

# Generation of twisted $\gamma$ -ray radiation by nonlinear Thomson scattering of twisted light

Cite as: Matter Radiat. Extremes 4, 024401 (2019); doi: 10.1063/1.5086347

Submitted: 27 July 2018 • Accepted: 25 October 2018 •

Published Online: 11 March 2019



View Online



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## ABSTRACT

Interaction of twisted strong laser radiation with electrons in the classical regime is considered. We investigate transfer of the angular momentum of absorbed laser photons to the emitted radiation. An interaction regime is considered where radiation reaction is negligible and the formation length of radiation is comparable to or larger than the laser wavelength. The latter condition ensures that the structure of the laser field plays a role in the electron dynamics during the formation of radiation. We distinguish the case of a single electron from that of an electron beam. For a single electron, the spin angular momentum of the driving laser photons is transferred to the radiation field, while the orbital angular momentum of the laser field is not. We conclude that in the classical regime, to imprint the angular momentum of twisted light on radiation, an electron beam is a prerequisite. In the latter case, nonlinear Thomson scattering of twisted light off an ultrarelativistic electron beam produces high-frequency radiation that is twisted, with a topological charge proportional to the harmonic order.

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## I. INTRODUCTION

Twisted light or vortex light is light with a well-defined orbital angular momentum (OAM) equal to  $\ell \hbar$  per photon, where  $\ell$  is the topological charge. Twisted light has a helical wavefront and a dark center associated with phase singularity. In 1992, Allen *et al.*<sup>1</sup> proposed that twisted light can be generated in the laboratory using astigmatic optical elements. Since then, growing attention has been paid to this fundamental property of light. The OAM of optical vortices can be transferred to atoms, molecules, and nanostructures,<sup>2–10</sup> thereby allowing diverse applications in the visible and infrared regimes, such as in quantum information,<sup>11</sup> microscopy,<sup>12</sup> and micromanipulation.<sup>13,14</sup> To bring these applications down to the nanometer scale,<sup>15</sup> different methods have been proposed to generate extreme-ultraviolet and x-ray vortex beams via high-order harmonic generation in gas-phase atomic targets,<sup>16–18</sup> plasmas,<sup>19–21</sup> and helical undulators.<sup>22–24</sup> Recently, extensive studies have been devoted to the generation of  $\gamma$  photons,<sup>25–30</sup> which could have promising applications in nuclear physics and astrophysics. Twisted photons of MeV energies can be used to study nuclear resonance fluorescence and photonuclear reactions that depend on angular momentum selection rules<sup>30</sup> and to manipulate the rotation of nuclei by absorbing high-order

twisted photons at energies below those of giant dipole resonances.<sup>26</sup> Twisted photons in the MeV to GeV range may play a significant role in investigating the spin puzzle<sup>31</sup> and phenomena around rotating massive astrophysical objects.<sup>32–36</sup>

To generate a  $\gamma$ -ray vortex, Compton/Thomson scattering of light with angular momentum off ultrarelativistic electrons is a promising method. It has been analytically demonstrated that the radiation field of an electron moving along the ideal spiral trajectory has a periodic azimuthal dependence, i.e., accommodates OAM, and the ratio of angular momentum to energy is  $\ell/\omega_0$  for all harmonics.<sup>37</sup> Physically, this is realized for electron motion in a circularly polarized plane laser field<sup>30</sup> or in a helical undulator.<sup>22</sup> To upshift the energy of the radiation to the  $\gamma$ -photon energy range, nonlinear Thomson scattering of a circularly polarized laser beam on an ultrarelativistic electron is considered.<sup>30</sup> Photons of circularly polarized light carry  $\hbar$  spin angular momentum (SAM), and during multiphoton Thomson scattering the SAM of  $\tilde{n}$  absorbed laser photons ( $\tilde{n}\hbar$ ) is transferred to the emitted photon. Since the total absorbed angular momentum is transferred to both the spin and orbital angular momentum of the radiation, each photon of the  $n$ th harmonic can only carry  $(n - 1)\hbar$  OAM. Unlike light with SAM, twisted light with OAM can provide a more efficient way to generate

high-order twisted photons. Each twisted laser photon has  $\ell \hbar$  ( $\ell = \pm 1, \pm 2, \dots$ ) OAM, which can be directly transferred to radiation. Linear Thomson/Compton scattering of twisted light/photons has been demonstrated with a single plane wave electron in a quantum description<sup>26</sup> and with an electron beam in a classical treatment.<sup>29</sup> The information of the OAM of the driving laser field is encoded in the transverse structure of the field, which can be successfully sensed by a single plane wave electron in the quantum description<sup>26</sup> or by the electron beam in the classical description.<sup>29</sup> Therefore, in both cases, the OAM of the driving field is transferred to the radiation field. A question arises as to how the OAM transfer from the driving field to the radiation in Thomson scattering will change in the case of a non-plane-wave single electron, described quantum mechanically as a limited transverse wavepacket, or equivalently in the case of a single electron in the classical description.

In this paper, we study nonlinear Thomson scattering of twisted light in the classical regime in the cases of a single electron and of an electron beam. The laser intensity parameter  $\xi = eE_0/mc\omega_0 = 1$  is assumed, with laser field amplitude  $E_0$  and frequency  $\omega_0$ , which allows nonlinear interaction, while, at the same time, the formation length of the radiation can be comparable to the laser wavelength. Whereas in the single-electron case, only SAM of the driving field is transferred to the radiation, in the case of an electron beam with a large transfer profile, both SAM and OAM are transmitted to the emitted radiation. In the latter case, the twisted radiation consists of harmonic components that carry  $\ell \hbar$  OAM per photon of the  $n$ th harmonic.

## II. THOMSON SCATTERING OFF A SINGLE ELECTRON

The twisted light considered here is a linearly polarized Laguerre–Gaussian (LG) propagating along the  $z$  direction. The laser beam under the paraxial approximation takes the form<sup>38</sup>

$$E_{p\ell}(r, \phi, z) = i\omega_0 \left[ (\alpha\hat{x} + \beta\hat{y})u_{p\ell} - \frac{i}{k_0} \left( \alpha \frac{\partial u_{p\ell}}{\partial x} + \beta \frac{\partial u_{p\ell}}{\partial y} \right) \hat{z} \right] \times \exp[i(\omega_0 t - k_0 z)] \exp \left[ - \left( \frac{2\sqrt{\ln 2} (\omega_0 t - k_0 z)}{\omega_0 \tau_p} \right)^2 \right], \quad (1)$$

where  $k_0 = 2\pi/\lambda_0 = \omega_0/c$  is the wavevector,  $\tau_p$  is the pulse duration,  $T = 2\pi/\omega_0$  is the laser period,  $\sigma_z \equiv i(\alpha\beta^* - \alpha^*\beta)$  (with  $\sigma_z = 0$  for linear and  $\mp 1$  for right/left-hand circular polarization), and

$$u_{p\ell}(r, \phi, z) = \frac{C}{(1 + z^2/z_R^2)^{1/2}} \left( \frac{r\sqrt{2}}{w(z)} \right)^\ell L_p^\ell \left( \frac{2r^2}{w^2(z)} \right) \times \exp \left[ - \frac{r^2}{w^2(z)} \right] \exp \left[ \frac{-ik_0 r^2 z}{2(z^2 + z_R^2)} \right] \exp(-i\ell\phi) \times \exp \left[ i(2p + \ell + 1) \tan^{-1} \left( \frac{z}{z_R} \right) \right], \quad (2)$$

where  $w(z) = \sqrt{1 + z^2/z_R^2}$ , the Rayleigh length  $z_R = \pi w_0^2/\lambda_0^2$  (with  $w_0$  the laser waist size), and

$$C = \sqrt{\frac{2p!}{w_0^2 \pi (p + |l|)!}}.$$

The laser parameters are set as  $\xi = 1$ ,  $\lambda_0 = 1 \mu\text{m}$ ,  $\ell = 1$ ,  $w_0 = 2\lambda_0$ , and  $\omega_0 \tau_p/2\pi = 6$ .

We consider that the laser beam is scattered by a counterpropagating ultrarelativistic single electron initially located at  $x = \lambda_0$ ,  $y = \lambda_0$  with energy  $\gamma = 10$ . The ratio of angular momentum to energy of the radiation is calculated as<sup>39,40</sup>

$$\frac{\langle dL_z/dt \rangle}{\langle dW/dt \rangle} = \frac{\int_{\text{shell}} (\vec{r} \times \langle \vec{S} \rangle) r^2 d\Omega}{c \oint \langle \vec{S} \rangle \cdot d\vec{\sigma}}, \quad (3)$$

where  $\langle \vec{S} \rangle = (c/4\pi) \text{Re}(\vec{E} \times \vec{H}^*)$  is the time-averaged Poynting vector,  $d\vec{\sigma} = r^2 \sin\theta d\theta d\phi \vec{n}$  is the surface element of the spherical surface at distance  $r$  from the origin,  $\vec{n} = \vec{r}/r$  is the direction toward the observer, and  $\Omega$  is the solid angle. For the  $n$ th harmonic,<sup>37</sup>

$$\frac{\langle dL_{nz}/dt \rangle}{\langle dW_n/dt \rangle} = \frac{\int_{\text{shell}} (\vec{r} \times [\text{Re}(\vec{E}_n \times \vec{H}_n^*)]) r^2 d\Omega}{c \oint [\text{Re}(\vec{E}_n \times \vec{H}_n^*)] \cdot d\vec{\sigma}}. \quad (4)$$

$\vec{E}_n$  and  $\vec{H}_n$  are the Fourier components of the electric and magnetic fields, which can be expressed as

$$\vec{H}_n = \vec{A}_n \times \vec{n} \left( \frac{1}{r^2} - \frac{ik}{r} \right) e^{ikr}$$

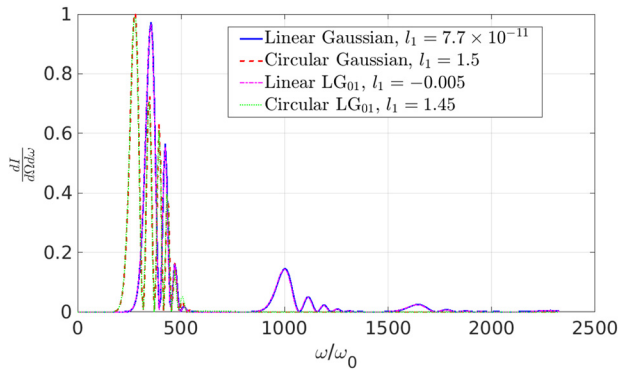
$$\vec{E}_n = \frac{i}{k} \left[ \vec{A}_n \left( \frac{k^2}{r} + \frac{ik}{r^2} - \frac{1}{r^3} \right) e^{ikr} + \vec{n} \left( \vec{n} \cdot \vec{A}_n \right) \left( - \frac{k^2}{r} - \frac{3ik}{r^2} + \frac{3}{r^3} \right) e^{ikr} \right], \quad (5)$$

where  $k = n\omega_0/c$  is the wavevector of the  $n$ th harmonic, and

$$\vec{A}_n = \frac{e}{c} \int_{-\infty}^{+\infty} \vec{v} e^{in\omega_0(t - \vec{n} \cdot \vec{r}/c)} dt, \quad (6)$$

with electron coordinate  $\vec{r}(t)$  and velocity  $\vec{v}(t)$ . In calculating the angular momentum of radiation, one cannot drop the terms of high order in  $1/r$ , even in the far field. Otherwise, the radiation will be approximated as a transverse field, leading to a linear momentum along the propagation direction, with a vanishing angular momentum of radiation. High-order terms give a small component along  $\vec{n}$ , resulting in an azimuthal component of linear momentum. Therefore, a nontrivial angular momentum independent of distance  $r$  is obtained when  $1/r^2$  terms are included.

The ratio between the angular momentum and the energy of the fundamental frequency for a linear and circular Gaussian laser beam and Laguerre–Gaussian laser field in the LG<sub>01</sub> mode are calculated respectively with Eq. (4), and the results are shown in Fig. 1. The angular momentum of radiation for a circular Gaussian beam and a circular LG<sub>01</sub> beam is qualitatively consistent with the prediction of the analytical solution given in Ref. 37, indicating that SAM of the laser photons is transferred to the angular momentum of the radiation field. However, the ratio  $\omega_0 \langle dL_{nz}/dt \rangle / \langle dW_n/dt \rangle \ll 1$  for both the linearly polarized Gaussian and LG<sub>01</sub> beams. The



**FIG. 1.** Normalized radiation intensities of linearly polarized Gaussian (solid line), linearly polarized LG<sub>01</sub> (dot-dashed line), circularly polarized Gaussian (dashed line), and circularly polarized LG<sub>01</sub> (dotted line) laser beams. The data show the ratio of angular momentum and energy  $R = \langle dL_{nz}/dt \rangle / \langle dW_r/dt \rangle$  multiplied by the fundamental frequency of the laser mode,  $\omega_0$ . The laser and electron parameters are  $\xi = 1$  and  $\gamma = 10$ .

latter indicates that in the case of a single electron, the OAM of the driving laser photons is not transferred to the radiation field, although the formation length of the photon emission is comparable to the laser wavelength.

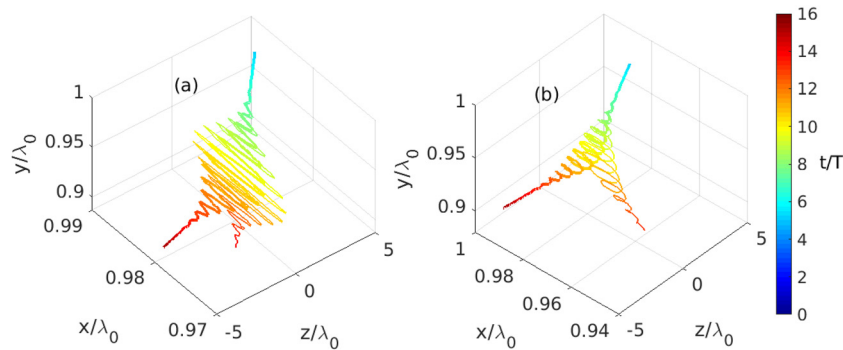
For understanding of the last property, we analyze the electron trajectories. In Fig. 2, the trajectories of the electron initially located at  $x_0 = 0.5w_0$ ,  $y_0 = 0.5w_0$  (the peak of the LG<sub>01</sub> beam) are shown in different laser fields. It appears that the trajectories of an electron in twisted and nontwisted light are quite similar as long as the lasers have the same polarization. Specifically, an electron in a linear Gaussian or linear LG<sub>01</sub> beam oscillates along the  $x$  direction and moves in the  $z$  direction, as shown in Fig. 2(a). An electron in a circular Gaussian or circular LG<sub>01</sub> beam rotates in the polarization plane and moves in the  $z$  direction, as shown in Fig. 2(b). Therefore, in the case of a single electron, its classical trajectories are mostly determined by the polarization property of the driving laser field (which defines the SAM of the field), but not by the large-scale transverse property of the field (which defines the OAM of the field). SAM of

the laser field photons can be transferred to radiation by a single classical electron, but OAM cannot. The spiral motion of an electron in a circularly polarized Gaussian beam can produce a twist in radiation. Meanwhile, the OAM is a property encoded in the phase structure of the laser beam. This property of the laser beam can be transferred to the radiation if the electron is exposed to the full phase structure of the laser beam with OAM. In the case of a single electron, this is possible only in a quantum description with a large size of the electron wavepacket encompassing the field structure.

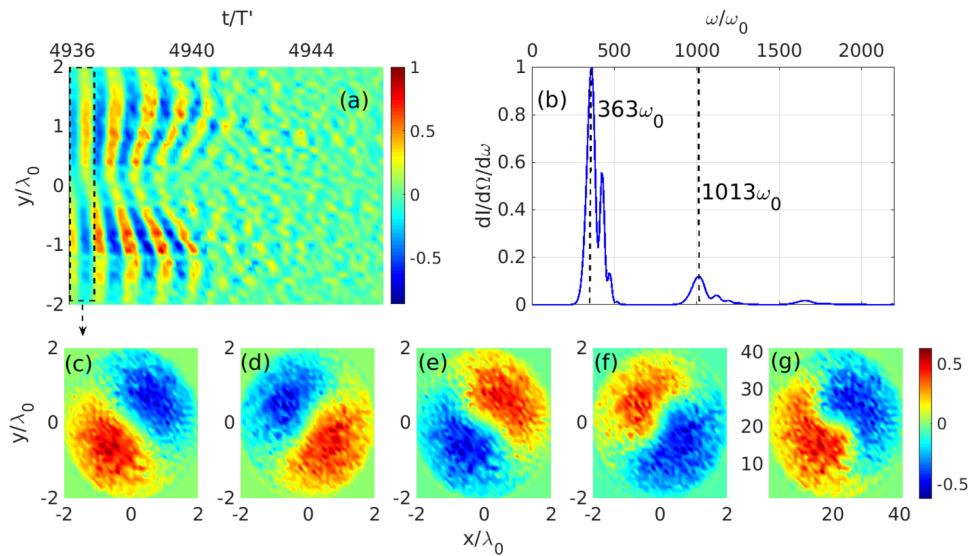
For linear Compton scattering, Thomson/Compton scattering of twisted light by an ultrarelativistic plane wave electron (an infinite transverse wavepacket) has been investigated quantum mechanically in Refs. 25–27, and the results suggest that the projection of the angular momentum of the emitted twisted photon on the propagation direction is the same as that of the incoming photon. Nonlinear Thomson/Compton scattering of an intense twisted laser field is difficult to treat fully quantum mechanically, but classical simulation with an electron beam, which mimics the plane wave case, may shed some light on this problem.

### III. THOMSON SCATTERING OFF AN ELECTRON BEAM

In this section, we consider nonlinear Thomson scattering of a twisted strong laser beam off an electron beam. The laser considered here is a linearly polarized LG<sub>01</sub> beam, with  $\xi = 1$ ,  $\lambda_0 = 1 \mu\text{m}$ ,  $\ell = 1$ ,  $w_0 = 2\lambda_0$ , and  $\omega_0\tau_p/2\pi = 6$ . The  $2 \times 10^4$  electrons are initially distributed in a sheet located at  $z = 0$ . Electrons are randomly distributed with a transverse spatial Gaussian distribution  $f(x) = (2\pi\sigma_\perp^2)^{-1/2} \exp(-x^2/2\sigma_\perp^2)$ ,  $\sigma_\perp = 1.2\lambda_0$ . The radius of the beam is  $2\lambda_0$ , which means that only electrons with coordinates  $(x, y)$  that fulfill  $\sqrt{x^2 + y^2} < r$  are accepted. The initial energy of the electron is  $\gamma = 10$ . The radiation fields at  $z = -13.6\lambda_0$  are shown in Fig. 3. For the chosen parameters  $\gamma \gg \xi$ , the emission is well collimated and the frequency of the radiation is dominated by the Doppler shift corresponding to the forward emission. Harmonics in the not very strong intensity regime ( $\xi \sim 1$ ) are weaker than those at the fundamental frequency; see Fig. 3(b). Therefore, the radiation field is governed by the fundamental frequency of forward emission. In fact, the radiation spectrum in



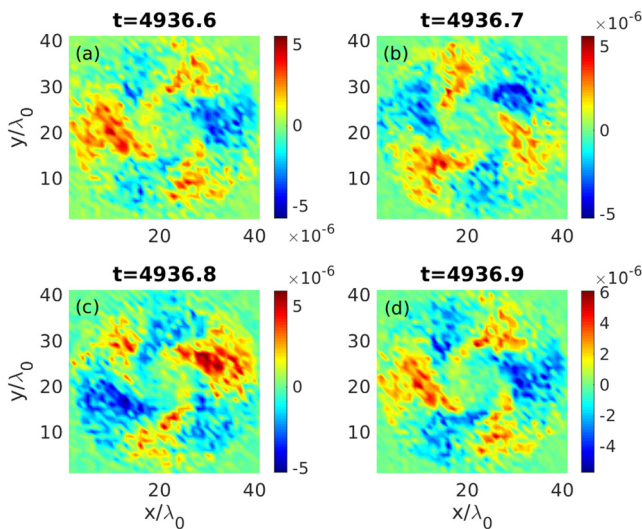
**FIG. 2.** Trajectory of an electron in linearly polarized (a) and circularly polarized (b) Gaussian (thick line) and LG<sub>01</sub> (thin line) beams for  $0 < t < 16T$ . The color scale represents time  $t$  in laser periods  $T$ .



**FIG. 3.** (a) Time evolution of the radiation field at  $x=0$ ,  $z=-13.6\lambda_0$ . (b) Spectrum of the electron beam. (c)–(f) Intensity distribution of the radiation field in the  $x$ - $y$  plane during one rotating loop [4936 $T'$ , 4937 $T'$ ], with a time interval  $dt = 1.0 \times 10^{-18}$  s.  $T' = T/363$  is the period of the radiation.

Fig. 3(b) is dominated by the corresponding peak at the upshifted fundamental frequency  $\omega' = 363\omega_0 \sim 4\gamma^2\omega_0$ . The transverse distribution exhibits a rotating pattern characterized by topological charge  $\ell = 1$ , which is repeated in a period  $T' \sim T/363$ . More interestingly, the third harmonic with frequency  $\omega'_3 \sim 1$  keV has an LG<sub>03</sub>-like transverse pattern, as shown in Fig. 3. Therefore, the radiation field of the nonlinear Thomson scattering of twisted light on an electron beam is twisted, and the harmonic order is proportional to the topological charge (Fig. 4). Thus, twisted  $\gamma$ -ray

radiation can be generated with ultrarelativistic electrons with energies in the several hundreds of MeV to GeV range, with a high OAM of radiation. One can conjecture from the classical calculation that nonlinear Compton/Thomson scattering of twisted photons off an ultrarelativistic single plane wave electron would provide high-order twisted photons with topological charge proportional to the harmonic order. The electron in the considered setup for OAM transfer can be quantum mechanically described as a plane wave, if the transverse size of the electron wavepacket  $\Delta r_{\perp e}$  is comparable to the characteristic transverse size  $r_{\perp f}$  of the driving field, which contains information on OAM. In this case, the uncertainty relation  $\Delta p_{\perp} \Delta r_{\perp e} \sim \hbar$  limits the momentum spread of the electron wavepacket,  $\Delta p_{\perp} \sim \hbar/r_{\perp f}$ . Assuming  $r_{\perp f} \sim \lambda_0 \sim 10^{-4}$  cm  $\sim 10^4$  a.u., one finds  $\Delta p_{\perp} \sim 10^{-4}$  a.u.



**FIG. 4.** Time evolution of the transverse distribution of the radiation of the third harmonic during a time interval  $T/3$ , where  $T'$  is the period of the fundamental frequency.

#### IV. CONCLUSION

Nonlinear Thomson scattering of light with OAM is significantly different from that of light with SAM. We provide a classical description of radiation at Thomson scattering. In the case of a single classical electron, only the SAM of the driving laser field can be transferred to radiation. Meanwhile, in the classical description, the OAM of the laser field can be transferred to radiation only when an electron beam is used. In the latter case, the radiation field has well-defined OAM with  $\langle dL_{nz}/dt \rangle / \langle dW_n/dt \rangle = \ell/\omega'_0$ . Each photon in the radiation field will be twisted, if, in addition, the formation length of the radiation is large enough that all phase structure of the driving field contributes coherently to the formation of the emitted photon. Otherwise, the produced photons are not twisted, and the OAM will constitute a collective property of the radiation beam. Moreover, a part of the OAM absorbed from the laser beam may also be shared with electrons.<sup>41</sup>



Based on these classical results, we make some conjectures regarding the quantum description. Specifically, in the quantum description, an electron with limited momentum spread has a large wavepacket and experiences the complete spatial structure of the laser beam. This corresponds physically to the classical case of an electron beam. In fact, quantum study of nonlinear Compton/Thomson scattering of twisted photons off a single ultrarelativistic plane wave electron<sup>25,26</sup> has shown the generation of twisted photons. In contrast, a single electron quantum wavepacket with a large momentum spread and a small wavepacket size cannot experience the complete structure of the laser beam, corresponding to the case of a single classical electron. In this case, the emitted photon will not be twisted.

Thus, nonlinear Thomson scattering of twisted light provides the possibility of generating highly twisted  $\gamma$  photons with much higher efficiency than schemes using circular polarized Gaussian beams. In fact, twisted light with topological charge  $l$  ( $=1, 2, 3, \dots$ ) carries  $lh$  OAM per photon, while each photon of a circularly polarized Gaussian beam has only  $\hbar$  SAM. Thus, if the electrons absorb the same number of photons, twisted light can transfer more angular momentum to radiation. Moreover, in a circularly polarized Gaussian beam, the OAM of the radiated photon due to the  $n$ -photon Thomson process is only  $(n - 1)\hbar$ , because the emitted photon itself has  $\hbar$  SAM. Since in the considered classical regime  $\xi \sim 1$  the fundamental frequency component of the radiation is generally more intense than the harmonics, the emission of twisted  $\gamma$  photons with a circular Gaussian beam would be inefficient. The generated twisted  $\gamma$  photons may have potential applications in nuclear physics and laboratory astrophysics.

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