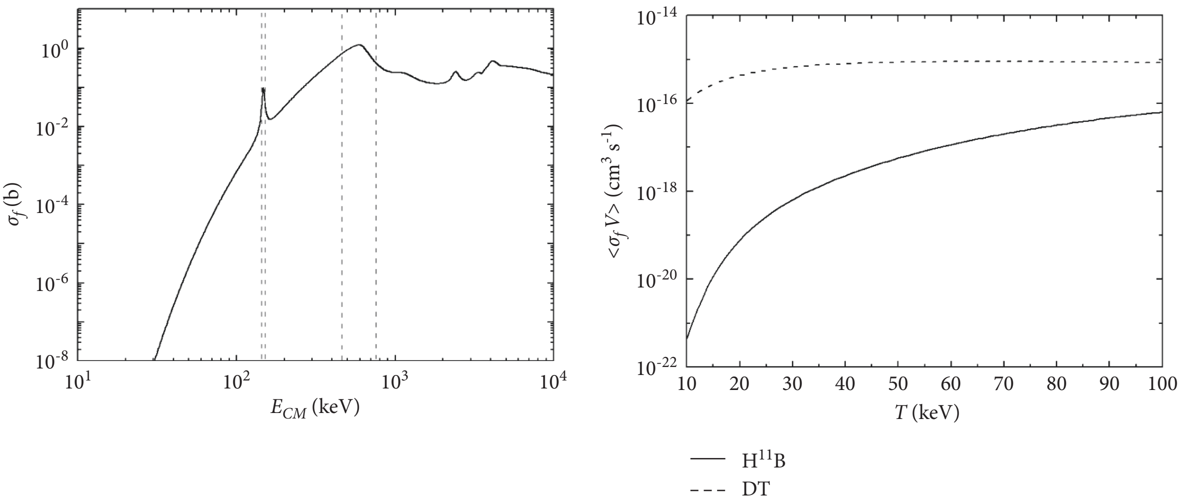
The p-^{11}B fusion reaction produces 3 α-particles with a Q-value of about 8.7 MeV. A mixture of H and ^{11}B has been proposed as an advanced fusion fuel because of certain attractive features [1, 2]. With regard to the reactants, they are abundant in nature (implying that no breeding would be needed), stable (meaning that issues like those associated with the radioactivity of tritium in DT fusion would be avoided), and cheap. With regard to the fusion products, there are only charged particles so that all the reaction energy can be released to the fuel (it is also worth noticing the possibility of direct energy conversion into electricity, without passing through a thermodynamic cycle). More importantly, no neutron is generated, meaning no induced activation of the environment surrounding the fuel (actually, there is still a residual neutron production in the fuel through the (p,n) and (α,n) side reactions on ^{11}B, though the rate is very low). Finally, as an inertial confinement fusion (ICF) fuel, the target would not need to be cryogenic.

A plot of the fusion cross section, σ_f, as a function of the centre of mass (CM) energy, E_{CM}, is shown in Figure 1(a). Resonances of major interest for H-^{11}B fusion are bounded by dashed lines: at 148 keV (with a width of just 5 keV) and 612 keV (with a width of 300 keV) [3]. At 612 keV, σ_f reaches its maximum value, 1.4 barn [6]. Below approximately 3.5 MeV, the reaction proceeds through 3 channels [6, 7]: the low branching-ratio ^{12}C direct breakup and the sequential decays via the first excited state or the ground state of ^{8}Be, i.e.:

$$p + {}^{11}B \rightarrow {}^{12}C^{*} \begin{array}{ll} \nearrow & \alpha_1 + {}^{8}Be_{(1)} \rightarrow \alpha_1 + \alpha_{11} + \alpha_{12} \\ \rightarrow & \alpha_0 + {}^{8}Be_{(0)} \rightarrow \alpha_0 + \alpha_{01} + \alpha_{02} \cdot \\ \searrow & 3\alpha \end{array} \qquad (1)$$

Summed over the reaction channels, the energy spectrum of the generated α-particles is a continuum; for an incoming proton with energy at the cross section maximum, it extends up to about 6.7 MeV in the laboratory. The spectrum is strongly peaked around 4 MeV. One can say that, on average, two α's are emitted at 4 MeV, while one is emitted at 1 MeV [8]. The α's angular distribution is isotropic in the CM system (nearly in the laboratory, because of the proton momentum).

In Figure 1(b), the reactivity, $<\sigma_f v>$, is shown as a function of ion temperature for H-^{11}B fuel and, as a term of comparison, DT fuel [3, 4]. We recall that the thermonuclear specific reaction rate is given by $R \overset{\text{def}}{=} n_X n_Y \langle \sigma_f v \rangle$, where n_X and n_Y are the number densities of the fusing species. The low reactivity represents a major drawback of H-^{11}B fuel: for



(a) (b)

Figure 1: Fusion cross section and reactivity of H-^{11}B fuel. (a) Fusion cross section as a function of the CM energy, based on the analytic approximation of Nevins and Swain [3] below 3.5 MeV and, above, on TENDL evaluated data. Resonances of major interest for H-^{11}B fusion are bounded by dashed lines. (b) Reactivity as a function of ion temperature for H-^{11}B fuel and, as a term of comparison, DT fuel. Plots are based on the analytic approximations of Nevins and Swain [3] and Bosch and Hale [4], respectively. Republished from Belloni [5]. © IOP Publishing Ltd. 2021.

instance, at temperatures of the order of those currently achievable in magnetic and inertial confinement experiments (around 10 keV), the H-^{11}B reactivity is 5 orders of magnitude lower than the DT reactivity.

The first comprehensive assessment of the viability of H-^{11}B fuel for thermonuclear fusion was carried out by Moreau [9] at the Princeton Plasma Physics Laboratory, mid-seventies. A steady-state, two-temperature (T_e, T_i) analysis of the balance of the power fluxes was applied: namely, from the fusion products the heat goes entirely to the fuel ions, then from the ions to the electrons, finally into radiation. Note that this power flow scheme requires $T_i > T_e$. In the case $T_i = T_e$, the plasma was found to not ignite because of the predominance of radiation losses. We recall that the fusion power per unit volume, P_F, is given by the product RQ, where Q is the reaction Q-value; the specific power transfer from ions to electrons, $-\mathrm{d}W_i/\mathrm{d}t$, is proportional to $n_e^2\,(T_i - T_e)/T_e^{3/2}$, while the radiation power lost by bremsstrahlung, P_B, scales as $n_e^2 T_e^{1/2}$ and the synchrotron radiation power, P_S, as $n_e T_e B^2$, where B is the magnetic field. Confinement requires that $B^2 \propto (n_e T_e + n_i T_i)/\beta$, where β is the *beta ratio*, the main parameter for confinement efficiency (typically, $\beta \ll 1$ in tokamaks, though higher values are desirable for fusion power production). For ignition to occur, the curves $P_F = -\mathrm{d}W_i/\mathrm{d}t$ and $P_F = (P_B + P_S)(1 - \eta)$, where η is the recirculating power fraction, must intersect in the T_e-T_i plane.

For the case of magnetic confinement, upon realistic assumptions for the recirculating power and confinement conditions, this analysis showed that no ignition point could arise in the T_e-T_i plane when, in addition to bremsstrahlung, synchrotron radiation losses were also taken into account. The conclusion was that H-^{11}B fusion is unfeasible in tokamaks. A chance, however, could come from inertial confinement, where only bremsstrahlung losses count. In this case, working at much higher densities, ignition points do exist in the T_e-T_i plane (Figure 2). For example, upon the hypothesis of a 70% fraction of fusion power to fuel ions, an ignition point exists for $T_e = 140$ keV and $T_i = 280$ keV, though these values are very high. At a fuel density of 10^{27} ion cm^{-3} (boron-to-proton concentration just lower than 10%), the Lawson criterion requires a minimum confinement time, τ_0, of 16 ps. Then, igniting the smallest possible pellet, with radius τ_0 times the speed of sound, would require a laser energy of 7 MJ. Such a figure is still challenging today, 40 years after Moreau's study. This explains why H-^{11}B fusion was put on hold, at least on the ground of experiment, and it took almost 30 years before having its first demonstration by lasers [10]. Moreover, the demonstration was achieved very far from the prescribed thermonuclear regime, indeed by exploiting an effect unknown at the time of Moreau's work, which is laser acceleration of ions.

This article is intended to give an overview of the nonthermal effects which can complement and supplement the thermonuclear burn of H-^{11}B fuel, through fusion events' multiplication or chain-like mechanisms. The following processes and concepts are reviewed:

(a) Fusion chain progressing via intermediate nuclear reactions;

(b) Suprathermal fusion chain, i.e., the chain sustained by suprathermal fuel ions elastically scattered by the fusion-born α's; and

(c) The *energy multiplication* (or *amplification*) *factor* in a beam-driven fusion scheme, which is relevant to proton fast ignition.

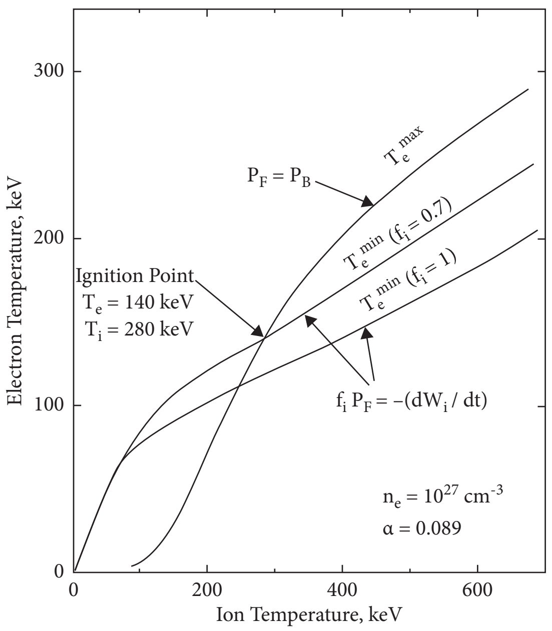


FIGURE 2: Ignition points in the T_e-T_i plane for high-density H-^{11}B plasma. f_i is the fraction of fusion power to plasma ions; α is the boron-to-proton ion concentration. Reproduced from Moreau [9] with the permission of the publisher. © IAEA 1977.

2. Multiplication Processes

2.1. Fusion Chain via Intermediate Nuclear Reactions. The possibility of a fusion chain progressing via intermediate nuclear reactions in H-B targets has been presented in two works by Belyaev et al. [11, 12]. The basic chain relies on the fact that a fusion-born α-particle can in turn react with ^{11}B through the reaction:

$$\alpha + {}^{11}\text{B} \longrightarrow \text{p} + {}^{14}\text{C} + 0.8\,\text{MeV}, \quad (2)$$

which generates a high-energy proton. This proton can in turn fuse with ^{11}B, possibly giving rise to a chain reaction. However, there is the competing reaction:

$$\alpha + {}^{11}\text{B} \longrightarrow \text{n} + {}^{14}\text{N}, \quad (3)$$

which acts as a sink of α's and generates neutrons. The neutron- and proton-generating reactions have a comparable cross section.

To compensate for the loss of α-particles in the fuel due to the neutron channel, and to reabsorb the neutron inside the fuel, one should exploit neutron capture on ^{10}B, i.e.:

$$\text{n} + {}^{10}\text{B} \longrightarrow \alpha + {}^{7}\text{Li}. \quad (4)$$

This means that natural boron should be used in the fuel, or ^{11}B should be adequately supplemented with ^{10}B.

Along all the reaction pathways in the chain, the authors find that the number of protons, α-particles, and neutrons in the fuel grows up as an avalanche over times of the order of 1 μs, approximately; cfr. Figure 1 in ref. [12]. One has to remark, however, that this approach lies upon highly idealised assumptions, in particular,

(i) maximum values of the reaction cross sections have been used,
(ii) particles' energy losses have been neglected, and
(iii) a too long confinement time is required, which is unrealistic for warm, solid-density fuel.

Shmatov [13] has finally shown that at least for temperatures up to 100 keV, only a tiny fraction of α-particles would be capable to react with ^{11}B because of their loss of energy in the fuel, thus preventing the development of the chain. On another note, if Belyaev et al.'s chain developed, fuel neutronicity would become considerably high, which would jeopardise the most appealing feature of H-^{11}B fusion.

It is also worth mentioning that experiments have been done by Labaune et al. [14] at LULI, France, to test the possibility of inducing a chain reaction in natural-boron or boron-nitride targets under irradiation by laser-accelerated protons (generated from a thin foil). Targets were solid or conditioned in a plasma state by laser irradiation. The authors intended to exploit several nuclear reaction pathways, as detailed in Figure 3. Even in the absence of a self-sustaining chain, they hoped that secondary reactions could substantially increase the energy yield compared with a pure p-^{11}B fusion scenario. While secondary reactions have successfully been induced and measured under such schemes, one has nevertheless to conclude that their rate is too low to induce any significant avalanche process or increase in the energy yield. For instance, in the case of a solid boron-nitride target, the overall number of the X(p,^{11}C) reactions, with X = ^{11}B or ^{14}N (Figure 3(b)), was estimated at 10^6 per shot by means of ^{11}C decay measurements. This figure appears to be at least 1000 times smaller than the number of ^{11}B(p, 3α) fusion reactions (>10^9 per shot).

2.2. Suprathermal Fusion Chain. The fact that three charged, massive, energetic particles are produced in the p-^{11}B reaction suggests that the fusion yield could effectively be enhanced by the elastic scattering of fuel ions to energies corresponding to the highest values of σ_f. This particularly applies to protons because of their higher charge-to-mass ratio compared with ^{11}B ions. While thermalising, some of the protons in these showers can undergo fusion, eventually setting a chain reaction up. At high matter density, moreover, α's tend to lose energy mostly to plasma ions rather than to electrons. This happens when the electrons' Fermi velocity (or their thermal velocity) becomes comparable to the α-particle velocity, while the ion thermal velocity remains substantially lower [15–17].

The suprathermal fusion chain in an infinite, homogeneous H-^{11}B plasma can be effectively parameterised in terms of two multiplication (or reproduction) factors. An α-particle emitted at a certain energy $E_{\alpha,0}$ in a primary fusion event is characterised by the multiplication factor $k_\alpha(E_{\alpha,0})$, i.e., the average number of secondary α-particles generated via suprathermal processes during the slowing down of the

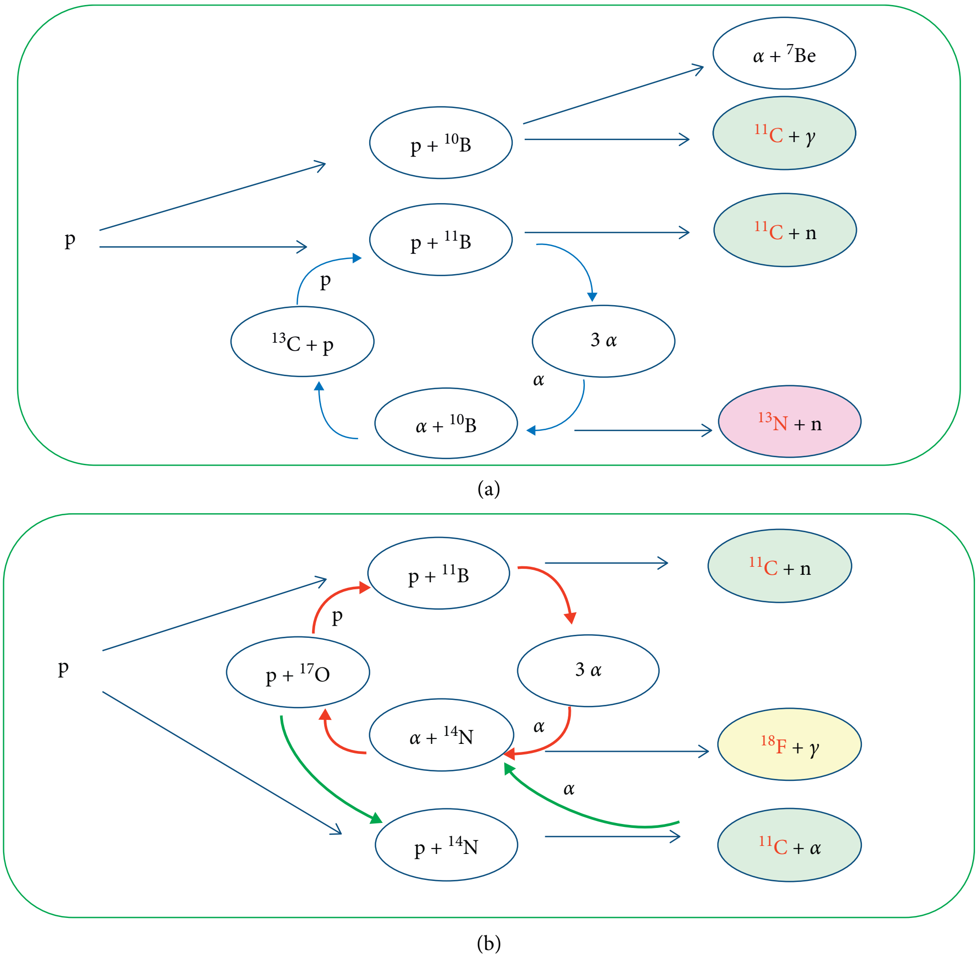


FIGURE 3: Scheme of the main primary and secondary nuclear reactions produced by the interaction between a laser-accelerated proton beam and (a) a natural boron target and (b) a boron-nitride target. Reproduced from Labaune et al. [14], under the terms of the Creative Commons CC BY License.

primary particle. Likewise, the multiplication factor can be expressed in terms of fusion events. Then, one defines k_∞ as the average number of secondary fusion events per primary event. k_∞ can be estimated through the integration of k_α over the α-emission spectrum, $\varphi(E_{\alpha,0})$ [5]:

$$k_\infty = \int k_\alpha\left(E_{\alpha,0}\right)\varphi\left(E_{\alpha,0}\right)\mathrm{d}E_{\alpha,0}, \qquad (5)$$

where $\int \varphi(E_{\alpha,0})\mathrm{d}E_{\alpha,0} = 1$. Strictly speaking, the concept of k_∞ is well grounded as long as the emitted α-particles have a comparable slowing-down time, and this quantity is in turn comparable to the (average) period between two consecutive generations of fusion events, τ_g. It is not difficult, then, to calculate the cumulative number of fusion events per unit volume at the time t, $n_f(t)$, in regime of multiplication, upon the thermonuclear specific rate R. Depending on the value of k_∞, one can distinguish three cases for the time evolution of n_f (hence, of the energy yield):

(1) $k_\infty < 1 \Rightarrow n_f$ increases linearly with time, asymptotically to $Rt/(1-k_\infty)$, for $t \gg \tau_g$;

(2) $k_\infty = 1 \Rightarrow n_f$ increases quadratically with time; in detail, $n_f(t) = R(t + t^2/2\tau_g)$; and

(3) $k_\infty > 1 \Rightarrow n_f$ diverges exponentially, with the growth rate $\ln k_\infty/\tau_g$.

It goes without saying that the capability to achieve a chain reaction with multiplicity k_∞ higher of or comparable to 1 would play a significant—if not indispensable—role in the possible exploitation of H-^{11}B fuel as an energy source. The question is also how and how much a weak multiplication regime, namely when $k_\infty < 1$ (and especially $k_\infty \ll 1$), can enhance the pure thermonuclear burn. Using the full expression of $n_f(t)$, it is easy to calculate the ratio I of the suprathermal-to-thermonuclear energy yield in the confinement time τ_c [5]; indeed, the total energy per unit volume is $n_f(\tau_c)Q$, the energy stemming from the sole thermonuclear burn is just $RQ\tau_c$, and the suprathermal yield is given by the difference between the first two. I is shown in Figure (4) as a function of k_∞ for several orders of magnitude of the parameter τ_c/τ_α, where $\tau_g \approx \tau_\alpha$ is assumed and τ_α is the thermalisation time of the α particle at its most probable emission energy. Note that in typical ICF conditions, the quantity τ_c/τ_α can reach the order of 10^3. One can distinguish the following noticeable limits for I. For $\tau_c/\tau_\alpha \gg 1$ and $k_\infty \ll 1$, I scales as k_∞. On the contrary, when

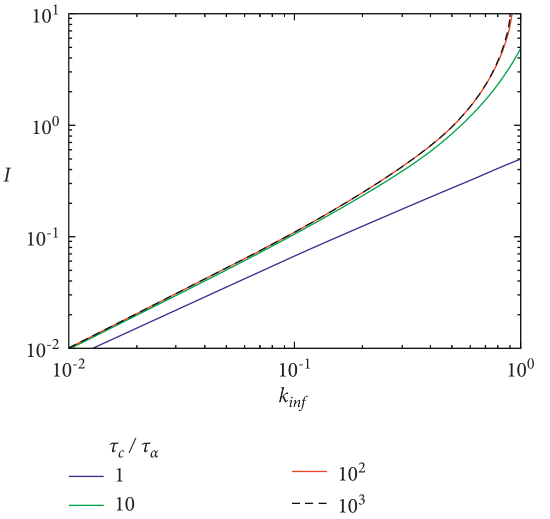


FIGURE 4: Suprathermal-to-thermonuclear energy ratio by the effect of a weak chain reaction. Republished from Belloni [5]. © IOP Publishing Ltd. 2021.

k_∞ approaches 1, I tends to $(1/2)\tau_c/\tau_\alpha$, which opens the possibility of very large increments in the energy output (consequently, high fusion gains). This means that k_∞ does not have to be necessarily larger than 1 to have a sizeable enhancement of the fusion yield.

The earliest studies were not encouraging, however. In the early 70s, Weaver et al. [18] estimated the increase in the H-^{11}B reaction rate due to nonthermal effects to vary between 5% and 15% in the density range of 10^{16}–$10^{26}\,\mathrm{cm}^{-3}$ and in the temperature range of 150–350 keV. Subsequent calculations by Moreau [9, 19] returned multiplication factors of the order of 10^{-2} in a plasma with $100 \leq T_e \leq 300$ keV, cold ions, and Coulomb logarithm $\ln\Lambda = 5$. In both cases, however, important details have not been given; moreover, only the Coulomb interaction has been taken into account in the α-ion scattering in the case of Moreau, or poorly known nuclear data have been used for this purpose in the case of Weaver et al.

The recent study of Putvinski et al. [20] has substantially confirmed the findings of Weaver et al. [18]. The H-^{11}B reactivity has been calculated using a proton spectrum, which included *kinetic* effects at high energy: besides α-scattering, cooling on colder electrons ($T_e < T_i$) and depletion of the spectrum tail by the fusion burn. The proton spectrum was self-consistently calculated by solving the steady-state Fokker–Planck equation upon a simple burn model. For reference parameters $T_i = 300$ keV, $T_e = 150$ keV, $n_B/n_p = 0.15$, and $E_{\alpha,0} = 4$ MeV, the resulting reactivity showed a 10% increase compared with its purely Maxwellian form. Note that this treatment is formally independent of absolute densities as long as a fixed value is used for $\ln\Lambda$ (details are not given, however). Also in this case, the nuclear interaction does not appear to have been taken into account in the α-ion scattering.

Recently, a supposed experimental manifestation of the suprathermal chain reaction has been the subject of some controversies [21–26], which have finally been resolved in favour of the impossibility to induce this effect in plasma conditions such as those achievable at the Prague Asterix Laser System (PALS), Czech Republic [27, 28]. A later study [5] has confirmed that in high-density, nondegenerate H-^{11}B plasma, k_∞ turns out to be of the order of 10^{-2} at most. The domain investigated is given by $10^{24} \leq n_e \leq 10^{28}\,\mathrm{cm}^{-3}$, $T_i = 1$ keV, $\max[T_i, 5E_F(n_e)] \leq T_e \leq 100$ keV, where E_F is the Fermi energy; $E_F[\mathrm{keV}] = 3.65 \times 10^{-18}\,(n_e[\mathrm{cm}^{-3}])^{2/3}$. This represents a low-$T_i$ regime, where the thermonuclear burn is very modest and is just used to seed the chain reaction; the hope was that the suprathermal chain could drive the plasma burn towards ignition, by increasing T_i quickly.

If T_i is sufficiently low, one can assume that, at least for the first few generations, the suprathermal showers elicited by the α particles do not interact with each other and do not significantly affect the background (thermal) Maxwell–Boltzmann distribution of plasma ions. In a scenario of this kind, each primary α-particle or fusion event can be treated independently through a simplified model compared with more sophisticated kinetic-theory approaches, which are indispensable at high reaction rates; see, e.g., refs. [29–31] for the case of DT fusion, and [20] for H-^{11}B fusion. The simplified approach of ref. [5], in particular, assesses whether the medium is *multiplicative* or not. Without entering details, the contribution to k_α of suprathermal H and ^{11}B ions ($k_{\alpha p}$ and $k_{\alpha B}$, respectively) is calculated separately, i.e.:

$$k_\alpha\left(E_{\alpha,0}\right) = k_{\alpha p}\left(E_{\alpha,0}\right) + k_{\alpha B}\left(E_{\alpha,0}\right). \tag{6}$$

Denoting by j the generic ion species and by $E_{j,0}$ its energy just after the scattering by an α particle, $k_{\alpha j}$ is related to the scattered ion spectrum, $\mathrm{d}N_j/\mathrm{d}E_{j,0}$, and the fusion probability of j, P_j, by the relation:

$$\frac{\mathrm{d}k_{\alpha j}}{\mathrm{d}E_{j,0}}\left(E_{j,0}; E_{\alpha,0}\right) = 3P_j\left(E_{j,0}\right)\frac{dN_j}{dE_{j,0}}\left(E_{j,0}; E_{\alpha,0}\right). \tag{7}$$

The spectrum $\mathrm{d}N_j/\mathrm{d}E_{j,0}$ in turn depends on the α-ion differential scattering cross section, $\sigma_{\alpha j}$, and the α-particle stopping power, $\mathrm{d}E_\alpha/\mathrm{d}x$, according to the relation:

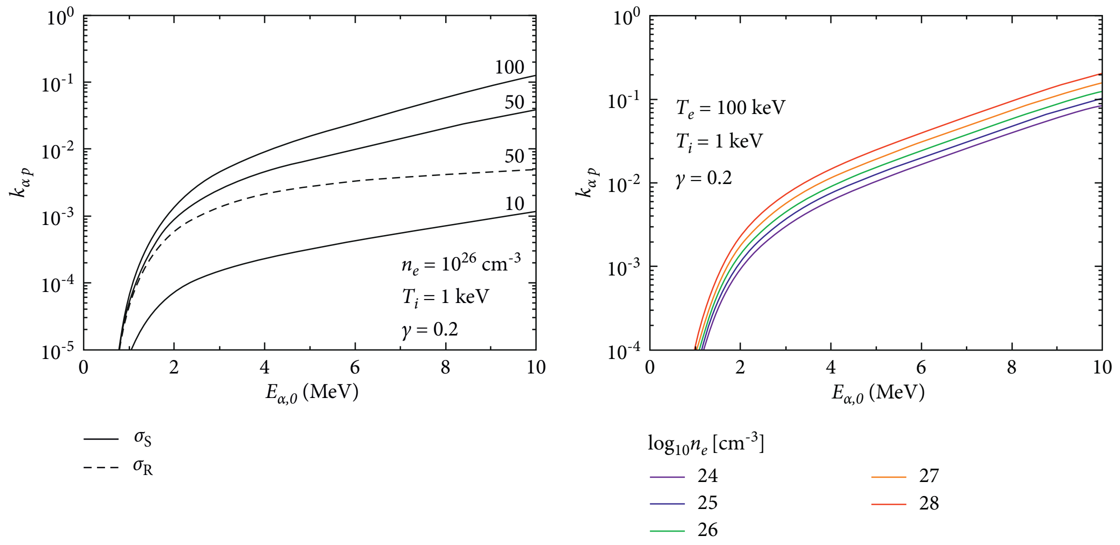
$$\frac{\mathrm{d}N_j}{\mathrm{d}E_{j,0}}\left(E_{j,0}; E_{\alpha,0}\right) = n_j \int_{(3/2)T_i}^{E_{\alpha,0}} \sigma_{\alpha j}\left(E_\alpha, E_{j,0}\right)\left(\frac{dE_\alpha}{dx}\right)^{-1} \mathrm{d}E_\alpha. \tag{8}$$

P_j depends on σ_f and the stopping power of j, $\mathrm{d}E_j/\mathrm{d}x$, in a similar fashion:

$$P_{p(B)}\left(E_{p(B),0}\right) = n_{B(p)} \int_0^{E_{p(B),0}} \sigma_f\left(E_{CM}\right)\left(\frac{\mathrm{d}E_{p(B)}}{\mathrm{d}x}\right)^{-1} \mathrm{d}E_{p(B)}. \tag{9}$$

From ref. [5], there are several points to remark and discuss. First of all, the contribution of suprathermal ^{11}B ions to k_α and k_∞ is of the order of 1% only. Nevertheless, the effect of the nuclear interaction in the α-^{11}B scattering should be counterchecked, in the light of the elastic cross section measurements at $E_\alpha < 5$ MeV performed by Spraker et al. [32]. In ref. [5], $\sigma_{\alpha B}$ is calculated as the Rutherford cross section only.

In the case of the scattered proton, the complete elastic cross section, accounting also for the nuclear interaction, must definitely be used in calculations. In Figure 5(a), $k_{\alpha p}$ is



(a) (b)

Figure 5: α-particle multiplication factor via suprathermal protons as a function of the initial α-energy, for different values of T_e at a fixed n_e (a), and of n_e at a fixed T_e (b). In (a), T_e values (in keV) are indicated next to the curves; σ_s is the complete α-p elastic scattering cross section, accounting also for the nuclear interaction. In (b), γ is the boron-to-proton ion concentration. Republished from Belloni [5]. © IOP Publishing Ltd. 2021.

plotted as a function of $E_{\alpha,0}$ for different values of T_e and $n_e = 10^{26}\,\mathrm{cm}^{-3}$. As a term of comparison, a curve based only on the Rutherford α-p scattering cross section, σ_R, is shown for $T_e = 50$ keV. One can appreciate that for the most probable α-emission energy, 4 MeV, $k_{\alpha p}$ is more than twice that found for a pure Coulomb scattering. At $E_{\alpha,0} = 10$ MeV, the difference reaches a factor of 8.

A parametric analysis shows that $k_{\alpha p}$ increases with both T_e and n_e, though it is much more sensitive to T_e. From Figure 5(b), one notes that $k_{\alpha p}$ drops quickly below $E_{\alpha,0} \simeq 2$ MeV, while above 4 MeV, the shape of the curves is approximately linear in the semilog plot, meaning an exponential increase with $E_{\alpha,0}$ (up to at least 10 MeV). Even at the highest values of n_e and T_e considered ($10^{28}\,\mathrm{cm}^{-3}$ and 100 keV, respectively), $k_{\alpha p}$ (hence, k_∞) remains significantly lower than 1; for instance, $k_{\alpha p} = 0.2$ for $E_{\alpha,0} = 10$ MeV, and $k_\infty \approx 0.01$ over the actual fusion spectrum. A fit of $k_{\alpha p}$-vs-$E_{\alpha,0}$ curves with an exponential function returns a common growth rate such that $k_{\alpha p}$ increases by a factor of about 2.5 each time $E_{\alpha,0}$ increases by 2 MeV. At $n_e = 10^{28}\, cm^{-3}$ and $T_e = 100$ keV, one extrapolates $k_{\alpha p} = 1$ for $E_{\alpha,0} \approx 13.6$ MeV.

This means that if we could boost the energy of α's—let us say—above 10 MeV, we could substantially increase the multiplication factor. How might this be achieved? In principle, two ways can be identified at present. One way is to use high-energy protons to trigger the fusion reaction, which can be referred to as a *kinematic boost*; α particles with energies up to 20 MeV have recently been generated by Bonvalet et al. [33] in a laser-driven pitcher-catcher experiment. Another way is to accelerate the fusion-born α's; for instance, in the same laser-induced electric field which accelerates the protons in direct-target-irradiation experiments. Evidence of this effect has recently been reported by Giuffrida et al. [34]. With either of these means, it is not obvious, however, how $E_{\alpha,0}$ could be kept so high for more than one generation. In the case of laser acceleration of the α particles, it is neither obvious how this effect, observed under irradiation of planar solid targets, could be reproduced on actual ICF targets.

It is also worth mentioning that the concept of a possible H-^{11}B fusion reactor has been proposed (but in a low-density plasma, in this case) [35,36], which is based on α's acceleration by the application of an external electric field to counterbalance the stopping power and induce an avalanche of reactions.

To summarise, values of k_∞ very close to 1 are needed in an ICF scheme to enhance the suprathermal-to-thermonuclear energy yield by factors of up to 10^3. Early computations by Weaver et al. [18] estimated the increase in the H-^{11}B reaction rate due to suprathermal effects to vary only between 5% and 15% in the density range of 10^{16}–$10^{26}\,\mathrm{cm}^{-3}$ ($n_e \approx n_i$) and temperature range of 150–350 keV ($T_e = T_i$). Subsequent calculations [9, 19] returned multiplication factors of the order of 10^{-2} in a plasma with $100 \le T_e \le 300$ keV, $T_i = 0$, and $\ln \Lambda = 5$. Recently, Putvinski et al. [20] have substantially confirmed the findings of Weaver et al., whereas in the high-density, low-T_i domain $10^{24} \lesssim n_e \lesssim 10^{28}\,\mathrm{cm}^{-3}$, $T_e \lesssim 100$ keV, and $T_i \sim 1$ keV (non-degenerate plasma), it has been found $k_\infty \sim 10^{-2}$ at most [5]. This latest work has also shown that particularly for the α-p scattering, the complete elastic cross section, which includes the nuclear interaction, is needed in calculations. Furthermore, k_α has been found to increase exponentially with the α-particle energy, at least in the range of 4–10 MeV, with a growth rate that is independent of n_e. This exponential growth could in principle be exploited in cases where the energy of the fusion-born α's is boosted, e.g., kinematically [33] or electrodynamically [34].

Finally, no experimental evidence of suprathermal multiplication has been achieved so far.

2.3. Energy Multiplication Factor in a Beam-Driven Fusion Scheme. Fast ignition may provide an option to ignite non-DT fuels inasmuch as it significantly relaxes implosion symmetry requirements (compared with central hot-spot ignition) and allows for nonspherical target configurations or fuel seeding [37]. Among the concepts for fast ignition, it has been proposed to use intense laser-accelerated proton beams [38]. In this approach, the energy deposited in the precompressed fuel by a proton beam generated outside the target bootstraps the fusion flame. Ideally, between 5 and 10% of the laser pulse energy is converted into kinetic energy of the beam as the result of the interaction of the pulse with a thin foil.

Here, we wish to emphasise that proton beam fast ignition is a particularly advantageous option for the case of H-^{11}B fuel. Indeed, while the protons transfer their energy to the plasma, additional heating is provided by in-flight fusion reactions. This is an exclusive effect of H-^{11}B fuel as it cannot obviously occur for other proposed fusion fuels under proton irradiation. On the quantitative ground, it is useful to make recourse to the so-called e*nergy multiplication factor*, a fundamental quantity in beam fusion. It is defined as the ratio of the fusion energy produced via in-flight reactions to the overall beam energy, i.e.:

$$F = \frac{P_p(E_0)\,Q}{E_0}, \tag{10}$$

where F is the energy multiplication factor, E_0 is the initial proton energy, Q is the fusion Q-value, and P_p is given by Equation (9) as long as it is sufficiently lower than 1. Denoting by E_b the beam energy, the overall energy deposited in the fuel, E_d, is then:

$$E_d = (1 + F)E_b. \tag{11}$$

An estimate of F is important to set the value of E_0 to be achieved in the laser acceleration of the protons.

A calculation of F vs E_0 for beam-driven H-^{11}B fusion has been carried out by Moreau [9]. This author considered protons injected into a ^{11}B plasma with warm electrons and cold ions. The results are shown in Figure 6 for several values of T_e. In all cases, there is a maximum at E_0 around 1 MeV; at high values of T_e, other two maxima appear just below 3 MeV and between 4 and 5 MeV, respectively. It is hard, however, to achieve a multiplication factor better than 30%. Anyway, Moreau's calculation should be redone with a more accurate fusion cross section (which was barely known at that time) and stopping power model (Sivukhin's model [39] was used).

Note that for a Maxwellian plasma, F is formally independent of density when the value of lnΛ is kept fixed. This comes from an implicit cancellation of the density in the product between n_B and $(\mathrm{d}E_p/\mathrm{d}x)^{-1}$ in Equation (9), with a residual density dependence holding through the expression of lnΛ in $\mathrm{d}E_p/\mathrm{d}x$ [5]. This residual dependence is very weak, however. It is also worth noticing that in a fully degenerate plasma, the electronic component of $\mathrm{d}E_p/\mathrm{d}x$ would become independent of n_e and proportional to the proton velocity, under certain conditions. (In general, the stopping power of an ion in a fully degenerate plasma scales roughly linearly with n_e. However, when the velocity of the ion is much smaller than the Fermi velocity and the parameter $r_s = (me^2/\hbar^2)(3/4\pi n_e)^{1/3}$—where m is the electron mass—is much smaller than 1, the stopping power becomes independent of n_e and proportional to the ion velocity. The condition $r_s \ll 1$ holds for $n_e \gg 10^{24}\mathrm{cm}^{-3}$) [17, 40, 41]. As long as the ion-ion component of $\mathrm{d}E_p/\mathrm{d}x$ can be neglected, P_p would then become truly proportional to n_B. This effect could boost P_p towards 1 even at low (possibly sub-MeV) values of E_0, given the linear velocity dependence of the electronic stopping power. As a consequence, F could rise up significantly, well above unity.

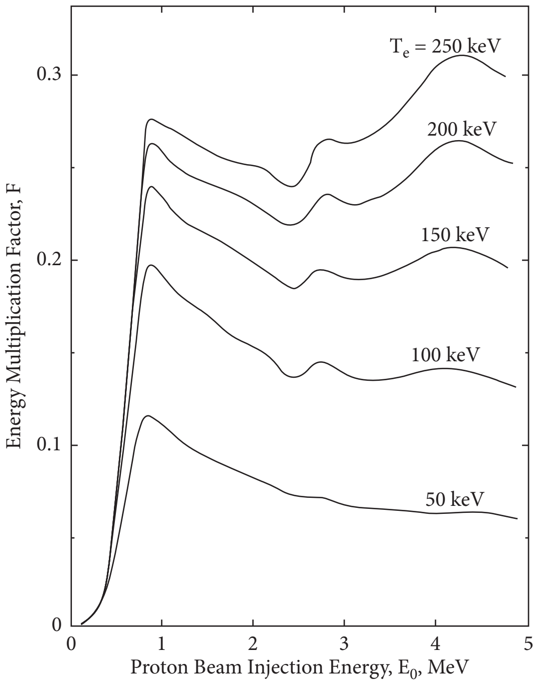


Figure 6: Energy multiplication factor vs proton injection energy for various electron temperatures. Protons are injected into a ^{11}B plasma with cold ions and warm electrons. The results are independent of plasma density when the value of *ln*Λ is fixed. Reproduced from Moreau [9] with the permission of the publisher. © IAEA 1977.

Another aspect to emphasise is that the energy multiplication factor can be further increased if suprathermal chain reaction effects take place. It is easy to show that in this case, E_d is augmented by a term $S_l F E_b$ compared with Equation (11), where the factor S_l depends on the number of generations, l, and is essentially a partial summation of the geometric series with common ratio k_∞, according to the relation:

$$S_l(k_\infty) + 1 = \sum_{i=0}^{l} k_\infty^i = \frac{1 - k_\infty^{l+1}}{1 - k_\infty}. \tag{12}$$

Explicitly,

$$E_d = \left(1 + F + S_l F\right) E_b, \tag{13}$$

where the energy multiplication factor can be redefined as follows:

$$F_{supr} \overset{\text{def}}{=} \left(1 + S_l\right) F. \tag{14}$$

Note that S_l converges to $k_\infty / (1 - k_\infty)$ when $k_\infty < 1$ and l is sufficiently large, while $S_l \longrightarrow l$ when $k_\infty \longrightarrow 1$. Now, if one was capable to keep the chain going for just two generations with k_∞ sufficiently close to 1, assuming $F = 0.3$, Equation (13) would return $E_d \approx 2E_b$, which is quite a significant amplification. It is particularly relevant to this case what has been mentioned in the previous section, namely, that k_∞ could be made close to 1 by exploiting the kinematic boost of the proton beam.

3. Conclusions

Recent laser-based experiments [27, 28, 34, 42, 43], basic ICF physics considerations, and current advances in laser technology suggest that a possible scheme to burn H-^{11}B fuel is based on laser-driven proton fast ignition. By itself, this scheme will likely not be enough to achieve high gains. We are confident, however, that it can be complemented by suprathermal effects and strategies for the containment of bremsstrahlung losses in order to increase the fusion yield and relax ignition and burn requirements. In this article, we have reviewed and discussed nonthermal processes of interest, such as the progression of fusion chains via intermediate nuclear reactions, suprathermal multiplication, and beam energy amplification in proton fast ignition.

Fusion chain processes based on intermediate nuclear reactions do not show the potential to make a substantial contribution to ignition and burn of H-^{11}B fuel. Increasing suprathermal fusion's k_∞ above the order of 10^{-2} also appears problematic in present-day laser-driven plasma conditions; nevertheless, promising directions for further investigation can be drawn. In particular, the work of Belloni [5] should be extended to calculate suprathermal effects.

(i) at higher T_i, by adopting more refined kinetic approaches to the problem (e.g., steady-state spectral conditions, via the so-called Boltzmann–Fokker–Planck equation [44]);

(ii) in (partially) degenerate plasmas, a regime that is also of interest for bremsstrahlung reduction [17, 45].

After Moreau [9], the energy multiplication factor for a proton ignitor should be reassessed against the latest measurements of the fusion cross section [6]. The energy multiplication factor should account not only for the in-flight fusion reactions but also for possible suprathermal multiplication of the fusion products. Calculations should include realistic fuel compositions and degenerate plasma regimes.

Finally, on the experimental side, it is to remark that accurate calculations of nonthermal effects, including beam fusion, need reliable fusion cross section measurements well beyond 3 MeV.

Data Availability

This is a review article. Underlying data can be found in the references.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this study.

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