

## Research Article

# High-Harmonic Generation and Correlated Electron Emission from Relativistic Plasma Mirrors at 1 kHz Repetition Rate

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We report evidence for the first generation of XUV spectra from relativistic surface high-harmonic generation (SHHG) on plasma mirrors at a kilohertz repetition rate, emitted simultaneously with energetic electrons. SHHG spectra and electron angular distributions are measured as a function of the experimentally controlled plasma density gradient scale length  $L_g$  for three increasingly short and intense driving pulses: 24 fs and  $a_0 = 1.1$ , 8 fs and  $a_0 = 1.6$ , and finally 4 fs and  $a_0 \approx 2.1$ , where  $a_0$  is the peak vector potential normalized by  $m_e c/e$  with the elementary charge  $e$ , the electron rest mass  $m_e$ , and the vacuum light velocity  $c$ . For all driver pulses, we observe correlated relativistic SHHG and electron emission in the range  $L_g \in [\lambda/20, \lambda/4]$ , with an optimum gradient scale length of  $L_g \approx \lambda/10$ . This universal optimal  $L_g$ -range is rationalized by deriving a direct intensity-independent link between the scale length  $L_g$  and an effective similarity parameter for relativistic laser-plasma interactions.

## 1. Introduction

Surface high-harmonic generation (SHHG) from relativistic plasma mirrors [1–4] is a promising method for greatly enhancing the power of attosecond light pulses. This is motivated by key properties that differentiate it from the established method based on high-harmonic generation in gases [5]. Since the medium is already highly ionized, SHHG has no inherent limitation for the driving intensity such that a large fraction of the energy of an ultrahigh intensity driving laser can be compressed into an attosecond electromagnetic pulse. This is expected to occur with extremely high, percent-level conversion efficiencies [6–9] when driven under strongly relativistic conditions with a normalized vector potential  $a_0 = eA_0/m_e c \gg 1$  (a practical formula is  $a_0 = \sqrt{I[\text{Wcm}^{-2}]\lambda^2[\mu\text{m}^2]/1.37 \times 10^{18}}$ ),

where  $e$  is the elementary charge,  $m_e$  the electron rest mass,  $c$  the vacuum light velocity,  $A_0$  the laser's peak vector potential,  $I$  its peak intensity, and  $\lambda$  its central wavelength. Reported experimental laser-to-XUV conversion efficiencies for plasma mirrors with  $a_0 \geq 1$  are  $\sim 10^{-4}$  [10–13] for photon energies  $> 20$  eV but are expected to increase with higher-intensity drivers. Finally, relativistic SHHG produces intrinsically Fourier-limited attosecond pulses, such that the complete spectral bandwidth, from the spectral cutoff in the XUV (or even hard X-rays [14]) all the way down to lowest harmonics and even the fundamental laser frequency, can contribute to the formation of attosecond pulses without costly spectral filtering and compression [15–17].

Reaching the relativistic SHHG regime with  $a_0 > 1$  requires an on-target intensity of  $\geq 10^{18}$  W/cm<sup>2</sup> for an

800 nm laser while retaining a very steep surface plasma density profile

$$n(x) = n_c \exp \left[ \frac{x}{L_g} \right], \quad (1)$$

with a scale length  $L_g$  of a small fraction of the driving laser wavelength  $\lambda$ . Here  $n_c$  is the nonrelativistic critical plasma density for the driving wavelength, and  $x$  is the coordinate in the target normal direction. Technically, this requires a highly focusable terawatt-class laser with a temporal contrast of  $\geq 10^{10}$ . These conditions are typically met by Joule-class amplifier chains with dedicated contrast filters [18–21] and operating at  $\leq 10$  Hz repetition rate [8, 10–13, 20, 22].

Many applications as well as parametric studies of this regime would benefit from a higher repetition rate. At LOA, we have developed a unique laser chain with a power-scaled hollow-core-fiber postcompression system [23, 24] operating at 1 kHz repetition rate. Using this kHz laser, which achieves ultrahigh intensities with few-mJ pulse energy and few-cycle pulse duration, we have demonstrated laser-plasma interaction in the relativistic regime through laser-wakefield acceleration of electrons, both in underdense gas jets [25, 26] and in the underdense part of a smooth plasma density gradient on a plasma mirror [27]. Here, we report on the first experimental demonstration of relativistic SHHG at kHz repetition rate, the arguably most demanding application in terms of laser performance, as it depends critically on the spatiotemporal pulse quality and the temporal contrast.

**1.1. High-Harmonic Generation on Plasma Mirrors.** For relativistic driving intensities,  $a_0 \geq 1$ , the SHHG emission mechanism is described by a push-pull process [28], also dubbed “relativistic electron spring” [29, 30], repeating once per driving laser period. The laser field first pushes electrons into the plasma, piling up a dense electron bunch and creating a restoring internal plasma field. As the laser field changes sign, the combined plasma and laser fields accelerate the electron bunch to a relativistic velocity towards the vacuum. SHHG emission then either results from a pure phase modulation of the incident laser field by the relativistically moving critical-density plasma surface (“relativistic oscillating mirror” (ROM)) [6, 15, 31] or as a coherent synchrotron emission (CSE) from the dense relativistic electron bunch as it gets accelerated orthogonally by the laser electric field [7, 9, 32].

For  $a_0 \sim 1$ , this relativistic SHHG emission needs to be discriminated from coherent wake emission (CWE) [4, 33], generated already at subrelativistic intensities,  $a_0 < 1$ . Once per laser field cycle, Brunel electrons [34] are accelerated out and back into the surface plasma, to form via collective trajectory crossings an electron density peak traversing the overdense part ( $n(x) > n_c$ ) of the plasma density gradient. In its wake, it excites plasma oscillations at the local plasma frequency which finally leads to the emission of one attosecond light pulse per laser cycle. Clear criteria are known for the discrimination of the two types of SHHG emission.

### (i) Intensity dependence

The laser-to-SHHG conversion efficiency of CWE is constant over a large intensity range  $> 10^{15}$  W/cm<sup>2</sup> [4], whereas that of relativistic SHHG increases rapidly over a large  $a_0$ -range from  $a_0 \sim 1$  until a saturation in the ultra-relativistic limit  $a_0 \sim 100$  [4, 9]. Note however that in this experimentally still out-of-reach ultra-relativistic regime, the radiation reaction force may significantly modify SHHG [35].

### (ii) Phase properties

The intensity dependence of the Brunel trajectories leads to increasing delays between successive attosecond pulses under the driving pulse envelope and thus to frequency-broadened negatively chirped individual CWE harmonics [33, 36]. A positively chirped driving pulse can partially compensate the effect and consequently lead to spectrally narrower CWE harmonics [33, 37, 38]. In contrast, for moderate  $a_0 \sim 1$ , the relativistic SHHG emission is Fourier-limited [4, 17] such that despite its much steeper intensity dependence and consequently shorter temporal envelope, its harmonic spectral width is narrower than that of CWE harmonics generated in the same conditions [4, 20, 39]. Only for higher  $a_0 \geq 10$ , while the individual attosecond pulses remain Fourier-limited, plasma surface denting will again lead to a varying pulse-spacing and thus induce an intrinsic harmonic chirp, with the opposite sign compared to that of CWE [4, 40, 41].

### (iii) Dependence on the plasma density gradient

The density gradient of the plasma-vacuum interface is a most crucial parameter [42], and its experimental control is a prerequisite for efficient SHHG [10, 22, 39]. In CWE, the electron density peak formation and conversion of plasma oscillations into XUV light are optimized for a very steep gradient with scale length  $L_g \sim \lambda/100$ . A growing  $L_g$  causes the CWE efficiency to drop rapidly [4] while the harmonic chirp and thus spectral width of the CWE-harmonic peaks increase [33, 37]. For the steep gradients optimizing CWE, relativistic SHHG is suppressed by the gyromagnetic effect [43] preventing electrons from escaping the plasma. Instead, relativistic SHHG has repeatedly been found in single-shot experiments to be optimized by smoother optimal density gradients [8, 10, 22, 39, 44], characterized by scale lengths  $L_g \approx \lambda/10$ . For further increased  $L_g \geq \lambda/4$ , SHHG disappears due to the onset of chaotic electron dynamics [8, 22].

### (iv) Simultaneous electron emission

Plasma mirrors also emit electron beams near the specular direction but slightly deviated toward the target normal [45, 46]. The same optimal  $L_g \approx \lambda/10$  has been found to maximize the charge of relativistic electron bunches emitted from plasma mirrors [47], which is thus found to be correlated with relativistic SHHG [8, 28] and anticorrelated with CWE [48].

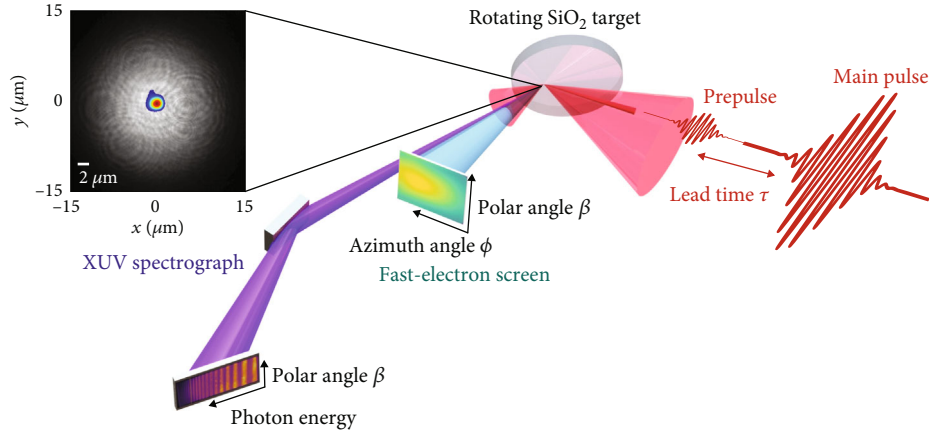


FIGURE 1: Schematics of the experimental setup for SHHG and electron acceleration on a kHz plasma mirror. The inset shows the on-target intensity profiles of the prepulse (in greyscale) and the main pulse (in colorscale).

## 2. Materials and Methods

The experiments were carried out using the 1 kHz “Salle Noire” laser system at LOA, delivering pulses with durations adjustable between 24 fs and <4 fs and a temporal contrast  $>10^{10}$  at 10 ps [24]. As illustrated in Figure 1, the p-polarized pulses are focused down to a  $\approx 1.8 \mu\text{m}$  FWHM spot by an  $f/1.3$ ,  $30^\circ$  off-axis parabola onto a rotating optically flat fused-silica target at an incidence of  $\theta = 55^\circ$ . With the pulse energy of 2.6 mJ on target, the measured spatial and temporal profiles yield, in the assumed absence of spatiotemporal couplings, peak intensities of  $2.5 \times 10^{18} \text{ W/cm}^2$  ( $a_0 = 1.1$  for  $\lambda = 800 \text{ nm}$ ),  $5.5 \times 10^{18} \text{ W/cm}^2$  ( $a_0 = 1.6$  for  $\lambda = 800 \text{ nm}$ ), and  $1.0 \times 10^{19} \text{ W/cm}^2$  ( $a_0 = 2.1$  for  $\lambda = 770 \text{ nm}$ ) for 24 fs, 8 fs, and 4- fs pulses, respectively.

We thus operate in an intensity regime where both CWE and relativistic SHHG are expected to give significant contributions to the generated harmonic signal. Controlling the plasma density gradient scale length lets us modulate their weights [39]. To this end, a spatially superposed prepulse with lead time  $\tau$  is focused to a  $13 \mu\text{m}$  FWHM spot in order to prepare a plasma density gradient scale length  $L_g = L_0 + c_s \tau$  [39, 49]. The expansion speed  $c_s$  is measured using spatial-domain interferometry (SDI) [49], and  $L_0$  is the scale length increase owing to the finite temporal contrast of the main pulse. Our observation of CWE (see below) justifies the assumption that  $L_0 \lesssim \lambda/50$  [4, 39].

The emitted SHHG radiation in the specular direction is recorded using an angle-resolving XUV spectrograph (cf. Figure 1), home-built with a gold-coated flat-field grating (600 lines/mm,  $85.3^\circ$  incidence) and a microchannel-plate (MCP) and phosphor screen detector. The MCP is time-gated for  $\approx 250 \text{ ns}$  synchronously with the laser pulses so as to suppress the detection of longer background plasma emission. The phosphor screen is finally imaged by a charged-coupled-device (CCD) camera to record angle-resolved ( $\beta \in [-35, 35]$  mrad) SHHG spectra. This only detects the central part of the SHHG beam, which in our conditions has a FWHM divergence of  $\approx 70 \text{ mrad}$ , owing in particular to the small source spot. The SHHG spectra shown in

the following are angle-integrated over the full detected range and represent the MCP signal without correction for the spectrometer’s spectral response so as to enhance the visibility of the higher harmonic orders.

Simultaneously, we detect the angular emission profile of electrons with energies  $>150 \text{ keV}$  with a lanex-screen [50] covered by a  $15 \mu\text{m}$  aluminium foil, placed between the specular and the target-normal directions and imaged by a CCD camera (cf. Figure 1). The distributions along the azimuthal angle  $\phi$  (in the incidence plane) shown in the following are integrated over the polar angle  $\beta \in [-15^\circ, 15^\circ]$  (perpendicular to the incidence plane).

All shown data are acquired by integrating over 100 ms long bursts of pulses at 1 kHz repetition rate. The carrier-envelope phase (CEP) of the laser pulses has not been locked in this work. Its rapid random variation during these 100-shot acquisitions thus averages out CEP effects on the SHHG [13, 16, 51] and electron emission.

## 3. Results

Figure 2(a) shows results obtained with a near-Fourier-limited 24 fs driver pulse. The prepulse lead time is scanned on the ps-timescale, which translates via an SDI measurement to a change of plasma density gradient scale length from  $L_0$  to  $L_0 + \lambda/5$ , where  $\lambda = 800 \text{ nm}$ . Two  $L_g$  regimes are clearly distinguished. At the shortest gradients,  $L_g - L_0 < \lambda/25$ , the harmonic peaks are broad and their intensity quickly drops with increasing  $L_g$ , as expected for CWE emission. At higher  $L_g$ , the harmonic peaks are remarkably narrower and their intensity goes through an optimum located at  $L_g - L_0 \approx \lambda/10$ , consistent with the theoretical expectations for relativistic SHHG as well as with earlier experimental observations with similar multicycle driver pulses [8, 10, 39, 44]. No harmonics are visible beyond the CWE cutoff at 30 eV, which is not very surprising as the laser intensity is only just around the relativistic threshold,  $a_0 = 1.1$ . Finally, we find a stronger electron emission at softer gradients, correlated to the (presumed) relativistic SHHG emission.

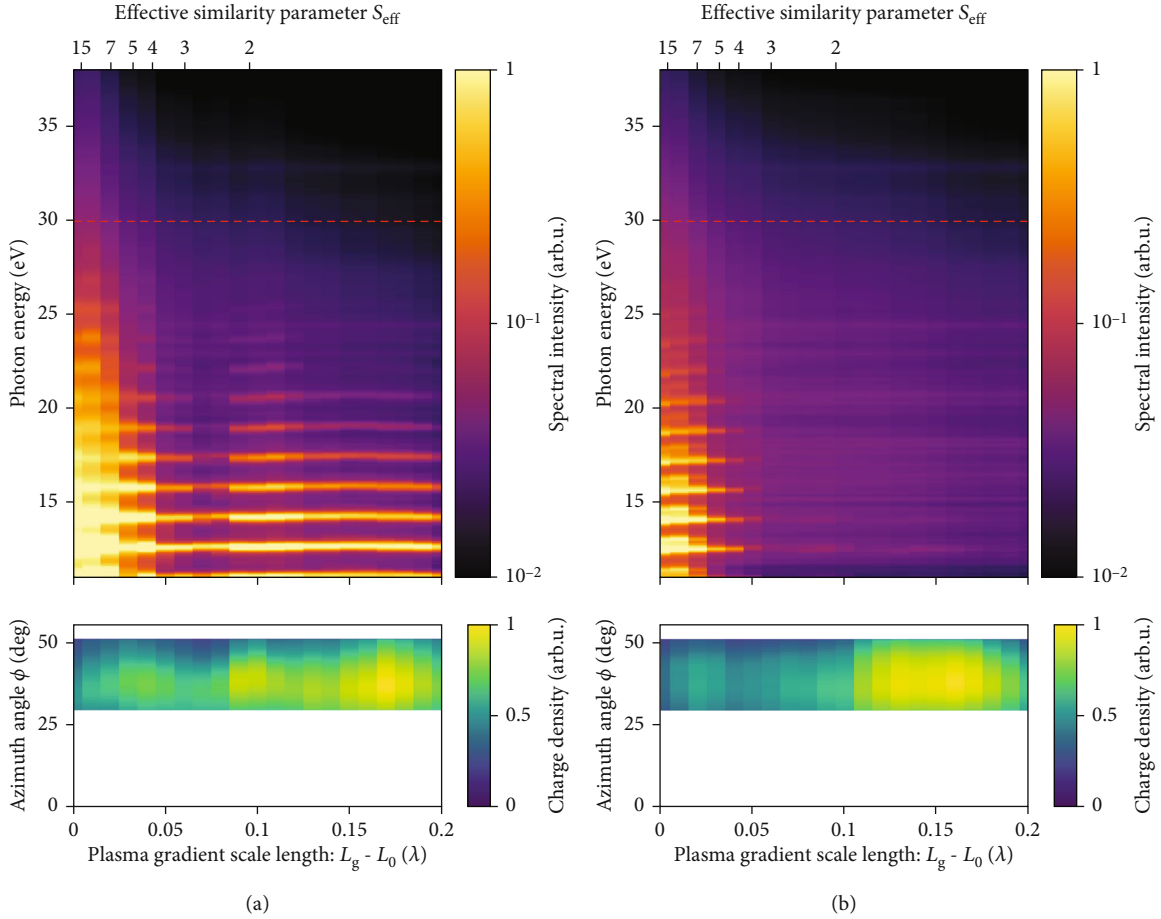


FIGURE 2: Synchronously measured SHHG (upper, log scale) and electron (lower) emission as a function of the plasma density gradient scale length. The dashed red line marks the CWE cutoff energy. The normal and specular directions correspond to  $\phi = 0$  and  $\phi = 55^\circ$ . (a) For a compressed, 24 fs,  $a_0 = 1.1$  driver pulse, and (b) for the same pulse with added 500 fs<sup>2</sup> GDD, 85 fs duration,  $a_0 = 0.6$ . The upper horizontal axis gives  $S_{\text{eff}}$  corresponding to  $L_g$  as given by equation (3) and assuming  $L_0 = \lambda/100$ .

We can corroborate this interpretation by using a “Dazzler” acousto-optic programmable dispersive filter [52] in our amplifier chain to add 500 fs<sup>2</sup> GDD to the driving pulses, thus chirping them positively. This stretches them to 85 fs duration and consequently decreases the intensity to  $0.7 \times 10^{18}$  W/cm<sup>2</sup> ( $a_0 = 0.6$  for  $\lambda = 800$  nm). As shown in Figure 2(b), the harmonic spectral widths decrease significantly at the shortest  $L_g$ , but remain similar in intensity. This is consistent with the compensation of the aperiodicity of the CWE attosecond pulse train and the constant CWE conversion efficiency. The disappearance of the SHHG emission at softer gradients is consistent with the highly nonlinear intensity-dependence of relativistic SHHG. Electrons are still emitted preferentially for the softer gradients and the resulting anti-correlation of harmonics and electrons is reminiscent of our earlier sub-relativistic-intensity results [48].

Reducing the driving pulse duration to 8 fs at constant pulse energy increases the peak normalized vector potential to  $a_0 = 1.5$  and leads to the results shown in Figure 3. As for the 24 fs driver, optimal relativistic SHHG is observed around  $L_g - L_0 \approx \lambda/10$ , with a clear correlation with electron emission for softer gradients. Additionally, we now observe

the spectral extent of the SHHG signal to reach and possibly slightly surpass the CWE cutoff. For longer  $L_g$ , the relativistic SHHG signal persists until  $L_g - L_0 \approx \lambda/4$  but its intensity and spectral extent gradually decrease.

An even shorter 4 fs driving pulse is expected to strongly boost the SHHG signal beyond the CWE cutoff. Self-steepening in the helium-filled hollow-core fiber used for postcompression however reduces the central wavelength to  $\lambda \approx 770$  nm so that the peak normalized vector potential only increases to  $a_0 = 2.1$  at constant pulse energy. Furthermore, such short pulses become extremely sensitive to spatiotemporal couplings which can drastically reduce the achievable peak intensity. On many days, we thus observe reduced spectral extent of the SHHG spectra generated with  $\leq 4$  fs pulses as compared to those obtained with 8 fs pulses shown in Figure 3, as was the case for the experiments reported in ref. [16].

We did nonetheless succeed in recording datasets like that shown in Figure 4 with a 4 fs driving pulse and shot-to-shot randomly varying CEP. Similarly to the above results, relativistic SHHG is observed for plasma density gradient scale lengths between  $L_g - L_0 \approx \lambda/20$  and  $\lambda/4$ , where

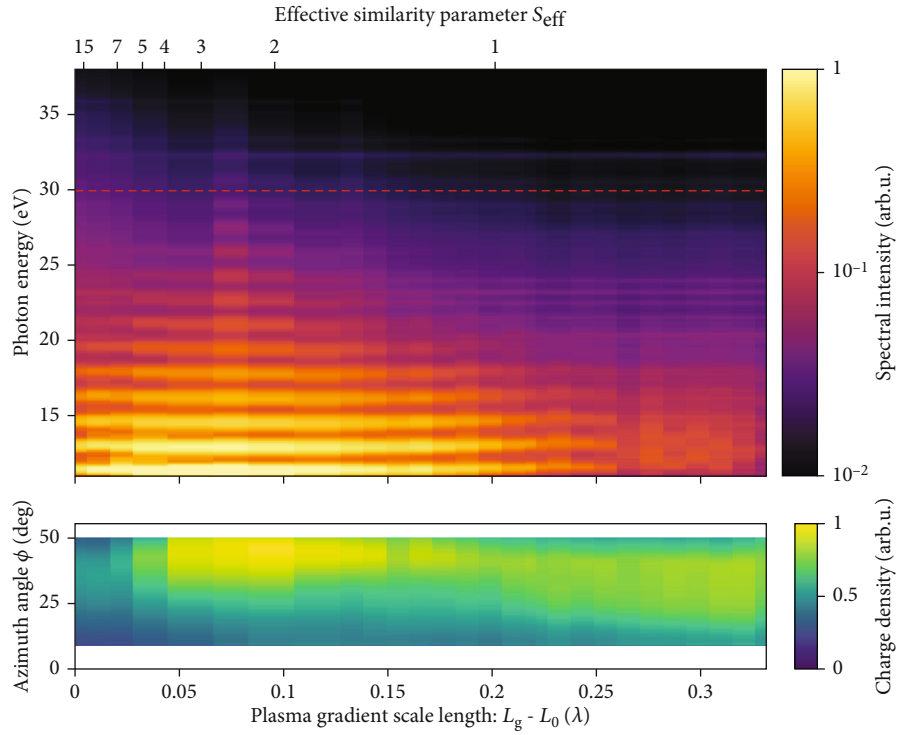


FIGURE 3: Synchronously measured SHHG (upper, log-scale) and electron (lower) emission as a function of the plasma density gradient scale length for an 8 fs,  $a_0 = 1.6$  driver pulse. The dashed red line marks the CWE cutoff energy. The normal and specular directions correspond to  $\phi = 0$  and  $\phi = 55^\circ$ . The upper horizontal axis gives  $S_{\text{eff}}$  corresponding to  $L_g$  as given by equation (3) and assuming  $L_0 = \lambda/100$ .

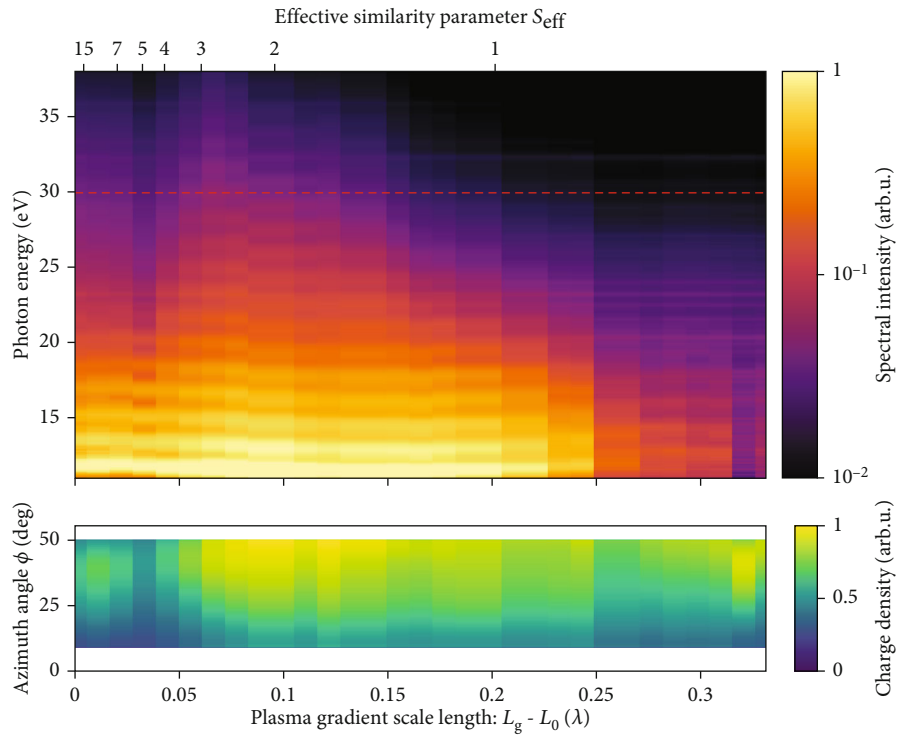


FIGURE 4: Synchronously measured SHHG (upper, log scale) and electron (lower) emission as a function of the plasma density gradient scale length for a 4 fs,  $a_0 = 2.1$  driver pulse with  $\lambda = 770$  nm central wavelength. The dashed red line marks the CWE cutoff energy. The normal and specular directions correspond to  $\phi = 0$  and  $\phi = 55^\circ$ . The upper horizontal axis gives  $S_{\text{eff}}$  corresponding to  $L_g$  as given by equation (3) and assuming  $L_0 = \lambda/100$ .



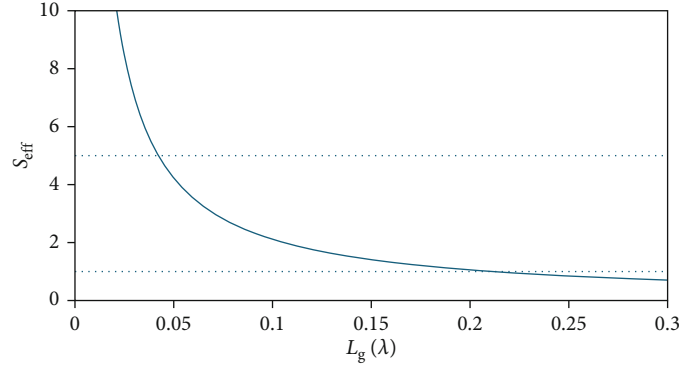


FIGURE 5: Theoretical intensity-independent effective similarity parameter  $S_{\text{eff}}$  (equation (3)) for  $\theta = 55^\circ$  as a function of the scale length  $L_g$  of an exponential plasma density gradient (equation (1)). Dashed lines mark  $S_{\text{eff}} = 1$  and  $S_{\text{eff}} = 5$ .

now  $\lambda = 770$  nm. Around the optimal  $L_g - L_0 \approx \lambda/10$ , the spectra now extend well beyond the CWE cutoff, which is definite proof for relativistic SHHG, reported here for the first time with a 1 kHz repetition rate. The spectra still present a discernible harmonic modulation, in particular for the softer gradients, and one can make out small gradual  $L_g$ -dependent shifts of the harmonic peaks. These may result from a varying temporal structure of the generated very short (essentially 2-pulse [16]) attosecond-pulse-train. All recorded spectra are continuous, i.e., the harmonic peaks spectrally overlap, but their averaging over of random driver CEPs precludes us from inferring a temporal structure from spectral features [13, 16, 51].

The electron emission is again correlated with relativistic SHHG. Compared to Figures 2 and 3, we can make out that the electron beam is less clearly pushed away from the  $55^\circ$  specular direction, because of less efficient ponderomotive scattering of the electrons by the sub-2-cycle laser pulse as compared to the multicycle 8 fs and 24 fs pulses.

#### 4. Discussion

We have found that relativistic SHHG occurs for plasma density gradient scale lengths  $L_g \in [\lambda/20, \lambda/4]$ , with an optimum at  $L_g \approx \lambda/10$ , with an assumed laser-contrast induced  $L_0 \approx \lambda/100$ . This optimum plasma gradient scale length is consistent with various earlier experiments with  $\approx 30$  fs laser drivers focused to higher  $a_0 > 3$  [8, 10, 39, 44]. Here we find, for three different driving laser durations from 24 to 4 fs and lower peak intensities corresponding to  $a_0 = 1.1$  to 2.1, the same optimal  $L_g$ -range. This apparent universality of the optimal plasma density gradient scale length is an intriguing observation.

In order to study density dependencies, particle-in-cell simulations often treat initially step-like plasma density profiles with variable peak density  $n$ . This adjusts the similarity parameter  $S = n/(n_c a_0)$ , which describes the relative strengths of the laser and induced internal plasma fields and scales laser-plasma interaction in the relativistic regime [53]. Optimal conditions for the relativistic push-pull process, relying on the formation and dynamics of a sharp electron density spike, are predicted for  $S \in [1, 5]$  [9, 29, 30].

For currently explored experimental laser intensities,  $a_0 \sim 1$ , few-times overcritical plasma densities should thus be optimal. In solid-target experiments however, the plasma rather has  $\sim 100n_c$  bulk density, and the interaction rather occurs in the smooth density gradient at the surface. In the push-phase, the laser pulse sweeps electrons forward and penetrates this gradient up to a depth  $x_b$ , where the laser's Lorentz force and the electrostatic restoring force exerted by the plasma ion background are balanced [29, 30]. The plasma ion density at the depth  $x_b$  can then be taken to define an effective similarity parameter  $S_{\text{eff}}$ . This was demonstrated in ref. [54] with numerical simulations for a linear density ramp, with the core result that properties of SHHG obtained for a given  $S_{\text{eff}}$  match those obtained with a step-like plasma profile whose density satisfies  $S = S_{\text{eff}}$ .

Here, we take the same approach but consider instead the experimentally relevant exponential density profile of equation (1). In this plasma, the laser-plasma force balance yields an instantaneous penetration depth [28, 41]  $x_b = L \ln [a(1 + \sin \theta) \cos \theta / \pi L / \lambda]$ , where  $\theta$  is the laser incidence angle on the plasma surface and  $a = a_0 \cos(\varphi)$  is the instantaneous value of the normalized vector potential during the pushing half-cycle, i.e.,  $\varphi \in [-\pi/2, \pi/2]$ . With the half-cycle averaged vector potential  $(2\pi)a_0$ , we can define an effective penetration depth of

$$\bar{x}_b = L_g \ln \left[ \frac{2a_0(1 + \sin \theta) \cos \theta}{\pi^2 (L_g/\lambda)} \right]. \quad (2)$$

Experimentally varying the scale length  $L_g$  thus corresponds to adjusting the effective plasma (ion) density  $n[\bar{x}_b(L_g)]$ , where a larger  $L_g$  corresponds to a lower density. With this density, we find an effective similarity parameter

$$S_{\text{eff}} = \frac{n(\bar{x}_b)}{n_c a_0} = \frac{2(1 + \sin \theta) \cos \theta}{\pi^2 (L_g/\lambda)}. \quad (3)$$

It is remarkable that this is independent of the laser intensity and, for a given incidence angle, only depends on the gradient scale length. For a given exponential density profile and for any intensity that is sufficiently high to be

considered relativistic, the laser thus piles up the electrons at the depth where the ion density corresponds to the *same* similarity parameter. This direct correspondence of  $L_g$  and  $S_{\text{eff}}$  is the reason for the intensity-independent optimal  $L_g$ -range that we find in our experiments.

We plot  $S_{\text{eff}}$  from equation (3) for our experimental incidence angle  $\theta = 55^\circ$  in Figure 5. The theoretically predicted optimal range  $S \in [1, 5]$  for relativistic SHHG [9, 29, 30] is thus achieved for  $L_g \in [\lambda/25, \lambda/5]$ . This is in excellent agreement with our experimentally observed intensity-independent  $L_g$ -range. Assuming  $L_0 = \lambda/100$ , equation (3) yields the  $S_{\text{eff}}$  axes in Figures 2–4.

## 5. Conclusion

We have reported clear evidence for the first observation of relativistic SHHG from plasma mirrors driven at kilohertz repetition rate. These are correlated to the emission of energetic electrons through the variation of the surface plasma density gradient scale length  $L_g$ . For three increasingly short and intense driving pulses (from 24 fs (9 optical cycles) and  $a_0 = 1.1$  to 4 fs (1.7 optical cycles) and  $a_0 \approx 2.1$ ), relativistic SHHG has been observed for the same  $L_g$ -range,  $L_g \in [\lambda/20, \lambda/4]$ , with an optimum gradient scale length of  $L_g \approx \lambda/10$ . The universality of the optimal  $L_g$ -range has been rationalized by deriving a direct intensity-independent link between the scale length  $L_g$  of an exponential plasma density profile and an effective similarity parameter for relativistic laser-plasma interactions. The observed optimal  $L_g$ -range corresponds very well to theoretically expected optimal similarity parameters.

These results open the route to a detailed examination and optimization of relativistic SHHG, exploiting the high kHz repetition rate and exceptional stability of our laser system [24] and next-generation systems in development [55]. In particular for the shortest pulse durations and with added CEP-locking, we should become able to harness the favorable phase properties of relativistic SHHG for the generation of powerful isolated attosecond pulses from ultraintense laser-driven plasma mirrors [13, 16, 51].

## Data Availability

The data that support the findings of this study are available from the corresponding author, S.H., upon reasonable request.

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

## Authors' Contributions

R.L.-M., J.F., and S.H. designed the research project. M.B., F.B., and S.H. built the SHHG setup, made preliminary experiments, and developed the data analysis routines. M.O., J.K. and A.V. operated the laser chain. M.O., J.K., L.D., D.L., and S.H. performed the experiments. S.H.,

M.O., and J.K. analyzed the data. S.H. wrote the paper, with support through comments from all authors.

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