Hindawi
Laser and Particle Beams
Volume 2022, Article ID 7571742, 12 pages
https://doi.org/10.1155/2022/7571742

CAMBRIDGE
UNIVERSITY PRESS

Research Article

Self-focusing/Defocusing of Hermite-Sinh-Gaussian Laser Beam in Underdense Inhomogeneous Plasmas

Kaijing Tian and Xiongping Xia

College of Science, Guilin University of Technology, Guilin 541004, China

Correspondence should be addressed to Xiongping Xia; xxpccp@163.com

Received 30 December 2021; Revised 13 February 2022; Accepted 9 March 2022; Published 28 March 2022

Academic Editor: Devki Nandan Gupta

The self-focusing/defocusing of Hermite-sinh-Gaussian (HshG) laser beam in underdense inhomogeneous plasmas is studied by using higher-order approximation theory. It is found that Hermite mode index and the fluctuation of the periodic plasma density have a significant effect on the dielectric constant and laser beam self-focusing/self-defocusing. With the increase of mode index, the high-order HshG laser beam is beneficial to suppress self-focusing and enhance self-defocusing. In addition, the effects of decentered parameters, beam intensity, and plasma non-uniformity on self-focusing/self-defocusing are discussed.

1. Introduction

There are a lot of non-linear phenomena in the interaction between intense laser and plasma, such as self-focusing/self-defocusing [1], filamentation [2], stimulated Raman scattering [3] and stimulated Brillouin scattering [4]. These nonlinearities have important effects on inertial confinement fusion [5] and laser particle accelerator [6]. In particular, plasma self-focusing can improve the yield of these applications by increasing the interaction length between the laser and the medium, but also increase the success of these applications by balancing the natural diffraction of the laser beam to transmit more Rayleigh lengths [7]. Therefore, the self-focusing generated by the interaction of laser pulses with plasma is a very worthy topic of study [8].

In interaction intense laser and plasms, the mechanism of self-focusing/defocusing mainly include collisional nonlinearity [9], ponderomotive nonlinearity [10, 11], relativistic nonlinearity [12]. Nanda and Kant [13] researched self-focusing of cosh-Gaussian laser beams in collisionless non-uniform magnetic plasmas. By comparing extraordinary mode and ordinary mode, it is found that increasing the plasma density ramp can make the former stronger self-focusing. Asgharnejad et al. [7] investigated self-focusing a Gaussian laser pulse in a weakly relativistic and ponderomotive regime inside a collisionless warm quantum plasma. Zhang et al. [14] analyzed relativistic self-focusing of Gaussian laser beam, the results show that relativistic effect can enhance the oscillation of dielectric function and increase the self-focusing. Abedi-Varaki and Jafari [15] have present that a decrease in plasma temperature and an increase in oblique magnetic field increases defocusing for left-handed polarization. Extending the High-harmonic generation phase matching cutoff by controlling the rapid self-defocusing effect of driving laser is experimentally and theoretically demonstrated by Sun et al. [16].

Lots of research of self-focusing/self-defocusing have focused on interaction of Gaussian beam and homogeneous/inhomogeneous plasmas. Studies by Gupta et al. [17] and Kant et al. [18] have shown that the existence of plasma density fluctuations can enhance self-focusing. Wani et al. [19] have found that electron temperature can reduce defocusing and plasma density transition increases the range of self-focusing effect. Recently, many researchers have analyzed the effects of different types of laser beams and plasma on self-focusing/self-defocusing. Laser beam extends from Gaussian type to the others type of beam, such as Hermite-Gaussian beam [20], cosh-Gaussian beam [21], super Gaussian beam [22]. Various types plasmas have been researched, such as cold quantum plasma with exponential density variation [23]. Effects of magnetic field on relativistic self-focusing, self-phase modulation and self-trapping of cosh-Gaussian laser beam is exploited by Gill et al. [24]. Kant et al. [25] observed relativistic self-focusing of Hermite-cosh-Gaussian laser pulse in plasma with exponential plasma density ramp. Devi and Malik [26] studied the relativistic self-focusing of super-Gaussian laser beams, they have found that the external magnetic field and the cyclotron motion of electrons is important for self-focusing. Based on the moment theory, Malik and Devi [27] developed a formulation to study the self-defocusing of super-Gaussian laser beam in tunnel ionized plasmas, and found that laser beam can get more defocused for higher intensity and smaller spot size. Kaur et al. [28] used paraxial approximation method to study propagation characteristics of Hermite-cosh-Gaussian laser beam in rippled density plasmas and strong oscillatory self-focusing and defocusing are observed in relativistic case.

In this paper, applying higher-order paraxial theory and WKB approximation, the self-focusing/self-defocusing are investigated in the interaction of Hermite-Sinh-Gaussian (HshG) laser beam and the non-uniform underdense plasma with periodic density fluctuations.

2. Propagation Theory of Laser Beam in Plasmas

In laser-plasma interaction, according to Maxwell's equation theory, the wave equation of laser propagation in plasma can be expressed as,

$$\nabla^2 E - \frac{\omega^2}{c^2}\varepsilon E + \nabla\left(\frac{E\cdot\nabla(\varepsilon)}{\varepsilon}\right) = 0. \tag{1}$$

For $k^{-2}\nabla^2 \ln(\varepsilon) \ll 1$ and $k^2 = \varepsilon\omega^2/c^2$, the wave equation can be converted to

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{\omega^2}{c^2}\varepsilon E = 0. \tag{2}$$

When the laser propagates in the plasma along the z-direction, the specific form of the field can be expressed as,

$$E = A(r,z)e^{-i(\omega t - kz)}. \tag{3}$$

In this paper, the interaction between HshG laser beam and inhomogeneous plasma is studied. For equation (3), the general solution of the field amplitude is HshG beam.

$$E(r,z) = \frac{E_0}{f(z)}\left[H_n\left(\frac{\sqrt{2}\,r}{r_0 f(z)}\right)\right]e^{b^2/4}\left(e^{-\left((r/r_0 f)+(b/2)\right)^2} - e^{-\left((r/r_0 f)-(b/2)\right)^2}\right). \tag{4}$$

E_0 is the amplitude of HshG laser beam at $r = z = 0$, Hermite polynomial $H_n(\sqrt{2}\,r/r_0 f(z))$, when $n = 0$, $H_n(\sqrt{2}\,r/r_0 f(z)) = 1$; when $n = 1$, $H_n(\sqrt{2}\,r/r_0 f(z)) = 2r$; when $n = 2$, $H_n(\sqrt{2}\,r/r_0 f(z)) = 4r^2 - 2$. r is the radial component in the cylindrical coordinate system, r_0 is the initial beam width, b represents decentred parameter, the dimensionless beam width parameter of laser beam is $f(z)$.

Non-uniform plasmas showing periodic variations have been found experimentally and theoretically by researchers in the process of laser-plasma interaction. The plasma system studied in this paper is based on the experimental results of Ouahid et al. [29]; electron density $N_e(z)$ is sinusoidal function. The expression is as follows,

$$N_e(z) = N_{0e}(1 + \alpha \sin qz), \tag{5}$$

where $N_e(z)$, N_{0e}, α, q represents electron density in equilibrium steady state, initial electron density, parameter of plasma fluctuation, wave number of plasma density fluctuation.

In laser-plasma interaction, when plasma density fluctuation exists, the dielectric function of plasma composed of linear part ε_0 and non-linear part $\phi(EE^*)$ can be expressed as $\varepsilon(r,z) = \varepsilon_0 - \phi(EE^*)$. In the limit $c^2/(r_0^2\omega_{p0}^2) < 1$, the linear and non-linear part can be presented as follows $\varepsilon_0(z) = 1 - \omega_{p0}^2(1 + \alpha \sin qz)/\gamma\omega^2$, $\phi(EE^*) = \omega_{p0}^2(1 + \alpha \sin qz)[1 - 1/(1 + \beta EE^*)^{1/2}]/\gamma\omega^2$. $\omega_{p0} = \sqrt{4\pi n_0 e^2/m}$ is the plasma frequency, e is the electronic charge and m shows the rest mass. Here the relativistic factor can be written as $\gamma = \sqrt{1 + \beta EE^*/2}$, where $\beta = (e/m\omega c)^2$. By applying higher-order paraxial theory and extending the dielectric function up to the fourth higher-order term.

$$\varepsilon(r,z) = \varepsilon_0(z) - \varepsilon_2(z)\frac{r^2}{r_0^2} - \varepsilon_4(z)\frac{r^4}{r_0^4}, \tag{6}$$

where $\varepsilon_0(z)$ is the linear part, $\varepsilon_2(z)$ and $\varepsilon_4(z)$ are expansion coefficients of $\varepsilon(r,z)$. Based on the research results of Pathak et al. [30]; using WKB approximation and higher-order paraxial theory, dielectric constant components of different Hermite mode indexes can be obtained in interaction of HshG beams and plasma.For mode index $n = 0$

$$\begin{cases} \varepsilon_0 = 1 - t + \frac{3t}{4}\frac{B}{f^2}e^{b^2/2}D - \frac{27t}{32}\frac{B^2}{f^4}e^{b^2}D^2, \\ \varepsilon_2 = t\frac{B^2}{f^6}e^{b^2}\left[\frac{3}{32}\left(4b^2-36\right)D + \frac{27}{16}a_2D^2\right] - \frac{tB}{4f^4}e^{b^2/2}\left[\left(7b^2-6\right)+3a_2D\right], \\ \varepsilon_4 = \frac{tB^2}{f^8}e^{b^2}\left[\frac{3}{32}\left(72+\frac{107}{2}b^4-102b^2\right)+\frac{27D^2}{32}\left(a_2^2+2a_4\right)+\frac{3}{16}\frac{D}{}a_2\left(42b^2-36\right)\right] \\ -\frac{tB}{4f^6}e^{b^2/2}\left[6+a_2\left(7b^2-6\right)+3a_4D\right]. \end{cases} \quad (7)$$

For mode index $n = 1$

$$\begin{cases} \varepsilon_0 = 1 - t, \\ \varepsilon_2 = -\frac{6tB}{f^4}e^{b^2/2}D, \\ \varepsilon_4 = \frac{54tB^2}{f^8}e^{b^2}D^2 - \frac{2tB}{f^6}e^{b^2/2}\left[\left(7b^2-6\right)+3a_2D\right]. \end{cases} \quad (8)$$

For mode index $n = 2$

$$\begin{cases} \varepsilon_0 = 1 - t + \frac{3tB}{f^2}e^{b^2/2}D - \frac{27t}{2}\frac{B^2}{f^4}e^{b^2}D^2, \\ \varepsilon_2 = \frac{9t}{4}\frac{B^2}{f^6}e^{b^2}\left[4\left(7b^2-6\right)-96\,D+12a_2D\right] - \frac{tB\,D}{4f^4}e^{b^2/2}\left[4\left(7b^2-6\right)-96\,D+12a_2D\right], \\ \varepsilon_4 = \frac{3tB^2}{32f^8}e^{b^2}\left[4\left(7b^2-6\right)-96\,D+12a_2D\right]^2 + \frac{9tB^2D}{4f^8}e^{b^2}\left[24+(192-96a_2+12a_4)D+(4a_2-32)\left(7b^2-6\right)\right] \\ -\frac{tB}{4f^6}e^{b^2/2}\left[24+(192-96a_2+12a_4)D+(4a_2-32)\left(7b^2-6\right)\right], \end{cases} \quad (9)$$

where in equations (7)–(9), $t = \omega_{p0}^2/\omega^2(1+\alpha\sin(q\xi))$ denotes the parameters related to plasma density, $B = \beta E_0^2$, $D = 1 - b^2/2 + b^4/8$ denote intensity dependence and intensity distribution dependence of dielectric constant. For wave equation (2), substitute (3) into equation (2), $\partial^2 A/\partial z^2$ be ignored, can get,

$$-2ik\frac{\partial A}{\partial z} - iA\frac{\partial k}{\partial z} + \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\omega^2}{c^2}\left[\varepsilon(r,z) - \varepsilon_0(z)\right]A = 0. \quad (10)$$

Following Nanda et al. [31]; complex amplitude A can be written as follows,

$$A(r,z) = A_0(r,z)e^{-ikS(r,z)}, \quad (11)$$

$A_0(r,z)$ and $S(r,z)$ are the real functions of space, according to Habibi and Ghamari [32]; they can be expressed this way

$$A_0^2 = \frac{E_0^2}{f^2} e^{b^2/2} \left(H_n \left(\frac{\sqrt{2} r}{r_0 f} \right) \right)^2 \left(1 + \frac{a_2 r^2}{r_0^2 f^2} + \frac{a_4 r^4}{r_0^4 f^4} \right) \left[e^{-2\left((r/r_0 f) + (b^2/2)\right)} - e^{-\left((2r^2/r_0^2 f^2) + (b^2/2)\right)} \right], \tag{12}$$

$$S(r,z) = S_0(z) + \frac{r^2}{r_0^2} S_2(z) + \frac{r^4}{r_0^4} S_4(z). \tag{13}$$

a_2, a_4, S_0, S_2, S_4 are the functions of z, S_0, S_2, S_4 are the components of the eikonal. $S_2 = (r_0^2/2f)(df/d\xi)$ corresponds to the spherical curvature of the wave front, S_4 indicates deviation from spherical curvature. a_2, a_4 are indicative of the departure of the beam from the HshG nature. Can get a complicated differential equation by bringing in equation (10) with equation (11), the following equations can be obtained if the real and imaginary parts are equal.

$$2\frac{\partial S}{\partial z} + \frac{2S}{k}\frac{\partial k}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) - \frac{r^2}{r_0^2}\frac{\varepsilon_2(z)}{\varepsilon_0(z)} - \frac{r^4}{r_0^4}\frac{\varepsilon_4(z)}{\varepsilon_0(z)}, \tag{14}$$

$$\frac{\partial A_0^2}{\partial z} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + \frac{A_0^2}{k}\frac{dk}{dz} = 0. \tag{15}$$

By substituting equations (12) and (13) in equation (15) for different Hermite mode indexes, and equating the coefficients of r^2 and r^4 on both sides of equation, the equation relation of parameter a_2 and a_4 is represented as.

For mode index $n = 0$

$$\frac{da_2}{d\xi} = -16 S_4' f^2, \tag{16a}$$

$$\frac{da_4}{d\xi} = \left[-24a_2 + 4\left(4 - 2b^2\right)\right] S_4' f^2. \tag{16b}$$

For mode index $n = 1$

$$\frac{da_2}{d\xi} = -24 S_4' f^2, \tag{17a}$$

$$\frac{da_4}{d\xi} = \left[-32a_2 + 4\left(4 - 2b^2\right)\right] S_4' f^2. \tag{17b}$$

For mode index $n = 2$

$$\frac{da_2}{d\xi} = -16 S_4' f^2, \tag{18a}$$

$$\frac{da_4}{d\xi} = \left[-24a_2 + 64 + 4\left(4 - 2b^2\right)\right] S_4' f^2, \tag{18b}$$

where $S_4' = S_4(\omega/c)$. Same method, equations (12) and (13) in equation (14) to get the following equations governing the beam width f and S_4'.

For mode index $n = 0$

$$\frac{d^2 f}{d\xi^2} = \left(\frac{-2a_2^2 + 8 - 8a_2 - 46b^4/3 + 4a_2 b^2 - 8b^2 + 8a_4}{\varepsilon_0 f^3}\right) - \frac{\varepsilon_2 f \rho^2}{\varepsilon_0} - \frac{1}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi}\frac{df}{d\xi}, \tag{19a}$$

$$\frac{dS_4'}{d\xi} = \frac{8a_2 a_4 - 4a_2^3 - 52a_2 b^4/3 - 16a_4 + 8a_2^2 + 104b^4/3 + 8b^2 a_4 - 4a_2^2 b^2 - 52b^6/3}{2\varepsilon_0 f^6} - \frac{4S_4'}{f}\frac{df}{d\xi} - \frac{S_4'}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi} - \frac{\varepsilon_4 \rho^2}{2\varepsilon_0}. \tag{19b}$$

For mode index $n = 1$

$$\frac{d^2 f}{d\xi^2} = \left(\frac{8 - 6a_2^2 - 8a_2 - 98b^4/3 + 4a_2b^2 - 8b^2 + 16a_4}{\varepsilon_0 f^3}\right) - \frac{\varepsilon_2 f \rho^2}{\varepsilon_0} - \frac{1}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi}\frac{df}{d\xi}, \tag{20a}$$

$$\frac{dS_4'}{d\xi} = \frac{\left(2a_2 - 4 + 2b^2\right)\left(4a_4 - 2a_2^2 - 26b^4/3\right)}{2\varepsilon_0 f^6} - \frac{4S_4'}{f}\frac{df}{d\xi} - \frac{S_4'}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi} - \frac{\varepsilon_4 \rho^2}{2\varepsilon_0}. \tag{20b}$$

For mode index $n = 2$

$$\frac{d^2 f}{d\xi^2} = \left(\frac{-2a_2^2 + 72 - 40a_2 - 46b^4/3 + 4a_2b^2 - 40b^2 + 8a_4}{\varepsilon_0 f^3}\right) - \frac{\varepsilon_2 f \rho^2}{\varepsilon_0} - \frac{1}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi}\frac{df}{d\xi}, \tag{21a}$$

$$\frac{dS_4'}{d\xi} = \frac{\left(2a_2 - 20 + 2b^2\right)\left(4a_4 - 64 - 2a_2^2 - 26b^4/3\right)}{2\varepsilon_0 f^6} - \frac{4S_4'}{f}\frac{df}{d\xi} - \frac{S_4'}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi} - \frac{\varepsilon_4 \rho^2}{2\varepsilon_0}. \tag{21b}$$

Normalize the above equations, $\xi = cz/\omega r_0^2 = z/R_d$ is the dimensionless propagation distance, $\rho = \omega r_0/c$ represents the dimensionless original beam width.

3. Discussion

In the paper, intensity of incident laser beam is 1.23×10^{17} W/cm^2, $\beta E_0^2 = 0.01, 0.02, 0.1$ and 0.2 (corresponding intensities are 1.23×10^{15} W/cm^2, 2.46×10^{15} W/cm^2, 1.23×10^{16} W/cm^2 and 2.46×10^{16} W/cm^2 respectively), the frequency and spot size of incident laser beam as $\omega = 1.78 \times 10^{15}$ rad/s, $r_0 = 1.68 \times 10^{-5}$ m in interaction system. Appling fourth-order Runge-Kutta method to solve equations (16a)–(21b), the phenomena of self-focusing/self-defocusing in the interaction of intense laser and the inhomogeneous plasma are discussed. The initial conditions are $f = 1$, $df/d\xi = 0$, $a_2 = 1$, as well as $da_2/d\xi = 0$ at $\xi = 0$, plasmas density is $N_e(z) = N_{0e}(1 + \alpha \sin qz)$.

Figures 1(a)–1(c) shows the linear component of the dielectric function of the HshG laser beam interaction of plasma with periodic density fluctuation at mode index $n = 0$, 1 and 2. The linear part of dielectric function shows sinusoidal periodic variations similar to oscillation in plasma of periodic density fluctuation. The peak value of the linear part decreases slightly with the increase of mode index, and the oscillation has intensified the tendency. For the non-linear part ε_2, ε_4 at different mode indexes, the value is much smaller compared to the linear part, and mode index has a more obvious influence on the non-linear part. In figure (d), (e) and (f), it can be seen that mode index obvious affect the peak value of non-linear part, the peak position drifts to the left as the mode increases, and finally all tend to stable. From the above linear and non-linear parts of the dielectric function, it shows an approximate periodic oscillatory variation due to the plasma periodicity, and its oscillatory variation trend to increase with the increase of mode index of Hermite polynomial. In addition, the mode index have more obvious effect on the non-linear part comparison linear part.

Figure 2 presents beam width variation for different Hermite mode indexes of HshG laser beams. In the periodic underdense inhomogeneous plasma, the beam width all show oscillatory changes, firstly, beam width becomes small and then increases. During the propagation of the laser beam, ponderomotive force influence on laser beam is obvious at the start, and beam width reduces and appears to convergence, resulting in self-focusing. Then, the influence ponderomotive force of laser beam decrease and beam width increase continuously with distance, and beam width presents diverge obviously, resulting in beam self-defocusing. However, at different mode indexes, the size and position of the minimum value of beam width are different. Increase of mode index, the size slightly increase and position appear early, and the beam diverges more quickly, which means the self-defocusing is more intense. For $n = 0$, reaches the minimum value 0.55 at $\xi \approx 0.14$. For $n = 1$, reaches the minimum value 0.6 at $\xi \approx 0.1$. For $n = 2$, reaches the minimum value 0.998 at $\xi \approx 0.02$. This is due to the combined effects of HshG laser beam and periodic underdense non-uniform plasma, causing a similar periodic oscillatory variation of dielectric function, which leads to self-focusing and self-defocusing. In addition, with the increase of mode index, the oscillation of the dielectric function is enhanced, indicating that the instability is enhanced, which will affect the coupling of the laser plasma, so that the quality of Gaussian beam decreases, self-focusing reduce and self-defocusing enhance.

Figure 3 presents the effect of decentered parameters b on beam width f in different modes indexes. It is shown that the decentered parameter has obvious influence on the beam width. For $n = 0$, the beam width slowly increases in $b = 0$, and in $b \neq 0$, it first decreases, then increases, and with the

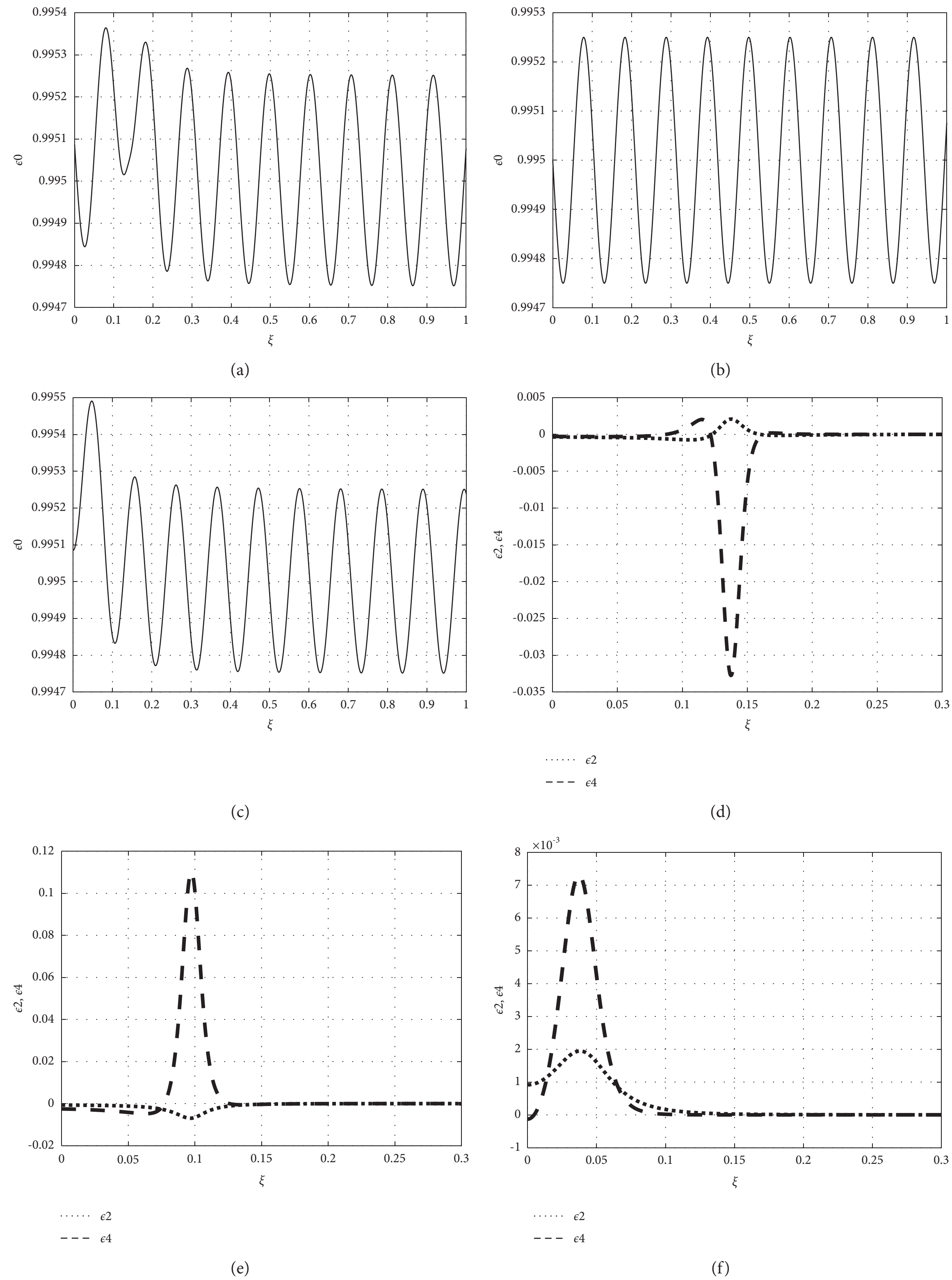

Figure 1: Variation of the linear ε_0 and non-linear componentsε_2, ε_4 of the dielectric constant, with normalized propagation distance ξ for different Hermite mode indexes. (a) and (d) ε_0 and ε_2, ε_4 at $n = 0$; (b) and (e) ε_0 and ε_2, ε_4 at $n = 1$; (c) and (f) ε_0 and ε_2, ε_4 at $n = 2$. The other parameters are $b = 1.3$, $\omega r_0/c = 100$, $\beta E_0^2 = 0.02$, $t = 0.005 \times (1 + 0.05 \times \sin(60\xi))$.

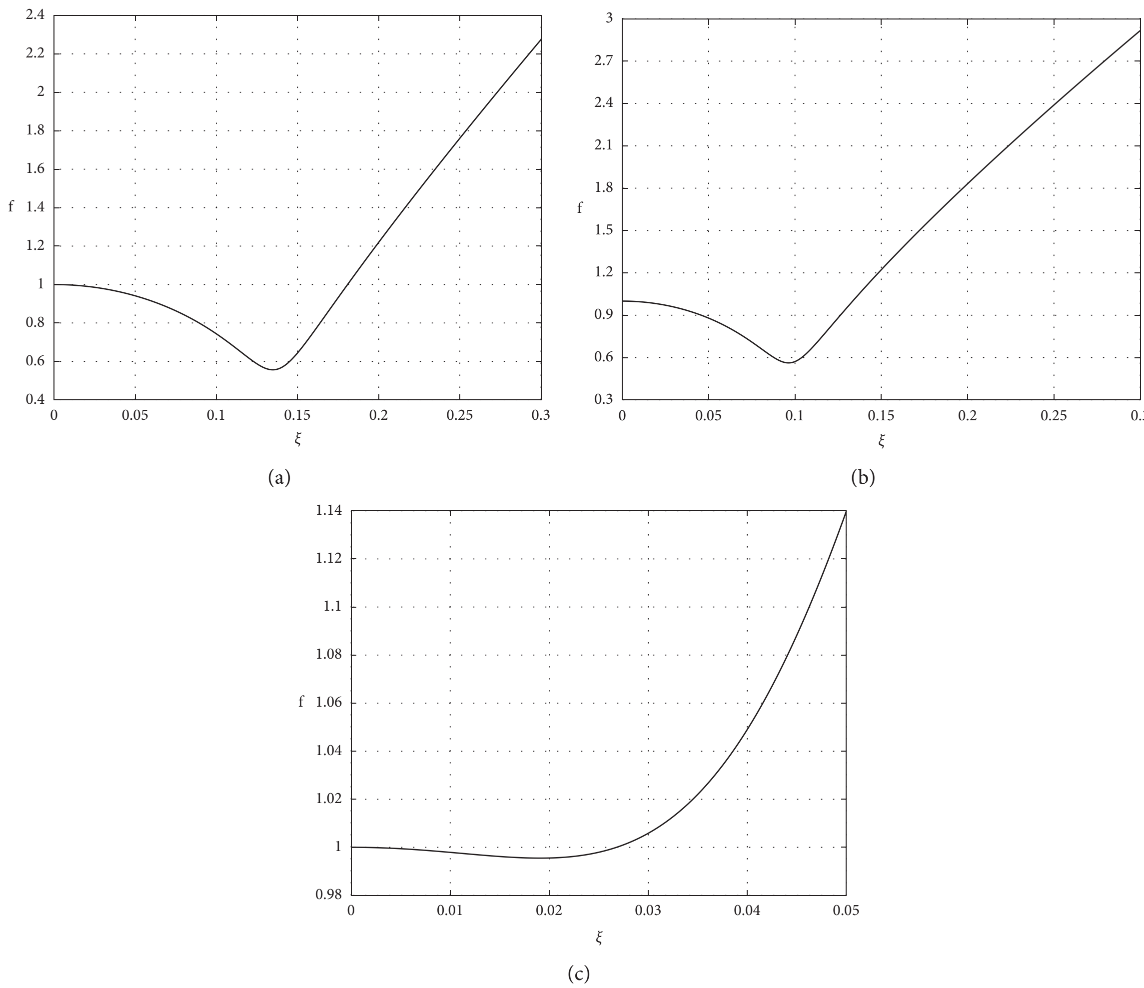

Figure 2: Beam width f with normalized propagation distance ξ for different Hermite mode indexes of HshG laser beams, (a) $n = 0$, (b) $n = 1$, (c) $n = 2$. Rest of parameters are considered as $b = 1.3$, $\omega r_0/c = 100$, $\beta E_0^2 = 0.02$, $t = 0.005 \times (1 + 0.05 \times \sin(60\xi))$.

increase of b, the variation of beam width is strengthened. $n = 1$ is similar to $n = 0$. For (e) $n = 2$, beam width increases slightly at first, and then almost linearly increases, moreover, beam width decreases slightly for the same propagation distance with increase of b. (a) (c) correspond to (b) (d), the rules are consistent, the purpose is to study the effect of smaller changes in b on the self-focusing, and optimize the value of the decentered parameter. (f) depicts the variation of the beam width when $\beta E_0^2 = 0.3$, the beam width shows an oscillation trend. Compared with (e), it not only shows the influence of decentered parameters on self-focusing, but also reflects the importance of laser intensity to self-focusing. In summary, (b) (d) (f) show that the decentered parameter is very sensitive to the self-focusing, which is consistent with that described in the paper of Nanda et al. [33]. For the zero-order and 1-order HshG beams, under the influence of the decentered parameter, there is a significant beam self-focusing at $b \neq 0$, however, for the 2-order HshG beam, it presents obvious beam self-defocusing, in this case, the effect of decentered parameter on self-defocusing is relatively slight. It can be clearly seen that the effect of decentered parameters on self-focusing of the HshG beam is significantly stronger than self-defocusing.

Figure 4 depicts the variation of the beam width for different light intensities. The figure is shown that, beam width increases at the same propagation distance with the increase of light intensity, it implies that self-focusing decrease and self-defocusing increase. In addition, it is easier to form self-focusing with the decrease of laser intensity, while self-focusing becomes weak when the light intensity

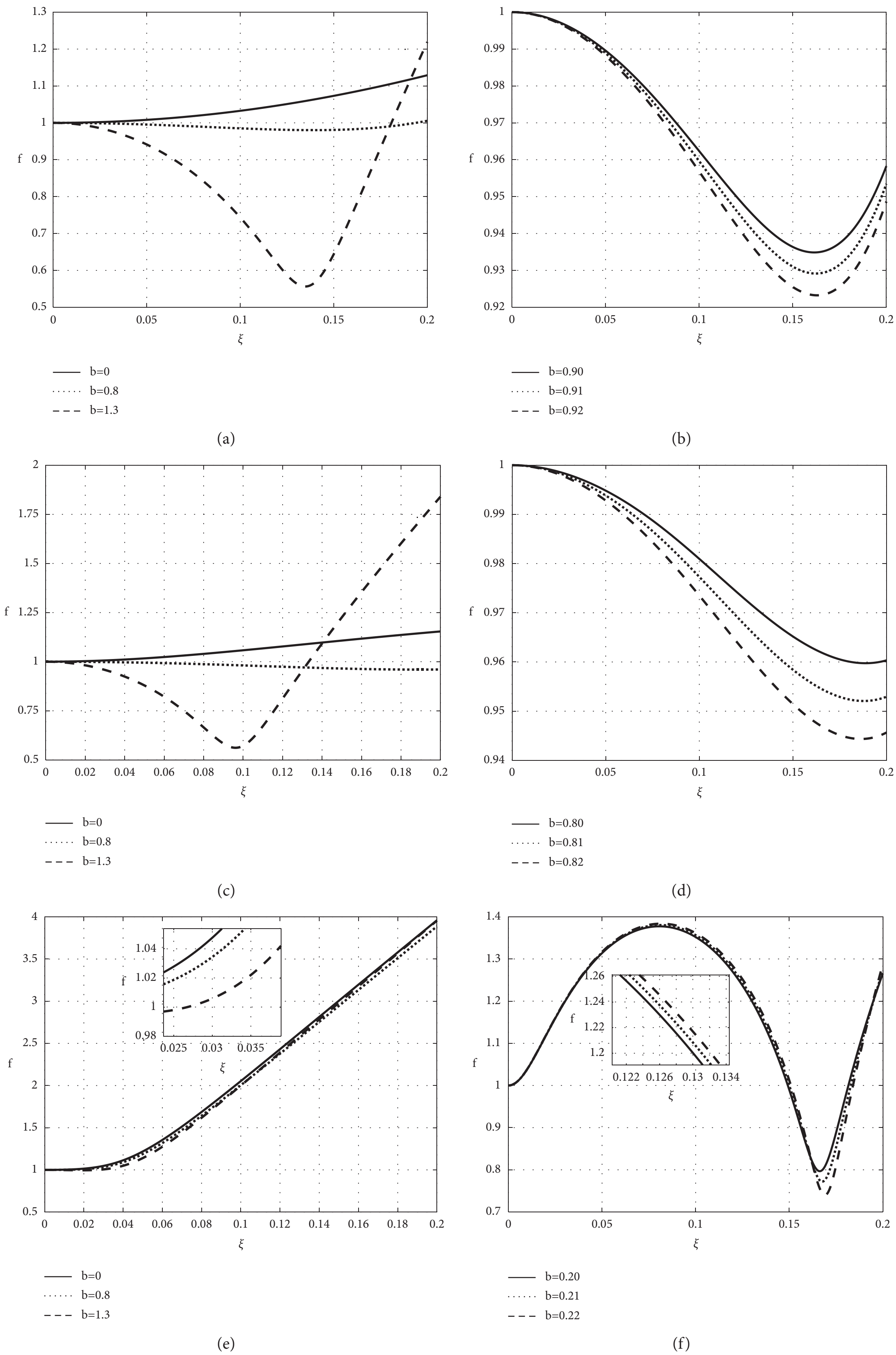

Figure 3: Variation of beam width f with normalized propagation distance ξ for the decentered parameters $b = 0$, 0.8 and 1.3, and for (a) (b) $n = 0$, (c) (d) $n = 1$, (e) (f) $n = 2$. The other parameters are taken as $\omega r_0/c = 100$, $\beta E_0^2 = 0.02$, $t = 0.005 \times (1 + 0.05 \times \sin(60\xi))$.

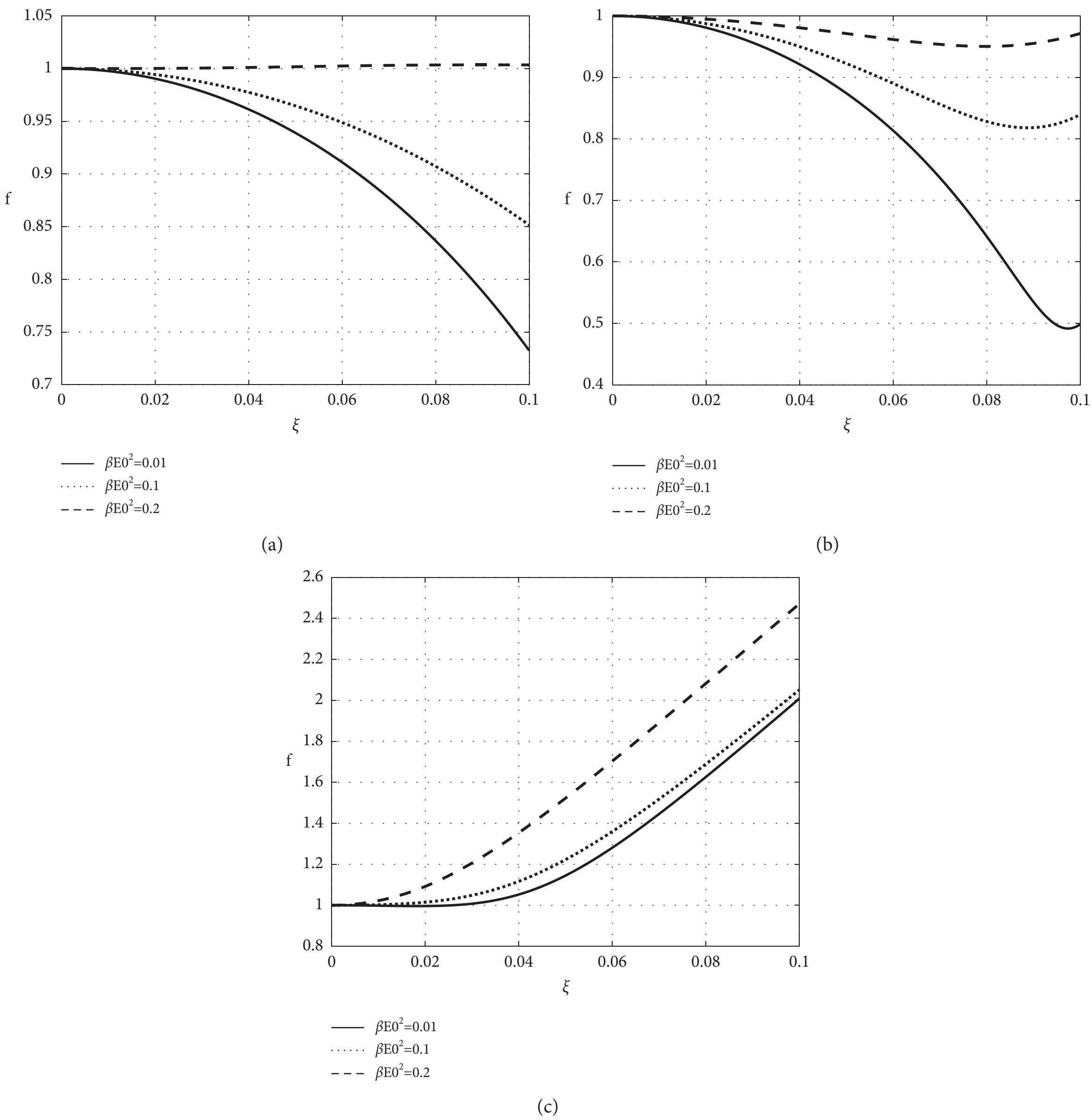

FIGURE 4: Variation of beam width f with dimensionless propagation distance ξ for different values of normalized light intensity $\beta E_0^2 = 0.01$, 0.1 and 0.2. (a) $n = 0$, (b) $n = 1$, (c) $n = 2$. The other parameters are $b = 1.3$, $\omega r_0/c = 100$, $t = 0.005 \times (1 + 0.05 \times \sin(60\xi))$.

increases to 0.2, it may be because laser intensity is close or exceed the threshold value of forming self-focusing, leading to self-focusing reduce or disappear. In Figure 4(c), for $n = 2$, when laser intensity increases, beam width continues to increase, self-focusing is very weak and produces obvious self-defocusing. Hence, it is shown that the high-order HshG beam and high beam intensity are benefit to restrain self-focusing and enhance self-defocusing.

Figure 5 depicts the variation of the beam width for different plasma density parameter. With the increase of plasma inhomogeneity, the beam width increases at $n = 0$ and $n = 1$. At $n = 2$, as the plasma inhomogeneity increases, beam width slowly decreases first and then increases. It implies that the inhomogeneity of plasma has a significant effect on the beam width because the plasma density fluctuation affects the dielectric function, which directly affects laser propagation process in plasma, lead to the change of beam width, and increase non-uniformity of plasmas, it can strength self-focusing/self-defocusing.

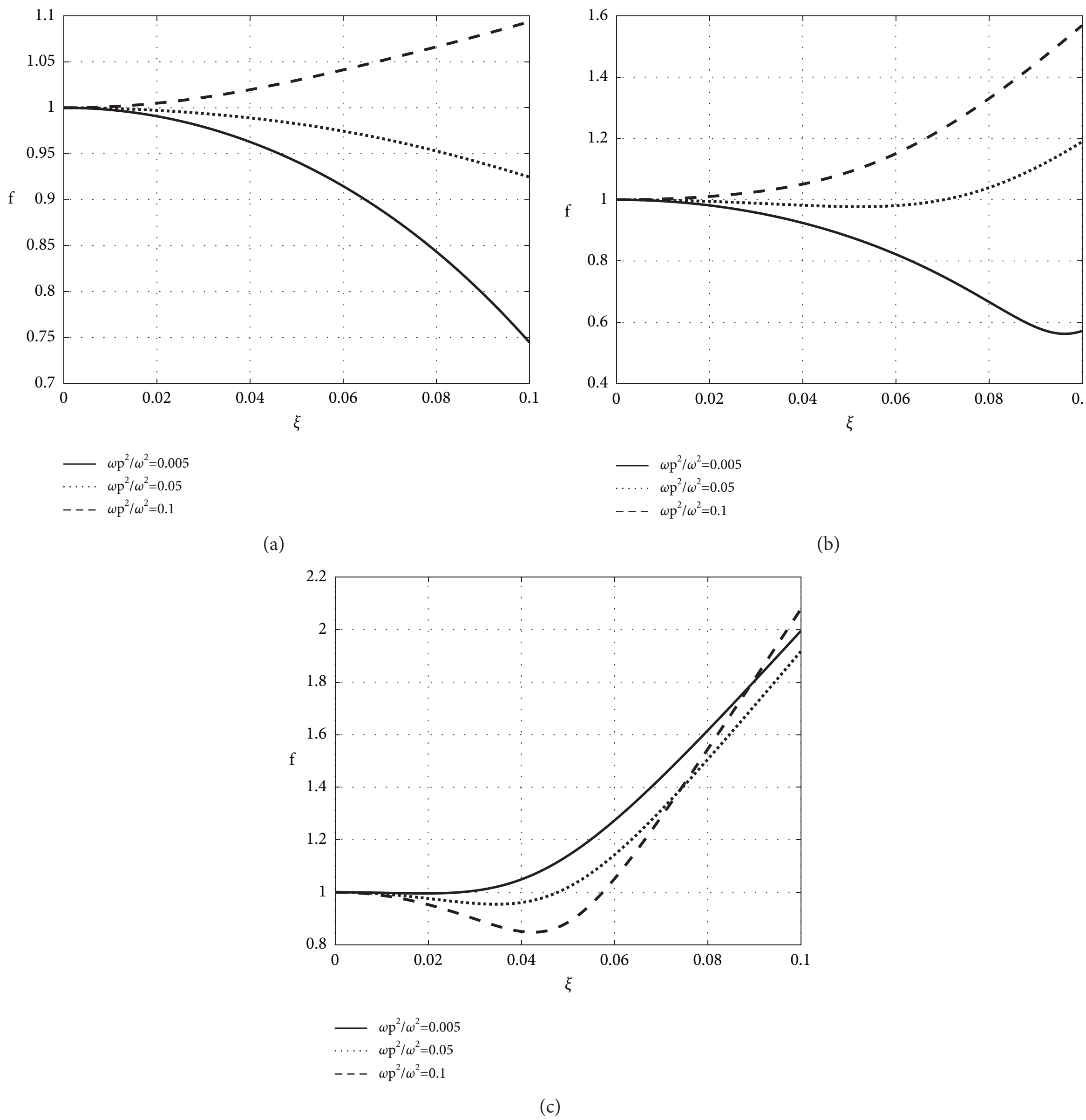

Figure 5: Variation of the beam width f with normalized propagation distance ξ for different values of plasma density. (a) $n = 0$, (b) $n = 1$, (c) $n = 2$. The other parameters are $b = 1.3$, $\omega r_0/c = 100$, $\beta E_0^2 = 0.02$, $t = 0.005 \times (1 + 0.05 \times \sin(60\xi))$ solid line, $t = 0.05 \times (1 + 0.05 \times \sin(60\xi))$ dotted line, $t = 0.1 \times (1 + 0.05 \times \sin(60\xi))$ point line.

4. Conclusion

In the paper, apply high-order paraxial theory to analyze the self-focusing/self-defocusing of HshG laser beam in underdense inhomogeneous plasma. Self-focusing and self-defocusing laws are explored by analyzing the variation of the dielectric constant and the influence of the decentered parameters, laser intensity and plasma density on beam width. The results show that, due to the combined effects of HshG laser beam and periodic underdense non-uniform plasma, lead to dielectric function present a similar periodic oscillation variation. Under the influence of ponderomotive force and diffraction effect, lead to beam self-focusing/self-defocusing. With increase of Hermite mode index, causing the position of self-focusing/self-defocusing is bring forward, and beam defocusing is strengthened, it implies that the higher mode index HshG beam is benefit to reduce self-focusing and enhance defocusing. In addition, the decentered parameter, beam intensity and non-uniformity of the plasma has a significant influence on the beam width. Influence of the decentered parameter on beam self-focusing is stronger than self-defocusing. In the high-order HshG beam, high beam intensity is benefit to restrain self-focusing and enhance self-defocusing, and increase non-uniformity of plasmas, it can strength self-focusing/self-defocusing.

Data Availability

Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Conflicts of Interest

Authors have no conflict of interest to declare.

Acknowledgments

The work is supported by the National Natural Science Foundation of China (Grant no. 11447169) and the Natural Science Foundation of Guangxi province (2018GXNSFAA138180, 2016GXNSFAA380071, and 2016GXNSFBA380204).

References

[1] V. F. Kovalev and V. Y. Bychenkov, "Analytic theory of relativistic self-focusing for a Gaussian light beam entering a plasma: renormalization-group approach," *Physical Review*, vol. 99, Article ID 043201, 2019.

[2] M. B. Hassan, I. J. Abd-Ali, and A. O. Soary, "The filamentation instability of nonparaxial laser beam inside magnetized plasma," *Results in Physics*, vol. 14, Article ID 102386, 2019.

[3] H. Peng, Y. L. Zuo, H. Y. Zhu, and J. Q. Su, "Forward Raman scattering of the seed pulse in strongly coupled stimulated Brillouin amplification in plasma," *Physics of Plasmas*, vol. 25, no. 1, Article ID 013118, 2018.

[4] Z. Li, Y. Zuo, J. Su, and S. Yang, "The filamentation effect in short pulse amplification by strong-coupling stimulated Brillouin scattering," *Physics of Plasmas*, vol. 26, no. 9, Article ID 093102, 2019.

[5] C. Lu, V. Tikhonchuk, and S. Weber, "Analytic solutions for delocalized heat transport," *Plasma Physics and Controlled Fusion*, vol. 63, no. 7, Article ID 075005, 2021.

[6] K. V. Lotov, A. P. Sosedkin, A. V. Petrenko et al., "Electron trapping and acceleration by the plasma wakefield of a self-modulating proton beam," *Physics of Plasmas*, vol. 21, no. 12, Article ID 123116, 2014.

[7] D. Asgharnejad, T. Mohsenpour, and S. Mirzanejhad, "Investigation of self-focusing a Gaussian laser pulse in a weakly relativistic and ponderomotive regime inside a collisionless warm quantum plasma," *Chinese Journal of Physics*, vol. 73, pp. 304–313, 2021.

[8] X. Xia and B. Xu, "Off-axial contribution of beam self-focusing in plasma with density ripple," *Optik*, vol. 125, no. 19, pp. 5899–5903, 2014.

[9] M. Singh and D. N. Gupta, "Laser-pulse compression in a collisional plasma under weak-relativistic ponderomotive nonlinearity," *Physics of Plasmas*, vol. 23, no. 5, Article ID 053119, 2016.

[10] S. D. Patil, P. P. Chikode, and M. V. Takale, "Turning point temperature of self-focusing at laser-plasma interaction with weak relativistic-ponderomotive nonlinearity: effect of light absorption," *Journal of Optics*, vol. 47, no. 2, pp. 174–179, 2018.

[11] P. Rawat and G. Purohit, "Self-focusing of a cosh-Gaussian laser beam in magnetized plasma under relativistic-ponderomotive regime," *Contributions to Plasma Physics*, vol. 59, no. 2, pp. 226–235, 2019.

[12] A. Sharma and V. K. Tripathi, "Relativistic and ponderomotive self-focusing of a laser pulse in magnetized plasma," *Laser and Particle Beams*, vol. 30, no. 4, pp. 659–664, 2012.

[13] V. Nanda and N. Kant, "Strong self-focusing of a cosh-Gaussian laser beam in collisionless magneto-plasma under plasma density ramp," *Physics of Plasmas*, vol. 21, no. 7, Article ID 072111, 2014.

[14] G. Zhang, Q. Liang, and X. Xia, "Relativistic self-focusing in the interaction of laser beam and plasma with periodical density ripple," *Laser and Particle Beams*, vol. 38, no. 4, pp. 244–250, 2020.

[15] M. Abedi-Varaki and S. Jafari, "Nonlinear interaction of intense left- and right-hand polarized laser pulse with hot magnetized plasma," *Journal of Plasma Physics*, vol. 83, no. 4, Article ID 655830401, 2017.

[16] H.-W. Sun, P.-C. Huang, Y.-H. Tzeng et al., "Extended phase matching of high harmonic generation by plasma-induced defocusing," *Optica*, vol. 4, no. 8, pp. 976–981, 2017.

[17] D. N. Gupta, M. S. Hur, and H. Suk, "Additional focusing of a high-intensity laser beam in a plasma with a density ramp and a magnetic field," *Applied Physics Letters*, vol. 91, no. 8, Article ID 081505, 2007.

[18] N. Kant, M. A. Wani, and A. Kumar, "Self-focusing of Hermite-Gaussian laser beams in plasma under plasma density ramp," *Optics Communications*, vol. 285, no. 21-22, pp. 4483–4487, 2012.

[19] M. A. Wani, V. Thakur, H. S. Ghotra, and N. Kant, "Effect of axial electron temperature and plasma density ramp on self-focusing/defocusing of a laser beam in plasma," *Optik*, vol. 192, Article ID 162963, 2019.

[20] J. Wadhwa and A. Singh, "Second harmonic generation of self-focused Hermite-Gaussian laser beam in collisional plasma," *Optik*, vol. 202, Article ID 162326, 2020.

[21] T. U. Urunkar, S. D. Patil, A. T. Valkunde, B. D. Vhanmore, K. M. Gavade, and M. V. Takale, "Effect of critical beam radius on self-focusing of cosh-Gaussian laser beams in collisionless magnetized plasma," *Communications in Theoretical Physics*, vol. 70, no. 2, pp. 220–224, 2018.

[22] H. K. Malik and L. Devi, "Relativistic self focusing and frequency shift of super-Gaussian laser beam in plasma," *Results in Physics*, vol. 17, Article ID 103070, 2020.

[23] V. Thakur and N. Kant, "Stronger self-focusing of a chirped pulse laser with exponential density ramp profile in cold quantum magnetoplasma," *Optik*, vol. 172, pp. 191–196, 2018.

[24] T. S. Gill, R. Kaur, and R. Mahajan, "Relativistic self-focusing and self-phase modulation of cosh-Gaussian laser beam in magnetoplasma," *Laser and Particle Beams*, vol. 29, no. 2, pp. 183–191, 2011.

[25] N. Kant, S. Vij, S. K. Chakravarti, J. P. Kushwaha, and V. Thakur, "Relativistic self-focusing of Hermite-cosh-Gaussian laser beam in magnetoplasma with exponential plasma density ramp," *Communications in Theoretical Physics*, vol. 71, no. 12, pp. 1469–1474, 2019.

[26] L. Devi and H. K. Malik, "Role of magnetic field on self focusing of super-Gaussian laser beam under relativistic effect," *Optik*, vol. 207, Article ID 164439, 2020.

[27] H. K. Malik and L. Devi, "Self-defocusing of super-Gaussian laser beam in tunnel ionized plasmas," *Optik*, vol. 222, Article ID 165357, 2020b.

[28] R. Kaur, T. Singh Gill, and R. Mahajan, "Relativistic effects on evolution of a q-Gaussian laser beam in magnetoplasma: application of higher order corrections," *Physics of Plasmas*, vol. 24, no. 5, Article ID 053105, 2017.

[29] L. Ouahid, L. Dalil-Essakali, and A. Belafhal, "Evolution of the beam-width parameter of zeroth-order Bessel-Gaussian beams in collisional plasma with density ripple," *Optical and Quantum Electronics*, vol. 51, no. 4, p. 109, 2019.

[30] N. Pathak, M. Kaur, S. Kaur, and T. S. Gill, "Non-paraxial theory of self-focusing/defocusing of Hermite cosh Gaussian laser beam in rippled density plasmas," *Contributions to Plasma Physics*, vol. 59, no. 10, Article ID e201900026, 2019.

[31] V. Nanda, H. S. Ghotra, and N. Kant, "Early and strong relativistic self-focusing of cosh-Gaussian laser beam in cold quantum plasma," *Optik*, vol. 156, pp. 191–196, 2018.

[32] M. Habibi and F. Ghamari, "Improved focusing of a cosh-Gaussian laser beam in quantum plasma: higher order paraxial theory," *IEEE Transactions on Plasma Science*, vol. 43, no. 7, pp. 2160–2165, 2015.

[33] V. Nanda, N. Kant, and M. A. Wani, "Sensitiveness of decentered parameter for relativistic self-focusing of Hermite-cosh-Gaussian laser beam in plasma," *IEEE Transactions on Plasma Science*, vol. 41, no. 8, pp. 2251–2256, 2013.