

CHARACTERISTIC FEATURES OF MUELLER MATRIX PATTERNS FOR POLARIZATION SCATTERING MODEL OF BIOLOGICAL TISSUES

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We developed a model to describe polarized photon scattering in biological tissues. In this model, tissues are simplified to a mixture of scatterers and surrounding medium. There are two types of scatterers in the model: solid spheres and infinitely long solid cylinders. Variables related to the scatterers include: the densities and sizes of the spheres and cylinders, the orientation and angular distribution of cylinders. Variables related to the surrounding medium include: the refractive index, absorption coefficient and birefringence. In this paper, as a development we introduce an optical activity effect to the model. By comparing experiments and Monte Carlo simulations, we analyze the backscattering Mueller matrix patterns of several tissue-like media, and summarize the different effects coming from anisotropic scattering and optical properties. In addition, we propose a possible method to extract the optical activity values for tissues. Both the experimental and simulated results show that, by analyzing the Mueller matrix patterns, the microstructure and optical properties of the medium can be obtained. The characteristic features of Mueller matrix patterns are potentially powerful tools for studying the contrast mechanisms of polarization imaging for medical diagnosis.

Keywords: Mueller matrix; scattering; polarization; tissue; anisotropy.

1. Introduction

Recently, there have been growing interests in the applications of polarization imaging techniques for biomedical purposes. Mueller matrix provides a

comprehensive representation for describing the polarization-related properties of the scattering media such as biological tissues. In the past decade, several groups have demonstrated that the Mueller

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matrix imaging techniques have good potential in cancer detection.^{1–5} However, the relations between Mueller matrix elements and structural properties of biological tissues are often not so clear, hindering further clinical applications of Mueller matrix polarimetric techniques to medical diagnosis. To interpret the relations adequately, we have to learn in detail the behaviors of polarized photons propagation in tissues. Monte Carlo simulations have been used as a powerful tool for analyzing the interactions between light and tissues. A proper optical scattering model is essential for such studies. A sphere only scattering model was used first for isotropic tissues such as blood, liver and fat.^{6,7} Several groups studied the two-dimensional (2D) backscattering Mueller matrix patterns of such model.^{8–10} Then the optical activity¹¹ and linear birefringence^{12–14} for the ambient media were taken into account in the sphere optical activity model and sphere birefringence model. Using these models and Monte Carlo simulations, optical anisotropy properties (optical activity, linear birefringence, dichroism) of biological objects can be generated and studied. For instance, Angelsky *et al.* discussed the anisotropic complex biological tissue models and related possible diagnostic criteria.^{15–19} To mimic the fibrous structures in biological tissues such as collagen fibers, muscle fibers and elastins, we proposed a sphere-cylinder scattering model (SCSM)^{20,21} and studied the characteristic features of its backscattering Mueller matrix patterns.^{22,23} Recently, SCSM has been expanded to the comprehensive sphere-cylinder birefringence model (SCBM) to take into account of birefringent ambient medium.²⁴ A good agreement between the experimental and Monte Carlo simulated results has shown that the SCBM is a potentially powerful tool for biomedical imaging and diagnostic studies.

In this paper, we will summarize the developments of scattering models for biological tissues and introduce optical activity into the existing model. Using the model and Monte Carlo simulations we study the characteristic features of tissue-like media in detail. By comparing experiments and Monte Carlo simulations, we analyze the backscattering Mueller matrix patterns of several tissue-like media, and summarize the different effects coming from anisotropic scattering and optical properties (optical activity, linear birefringence). In addition, we study the possible method to extract the optical activity values for tissues. Both the experimental

and simulated results show that, by analyzing the Mueller matrix patterns, the microstructures and optical properties of the medium can be obtained. The characteristic features of Mueller matrix patterns are potentially powerful tools for studying the mechanisms of polarization imaging for biomedical diagnosis.

2. Methods: Monte Carlo Simulation and Experiments

2.1. Scattering model for biological tissues

As shown in Fig. 1, the scattering model for biological tissues is a homogeneous medium consisting of microspheres and infinitely long cylindrical scatterers. Parameters of the scatterers, such as their sizes, refractive indices and densities can be varied independently. The orientations of the cylinders are also variable. It is assumed that the orientation angle of the cylinders follows a Gaussian distribution. In addition, parameters of the surrounding media, such as refractive index, absorption coefficient, optical rotation degree (ORD), birefringence value and birefringence axis, are all variables in the model.

2.2. Monte Carlo simulation

In our previous works, a Monte Carlo program has been developed to track down scattering and propagation behavior of each polarized photon.^{20,24} In Monte Carlo simulations for SCBM, the program starts by launching a normal incident photon at an initial position and direction. The polarization state of this photon is represented by a Stokes vector. The

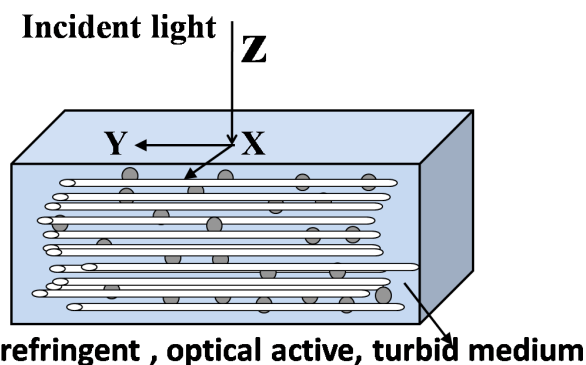


Fig. 1. Schematics of the scattering model for biological tissues.

photon propagates in a linearly birefringent medium until it is scattered or absorbed. The polarization state and position of the photon are both updated. At each scattering point, the photon drops part of its energy due to the absorption. A statistical method has been designed to determine whether the photon is scattered by the cylinders or the spherical particles. Then the scattering direction is calculated depending on the phase function with a random number. The simulation process will continue until a photon is completely absorbed or move out of the sample. All the information of the emitted photons will be stored, including their polarization states, positions and emitted directions.

In this paper, as a development to SCBM, we add a new module to the model and Monte Carlo program to calculate the effects of optical activity. Optical activity does not affect the scattering process, but it alters the polarization states of photons as their propagation between two successive scattering events.^{11,14} The new module calculates the changes of Stokes vector due to the optical activity medium as Eq. (1):

$$S' = R(\theta) \cdot S. \quad (1)$$

In Eq. (1), $R(\theta)$ represents the rotational matrix, which can be expressed as Eq. (2):

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

In Eq. (2), θ represents the rotational angle which can be obtained as Eq. (3):

$$\theta = \chi \cdot s = \text{ORD} \cdot \alpha \cdot s. \quad (3)$$

In Eq. (3), χ is the product of the ORD and the concentration of optical active molecules (α). ORD represents the rotatory power of the optical active molecules. S is the path length of light.

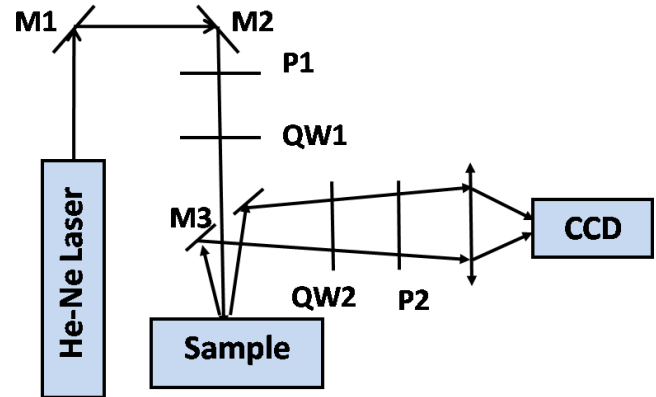


Fig. 2. A schematic of the experimental setup. P, polarizer; QW, quarter-wave plate; CCD, imaging camera.

2.3. Experimental setup

The typical backscattering Mueller matrix measurement experimental setup used in this paper is shown in Fig. 2.⁹ The light source is a 633 nm linearly polarized He–Ne laser (Daheng Optics, China, DH-HN600). A set of quarter-wave plate (QW1) (Daheng Optics, China, GCL-060402) and linear polarizer P1 (Daheng Optics, China, GCL-050003, extinction ratio 500:1) is used to change the polarization states of incident light. The polarization states of scattered light can be measured by a second set of quarter-wave plate (QW2) and linear polarizer (P2). The backscattering photons are recorded by an 8 bit CCD camera (Canon EOS 550D) to produce reflectance images of the samples. In this paper, for the measurements of Mueller matrix, six different polarization states are achieved for the incident light: 0° linear (H), 90° linear (V), 45° linear (P), 135° linear (M), right circular (R) and left circular (L), then six polarization components of scattered light corresponding to each incident state are recorded. A total of 36 reflectance images are captured and the Mueller matrix can be calculated from the reflectance images as Eq. (4). The Mueller matrix elements are all normalized by the maximum intensity of the m_{11} .

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} HH + HV + VH + VV & HH + HV - VH - VV & PH + PV - MP - MM & RH + RV - LH - LV \\ HH - HV + VH - VV & HH - HV - VH + VV & PH - PV - MH + MV & RH - RV - LH + LV \\ HP - HM + VP - VM & HP - HM - VP + VM & PP - PM - MP + MM & RP - RM - LP + LM \\ HR - LL + VR - RL & HR - VR + VL - HL & PR - MR + ML - PL & RR - RL - LR + LL \end{pmatrix}. \quad (4)$$

In Eq. (4), the first letter represents the input polarization state and the second letter represents the output polarization state. For example, “HV” represents a vertically-polarized (V) reflectance image with horizontally-polarized (H) incident light.

3. Results and Discussion

As introduced above, the polarization scattering model for biological tissues includes both spherical and infinitely long cylindrical scatterers, the optical activity and birefringence effects. According to characteristics of different biological tissues, the model can be adjusted. Based on this model, using both the experiments and Monte Carlo simulations, we studied the 2D backscattering Mueller matrix patterns for different types of scattering media, which include: the sphere scattering medium, the sphere birefringence medium, the sphere-cylinder scattering medium, the sphere-cylinder birefringence medium and the sphere optical activity

medium. We analyzed the characteristic features of 2D Mueller matrix elements patterns, and summarized the different effects coming from anisotropic scattering and optical properties.

3.1. Comparison of cylindrical scattering and birefringence effects

The experimental and Monte Carlo simulated results of different media are shown in Figs. 3 and 4. It can be observed that the simulated results agree well with the experimental measurements, showing the validity of the scattering model and Monte Carlo simulations. Figures 3(a) and 4(a) show the experimental and simulated 2D Mueller matrix patterns for the sphere scattering medium, which consists of $0.2\ \mu\text{m}$ diameter polystyrene microspheres (International Laboratory, USA, $\mu_s = 10\ \text{cm}^{-1}$, $n = 1.59$) suspending in water. Figures 3(b) and 4(b) show the experimental and simulated 2D Mueller matrix patterns for the sphere birefringence medium, which

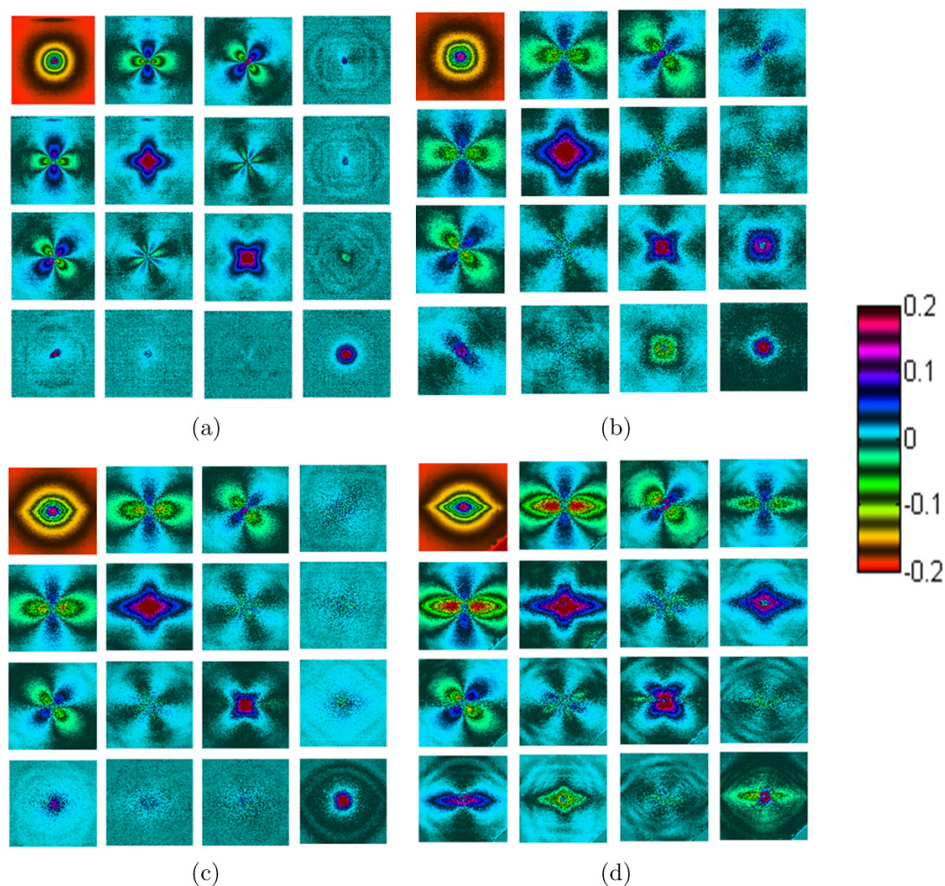


Fig. 3. Experimental backscattering Mueller matrices of (a) sphere scattering medium, (b) sphere birefringence medium, (c) sphere-cylinder scattering medium and (d) sphere-cylinder birefringence medium.

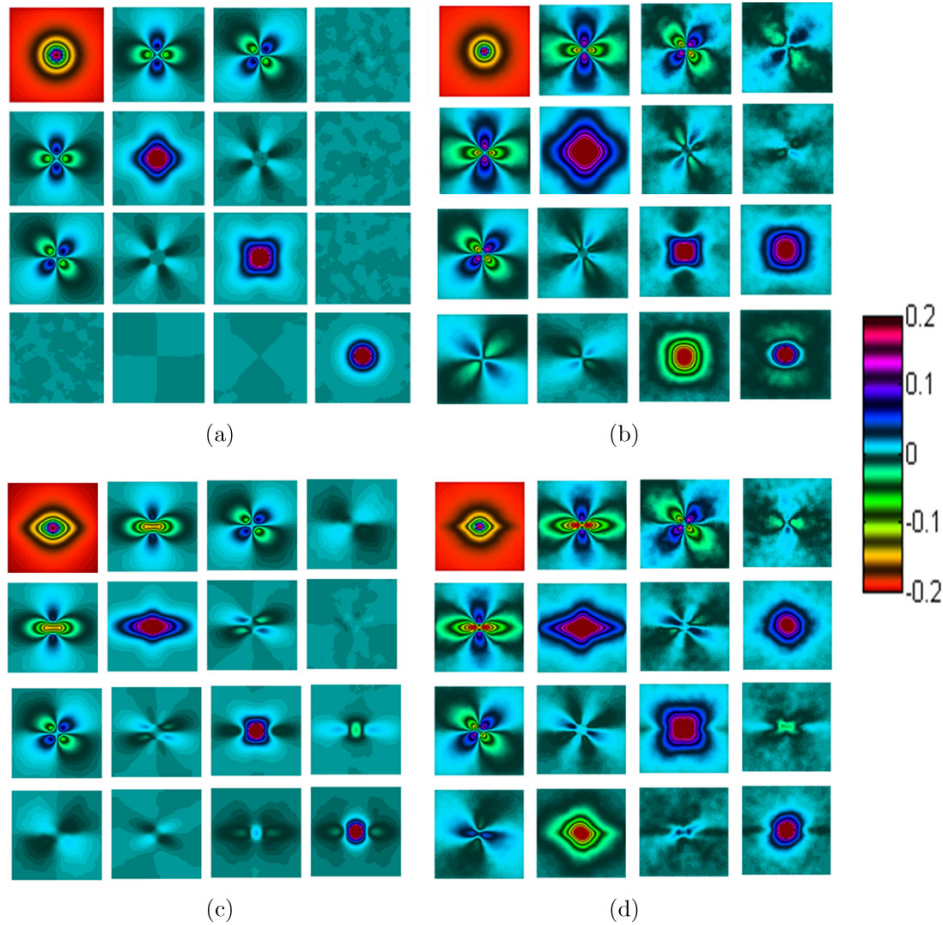


Fig. 4. Monte Carlo simulated backscattering Mueller matrices of (a) sphere scattering medium, (b) sphere birefringence medium, (c) sphere-cylinder scattering medium and (d) sphere-cylinder birefringence medium.

is an extended hydrogel cuboid with $0.2 \mu\text{m}$ diameter polystyrene microspheres ($\mu_s = 5 \text{ cm}^{-1}$, $n = 1.59$) immersed in it.¹⁴ The extraordinary axis of the birefringent sample (the extended direction of the hydrogel) is along the x -axis. The birefringence value of the elongated hydrogel sample is 1×10^{-5} responding to the extension of 5 mm. Figures 3(c) and 4(c) show the experimental and simulated 2D Mueller matrix patterns for the sphere-cylinder scattering media. The sample used in experiments is a three-layered medium consisting of both well-aligned silk fibers along the y -axis and microspheres submerged in water. The first and third layers are 5 mm thick solutions of $0.2 \mu\text{m}$ diameter polystyrene microspheres ($\mu_s = 5 \text{ cm}^{-1}$, $n = 1.59$). The second layer contains only well-aligned silk fibers (provided by Guangxi Institute of Supervision and Testing on Product Quality) along the y -axis. The thickness of the silk fibers is 1 mm. The diameters, scattering coefficient and refractive index of the silk fibers

are $1.5 \mu\text{m}$, 65 cm^{-1} and 1.56, respectively.²² Figures 3(d) and 4(d) show the experimental and simulated 2D Mueller matrix patterns for the sphere-cylinder birefringence media. The experimental sample is the three-layered microsphere-silk medium submerged in an extended hydrogel. The birefringence value of this sample is 1×10^{-5} responding to the extension of 5 mm. The birefringence axis (the direction of strain) is along 45° direction on the X - Y plane.²⁴

The experimental and simulated results shown in Figs. 3 and 4 demonstrate that the cylindrical scattering and birefringence effects affect Mueller matrix elements differently. The m_{11} element relates only to unpolarized light intensity, which is not affected by the linear birefringence. Therefore, comparing with the sphere scattering medium [see Figs. 3(a) and 4(a)], the m_{11} element for the sphere birefringence medium [see Figs. 3(b) and 4(b)] has the same circular characteristics. Since the m_{12} ,

m_{21} and m_{22} elements relate to horizontal and vertical polarization components, when the birefringence axis is along x or y direction, these elements are also not affected. However, compared to the sphere scattering medium, other Mueller matrix elements are affected by birefringence. The birefringence effects destroy the rotational symmetry of the m_{12}/m_{13} , m_{21}/m_{31} and m_{22}/m_{33} pairs which exists in the sphere scattering medium. The total intensity values of the m_{12} , m_{21} and m_{22} elements become larger than those of the m_{13} , m_{31} and m_{33} elements. The m_{14} (m_{41}), m_{24} (m_{42}) and m_{34} (m_{43}) elements have similar shapes as those of the m_{13} , m_{23} and m_{33} elements, respectively. This is because the transformation between circular polarization state and 45° linear polarization state in the medium is enhanced by the linear birefringence.¹² The m_{14} and m_{41} elements always keep the quatrefoil patterns, but rotate as the birefringence axis changes. Moreover, as the birefringence value varies, the absolute intensity of the m_{14} and m_{41} elements also changes. In short, the m_{14} and m_{41} elements can be used to determine the value and direction of birefringence.¹³

Different from the birefringence effects discussed above, the cylindrical scattering affects all the Mueller matrix elements. Comparing with the sphere scattering medium [see Figs. 3(a) and 4(a)], for the sphere-cylinder scattering medium [see Figs. 3(c) and 4(c)], the intensity distributions of the m_{11} , m_{12} , m_{21} and m_{22} elements along the x -axis become larger than those along the y -axis due

to the cylindrical scattering effect.^{21,22} For instance, the cylinders result in more photons being scattered to the x -axis direction, leading to a rhombic m_{11} shape in Figs. 3(c) and 4(c). Patterns of the m_{11} element can be used to judge the presence of the cylindrical scatterers.^{21–24} Besides, the cylindrical scattering effect also destroys the rotational symmetry of the m_{12}/m_{13} , m_{21}/m_{31} and m_{22}/m_{33} pairs. The total intensity values of the m_{12} , m_{21} and m_{22} elements become larger than those of the m_{13} , m_{31} and m_{33} . These changes are similar as those coming from the birefringence effect.

For the sphere-cylinder birefringence medium, both the cylindrical scattering and birefringence affect the Mueller matrix elements simultaneously. As shown in Figs. 3(d) and 4(d), the m_{11} element has rhombic shape due to the cylindrical scattering. The m_{14} and m_{41} elements keep the quatrefoil patterns and rotate by 45° with the birefringence axis. The Mueller matrix elements can be used to reveal the structural and optical properties of the media containing cylinders and birefringence.

3.2. Mueller matrix patterns of optical active medium

Besides spherical, cylindrical scatterers and birefringence effect, some tissues contain chiral molecules such as glucose. Therefore, the patterns of Mueller matrix for a sphere optical active medium should be studied. Figure 5(a) shows the experimental backscattering Mueller matrix of a sample

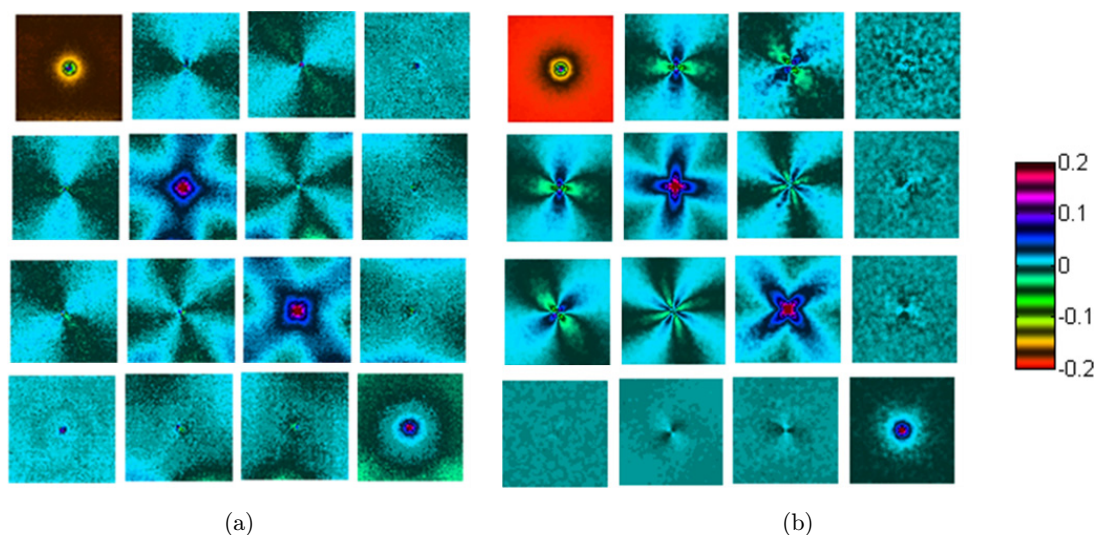


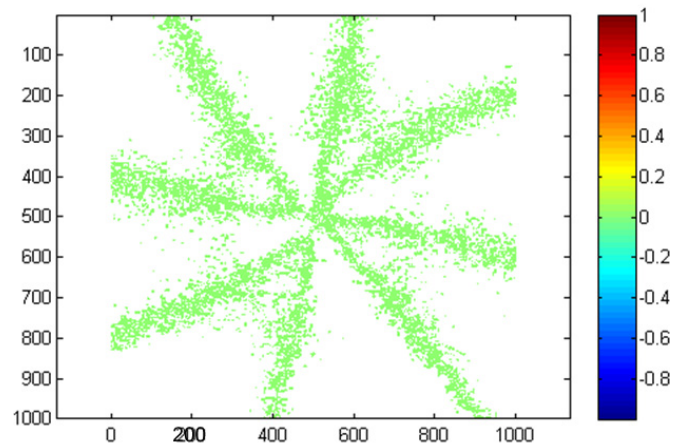
Fig. 5. Backscattering Mueller matrices for sphere optical active medium: (a) experimental result of microsphere sucrose solution and (b) Monte Carlo simulated results.

containing $2\ \mu\text{m}$ diameter polystyrene microspheres ($\mu_s = 5\ \text{cm}^{-1}$, $n = 1.59$) in sucrose solution ($\chi = 5.37\ \text{deg cm}^{-1}$, $n = 1.45$), Fig. 5(b) shows the corresponding Monte Carlo simulated results. The good agreement between experimental and simulated results shows the validity of the optical active module for the Monte Carlo program.

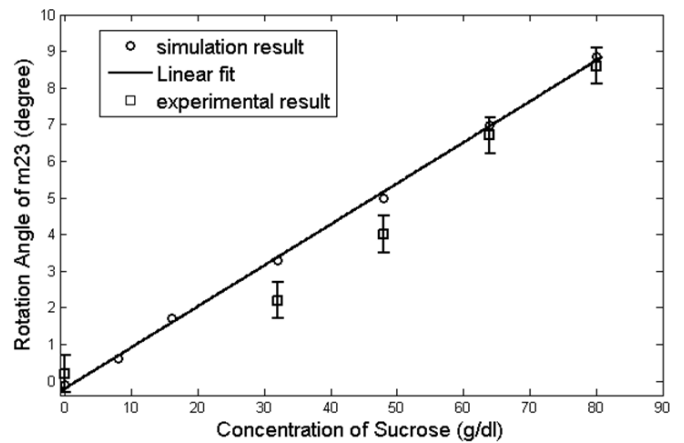
Similar as the sphere scattering medium [see Figs. 3(a) and 4(a)], for the sphere optical active medium the patterns of the m_{11} and m_{44} elements are groups of circles, the m_{14} and m_{41} elements are blanks. It can be observed that the optical activity does not affect the m_{11} , m_{14} , m_{41} and m_{44} elements, which only relate to unpolarized light and circular polarized light. The chiral sucrose molecules prominently change the polarization states of the linear polarized light. Hence, the optical activity affects the other Mueller matrix elements relating to the linear polarization states. The patterns of these elements have obvious different rotations around the incident point, which is at the center of each map. The diagonal symmetry properties of the Mueller matrix elements are destroyed because of the existence of chiral molecules. However, the optical activity effect does not destroy the rotational symmetry of the m_{12}/m_{13} , m_{21}/m_{31} and m_{22}/m_{33} elements pairs. While, the symmetry of the m_{12}/m_{21} , m_{13}/m_{31} , m_{24}/m_{42} and m_{34}/m_{43} elements pairs are damaged. The rotation angle of the $m_{21}/m_{31}/m_{24}/m_{34}$ becomes larger than that of the $m_{12}/m_{13}/m_{42}/m_{43}$ at the same detection position.¹¹ Among all the 16 Mueller matrix elements, the rotation effects of the m_{22} , m_{23} , m_{32} and m_{33} elements are the most obvious. Therefore, patterns of the $m_{22}/m_{33}/m_{23}/m_{32}$ elements can be used to detect the existence of the optical activity in a medium.

3.3. Effect of optical active molecules' concentration on Mueller matrix patterns

Furthermore, we analyze the effect of the optical active molecules' concentration on the 2D Mueller matrix patterns quantitatively. As mentioned in Sec. 3.2, the most sensitive Mueller matrix elements are the m_{22} , m_{23} , m_{33} and m_{32} . In this section, we choose the m_{23} as the analyzing example. Figure 6(a) shows the zero-intensity profiles of the m_{23} element. It can be observed that the equi-intensity profiles of the m_{23} are approximately



(a)



(b)

Fig. 6. Backscattering Mueller matrix element m_{23} for sphere optical active medium: (a) zero-intensity profile of the m_{23} and (b) rotation angle of the m_{23} change with the concentration of sucrose.

straight lines. We fit these lines using least square method and compare them with those of the sphere scattering medium. Then we can obtain the rotation angle of the m_{23} due to optical activity.

For the study of relationship between the rotation angle of the m_{23} and the value of optical activity, we change the concentration of sucrose in both the experiments and Monte Carlo simulations. The experimental and simulated results are shown in Fig. 6(b). It can be seen that as the concentration of sucrose increases, the rotation angle of the m_{23} also increases. Although there is discrepancy between the experimental and the simulated results because of the limited accuracy of measurements,

the results show that the rotation angle of the m_{23} correlates to the sucrose's concentration linearly. Therefore, the m_{23}/m_{32} can be used to determine the concentration of the optical active molecules quantitatively. However, it should be pointed out that such measurement works only for very high concentration. When the concentration of sucrose is less than 32 g/dl, the rotation of the m_{23} can hardly be observed.

4. Conclusions

We have developed a model to describe polarized photons scattering in biological tissues. In this model, tissues are simplified to a mixture of scatterers and surrounding medium. There are two types of scatterers: solid spheres and infinitely long solid cylinders. The densities and sizes of the spheres and cylinders, the orientation, angular distribution of the cylinders are variable. The refractive index, absorption coefficient and birefringence of the surrounding medium can also be varied. In this paper, we introduced optical activity into the existing module and summarized the characteristic features in the 2D backscattering Mueller matrix patterns due to different constituents in the model. By comparing experiments and Monte Carlo simulations, we analyzed the backscattering Mueller matrix patterns of several tissue-like media, which include the sphere scattering medium, the sphere birefringence medium, the sphere-cylinder scattering medium, the sphere-cylinder birefringence medium and the sphere optical active medium. Moreover, we summarized the different effects coming from anisotropic scattering and optical properties, and proposed a possible method to extract the optical activity values for tissues. For example, the m_{11} element can be used as an indicator for the concentration of cylindrical scatterers. The $m_{14}/41$ and $m_{22}/m_{33}/m_{23}/m_{32}$ elements can be used to indicate the existence of the birefringence and the optical activity effects, respectively. In summary, this polarization scattering model is a potentially powerful tool in studying the contrast mechanisms of polarization imaging techniques for biomedical purposes.

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