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Research Article

The theory of early nonlinear stage of $m=1$ instability with locally flattened q-profile

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Abstract

In this paper, we analyze the modification of fast particles on the nonlinear radial displacement of $m = 1$ internal kink mode with a shoulder-like equilibrium current theoretically. Using the matching method on the solutions of the outer and inner regions, we derive the analytical form of nonlinear radial displacement in the limit of $q' = q'' = 0$, which is valid to the cases of weak shear due to a slight flattening of the $q(r)$ profile around $q = 1$. We have taken into consideration the effects of the circulating and trapped fast particles on the nonlinear state of the mode. It is found that a fast particle can modify the nonlinear saturation level by the change of potential energy, depending on the fast particle properties. By the matching of linear dispersion relation to early nonlinear result, we also obtain the relations of radial displacement to the mode frequency and linear growth rate, and discuss the scaling for different stabilities of the MHD modes.

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1. Introduction

The shoulder-like current profile with the locally low shear at $q = 1$ surface has been firstly observed in TEXTOR tokamak plasmas [1]. In the absence of fast particles, the linear properties of internal kink mode for a locally flattened q-profile at $q = 1$ surface have been studied theoretically and numerically [2,3]. It is found that the linear internal kink mode is unstable due to the destabilization of both the current and pressure gradients [4]. More recently, under a similar equilibrium, the stability of kink/tearing mode was discussed by Connor et al. [5]. Theoretically, the linear internal kink instabilities, including the toroidal effect, have been widely studied by variational principle method for a finite magnetic shear, but the nonlinear theory of the mode for a locally zero magnetic shear has not been well developed [4,6]. In addition, there is little work to investigate the nonlinear saturation amplitude of the $m = 1$ instability with fast particles in this special configuration, which may be important in ITER-like plasmas with lower hybrid current drive (LHCD) [7].

In this paper, we will focus on the nonlinear $m = 1$ internal kink modes to derive a relation of the nonlinear mode amplitude and to further explore the role of fast particles for a locally modified current equilibrium. Then we will derive the modified saturated amplitude of the mode including kinetic effect of energetic particles (EPs) by matching the solution of inertial layer to that in the outer region, where inertia is negligible. We will also present the saturation amplitude of the mode modified by magnetohydrodynamic (MHD) and fast particle due to the wave–wave interactions.

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The paper is organized as follows. In Sec. 2, we will derive the effect of MHD nonlinearity on the modes by asymptotic matching and discuss the modification of locally flattened q-profile on mode amplitude by the fast particles in the limit of $q_s^{'} = q_s^{''} = 0$. To further clarify the effect of mode stability and frequency on the nonlinear mode amplitude, in Sec. 3, we will discuss the relation of different stabilities of the MHD modes. Finally, we will make a summary in Sec. 4.

2. Derivation of nonlinear mode amplitude with EPs

To begin the derivation, we use a locally flattened q-profile as [2].

$$q(r) = 1 + 2\left(r^2 - r_s^2\right) - 2\left(r^2 - r_s^2\right)\exp\left[-C_0\left(r^2 - r_s^2\right)^2\right] \qquad (1)$$

where $q(r_s) = 1$ and the constant C_0 is set to control the flatness of the q-profile, r_s is the radial location of $q = 1$ rational surface. For a slightly flattened q-profile as $C_0 \to \infty$, Eq. (1) becomes a parabolic function $q(r) \approx 1 + 2(r^2 - r_s^2)$, which is similar to the form of the monotonic q-profile. The pressure profile is $p(r) = p_0[1 - (r/a)]^2$ with p_0 being the pressure value at axis and a being the minor radius [2]. Due to the important role of EPs on the linear mode, it is necessary to include the EPs effect on the nonlinear analysis of mode. Following the previous works, the MHD and the kinetic contribution on potential energy can be written as [4,8,9].

$$\delta W_{\rm MHD} = \frac{1}{2}\int {\rm d}^3x\left[|\delta \boldsymbol{B}_\perp|^2/4 - \frac{j_\parallel}{c}\left(\boldsymbol{\xi}_\perp^* \times \boldsymbol{e}_\parallel\right)\cdot\delta\boldsymbol{B}_\perp - 2(\boldsymbol{\xi}_\perp\cdot\nabla p)\left(\boldsymbol{\xi}_\perp^*\cdot\boldsymbol{\kappa}\right) + B^2|\nabla\cdot\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2 + \Gamma p_c|\nabla\cdot\boldsymbol{\xi}_\perp|^2\right], \qquad (2)$$

$$\delta W_{\rm K} = -2^{9/2}\pi^3 m_h \int_0^{r_s} RBr{\rm d}r \int_{B_{\max}^{-1}}^{B_{\min}^{-1}} {\rm d}\alpha \int_0^{\infty} {\rm d}E\cdot E^{5/2}K_b\overline{J}^*\left(\frac{Q}{\omega - \overline{\omega}_{dh}}\right)\overline{J}, \qquad (3)$$

where $\boldsymbol{\xi}$ is the displacement vector, m_h is the energetic ion mass, $\boldsymbol{\kappa} = \partial\boldsymbol{e}_{||}/\partial l$ is the field line curvature, magnetic field $B = B_0(1 - r\cos\theta/R)$ with B_0 being B at axis and R being the major radius, Γ is the adiabatic index, p_c is the background plasma beta, $\overline{\omega}_{dh}$ is the averaged toroidal precession frequency, $K_b = \oint({\rm d}\theta/2\pi)(1 - \alpha B)^{-1/2}$, $\alpha = \mu/E$ with the magnetic moment μ and kinetic energy $E = v^2/2$, $Q = (\omega\partial_E + \widehat{\omega}_*)F_0$ with the equilibrium distribution of fast particles F_0, $\widehat{\omega}_* = -[i/\omega_c(\widehat{e}_{||} \times \nabla\ln F_0)]\cdot\nabla$ with the gyro-frequency ω_c, and $\overline{J}$ is the bounce average of $J = (\alpha B/2)\nabla\cdot\boldsymbol{\xi}_\perp - (1 - 3\alpha B/2)\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}$. $\omega = \omega_r + i\gamma$ with ω_r being the mode frequency and γ being the mode growth rate.

In previous work, the nonlinear wave–wave interaction in the inertial layer was found to play an important role in the dynamics of the $m = 1$ MHD mode [6]. In the finite positive shear case, the background ion in the layer can significantly modify the eigenvalue of the dispersion relation. Accordingly, we further study the local weak shear case and further derive the nonlinear saturation amplitude of the mode with energetic ions.

Following Ref. [6], flux function satisfies $\psi(r,\theta) = \psi[r - \xi(x,\theta)]$, where θ is the poloidal angle. Magnetic flux, thus, can be presented by Taylor formula expanded near the rational surface for the locally flattened q-profile as

$$\psi = \psi_0 + x\psi_0' + \frac{x^2}{2!}\psi_0'' + \frac{x^3}{3!}\psi_0^{(3)}..., \qquad (4)$$

where $\psi_0'' \equiv r_s(\boldsymbol{k}\cdot\boldsymbol{B})_s' = -B_\theta(r_s)q'(r_s)$, $x = (r - r_s)/r_s$. For the flattened q-profile, first non-vanishing derivative of $q(r_s)$ is up to the third order of the derivative, $q^{'''}(r_s)$, then we obtain ${\rm d}\psi/{\rm d}x = r_sB_0(x^3q^{'''}/6)/R$ by Taylor expanded near $q = 1$ surface for a resonant mode. Rescaling ψ by $(r_s/R)B_0q^{'''}/6$, it leads to

$${\rm d}\psi/{\rm d}x = x^3. \qquad (5)$$

In the layers, the nonlinear helical equilibrium is given by $\nabla^2\psi = J_z(\psi)$. By multiplying ${\rm d}\psi/{\rm d}r$, ${\rm d}({\rm d}\psi/{\rm d}r)^2/{\rm d}r = 2J_z(\psi)({\rm d}\psi/{\rm d}r)$ and integrating along r, we obtain $({\rm d}\psi/{\rm d}r)^2 = F(\psi) + G(\theta)$, where $F(\psi) = 2\int J_z(\psi'){\rm d}\psi'$, $G(\theta)$ are the function of integration. We further transform the unperturbed flux surface ψ to the equivalent surface $r = r_s$, and rewrite Eq. (5) as $({\rm d}\psi/{\rm d}x)({\rm d}x/{\rm d}r) = [f(x) + g(\theta)]^{1/2}$. Using $r - x = \xi$, the amplitude of radial displacement ξ has the form of

$$\partial\xi(x,\theta)/\partial x = x^3[f(x) + g(\theta)]^{-1/2} - 1. \qquad (6)$$

Using the limit of the incompressibility conditions, i.e. $\oint r{\rm d}r{\rm d}\theta = $ const., it requires $f(x)$ and $g(\theta)$ to satisfy

$$\oint [f(x) + g(\theta)]^{-1/2}{\rm d}\theta = x^{-3}. \qquad (7)$$

We integrate Eq. (6) on x and obtain the nonlinear displacement in the layer

$$\xi(x,\theta) = \int_0^x\left\{\frac{x^3}{[f(x) + g(\theta)]^{1/2}} - 1\right\}{\rm d}x + h(\theta), \qquad (8)$$

where the last term of right hand side of Eq. (8),$h(\theta)$, is the arbitrary function of θ. Further solution of matched asymptotic expansions is similar to Ref. [6], and it is found that $\xi(x,\theta)$ should be finite when $|x| \to \infty$ leading to $f(x) \to x^6$. Then the final form of Eq. (8) is

$$\xi(x,\theta) = \frac{1}{10}\frac{g(\theta)}{x^5} \pm \int_0^\infty\left\{\frac{x^3}{[f(x) + g(\theta)]^{1/2}} - 1\right\}{\rm d}x + h(\theta), \qquad (9)$$

where the sign "±" indicates $x \to \pm\infty$. This indicates the nonlinear perturbation in the inner layers around r_s for a weakly positive shear of $q_s^{'} = q_s^{''} = 0$. In order to match the solution to that in the exterior region, we also further give a linear ideal MHD solution for the outer plasma neglecting the inertia. Then the radial displacement in the region of $r<r_s$ for the $m=1$ Fourier component is determined by [6]

$$\frac{\partial\xi(r)}{\partial r}=\frac{\xi_0}{r^3(\boldsymbol{k}\cdot\boldsymbol{B})^2}\int_0^r\left[\hat{g}(r)+\hat{k}(r,\omega)\right]\mathrm{d}r, \tag{10}$$

and for $r>r_s$,

$$\frac{\partial\xi(r)}{\partial r}=\frac{\mathrm{const.}}{r^3(\boldsymbol{k}\cdot\boldsymbol{B})^2}, \tag{11}$$

where the MHD term has a form of $\hat{g}(r)=2k_z^2r^2p'+B_\theta^2k_z^2r(1-nq)(3+nq)$ and the nonlinear kinetic contribution $\hat{k}(r,\omega)$ is given by the kinetic energy principle [10], as

$$\hat{k}(r,\omega)=-2^{9/2}\pi^3m_hRBr\int_{B_{\max}}^{B_{\min}}\mathrm{d}\alpha\int_0^\infty\mathrm{d}E\cdot E^{5/2}K_b\bar{J}^*\frac{Q}{\omega-\bar{\omega}_{dh}}\bar{J}. \tag{12}$$

Since the kinetic effect of fast particles is only localized in the regions of $r<r_s$ for the internal kink modes [8,9], the asymptotic matching of different regions can be carried out. As the boundary conditions, when $|x|\to\infty$, the displacement in the inner layers satisfies $x\to\infty$, $\xi(\infty)=0$, and $x\to-\infty$, $\xi(-\infty)=\xi_0\cos\theta$, respectively. Using $\xi(r)=\xi_0+\tilde{\xi}(r)$, integrating Eq. (8), in the outer region when $r\to r_s^-$, we obtain the nonlinear displacement as

$$\xi(r)|_{r\to r_s^-}=\xi_0\left\{1-\frac{R_0^2}{5r_s^3B_0^2x^5(q_s'''/6)^2}\int_0^{r_s}\left[\hat{g}(r)+\hat{k}(r,\omega)\right]\mathrm{d}r\right\}\cos\theta. \tag{13}$$

On the other hand, if $r\to r_s^+$, the displacement with the boundary condition $\xi(r)=\tilde{\xi}(r)$ has a form of

$$\xi(r)|_{r\to r_s^+}=-\frac{R_0^2\xi_0}{5r_s^3B_0^2x^5({q'''}_s/6)^2}\int_0^{r_s}\left[\hat{g}(r)+\hat{k}(r,\omega)\right]\mathrm{d}r\cos\theta. \tag{14}$$

Further matching the outer layer solution to the inner layer solution, we find that the forms of $h(\theta)$ and $g(\theta)$ satisfy

$$h(\theta)=-\int_0^\infty\left\{\frac{x^3}{[f(x)+g(\theta)]^{1/2}}-1\right\}\mathrm{d}x=\frac{1}{2}\xi_0\cos\theta, \tag{15}$$

and

$$g(\theta)=-\frac{2R_0^2\xi_0}{r_s^3B_0^2(q_s'''/6)^2}\int_0^{r_s}\left[\hat{g}(r)+\hat{k}(r,\omega)\right]\mathrm{d}r\cos\theta, \tag{16}$$

respectively. To simplify the form of this equation and obtain an expression of ξ_0, we further expand $g(\theta)=\sum A_m\cos m\theta$ and use $g(\theta)\approx2\oint[g(\theta)\cos\theta]\mathrm{d}\theta\cos\theta$, then Eq. (16) becomes

$$\oint g\cos\theta\mathrm{d}\theta=-\frac{2\xi_0}{r_s({q'''}_s/6)^2}[\delta\hat{W}_{\mathrm{MHD}}+\delta\hat{W}_{\mathrm{K}}(\omega)], \tag{17}$$

where the potential energy is given in Eq. (3). Using $f(x)=f(-x)$ [6], the integral equation for $g(\theta)$ can be written as

$$\int_0^\infty\mathrm{d}f\left\{\left[\left(\oint\mathrm{d}\theta I_c^{-1/2}\right)^{-1}\right]^{-2/3}\left(\oint\mathrm{d}\theta I_c^{-3/2}\right)\left(\oint\mathrm{d}\theta I_c^{-1/2}\right)^{-3}\left(I_c^{-1/2}-\oint\mathrm{d}\theta I_c^{-1/2}\right)\right\}=-4\xi_0\cos\theta \tag{18}$$

where $I_c=[f(x)+g(\theta)]^{1/2}$. In order to derive nonlinear amplitude by Eq. (17), we multiply Eq. (18) by g and integrate it over θ, then we have

$$\langle g\cos\theta\rangle(\xi_0/r_s)^5=-\frac{2}{(r_s^3{q'''}_s/6)^2}\left[\delta\hat{W}_{\mathrm{MHD}}+\delta\hat{W}_{\mathrm{K}}(\omega)\right], \tag{19}$$

$$\oint g\mathrm{d}\theta\int_0^\infty\mathrm{d}f\left[\left(\langle I_c^{-1/2}\rangle^{-1}\right)^{-2/3}\langle I_c^{-3/2}\rangle\langle I_c^{-1/2}\rangle^{-3}\left(I_c^{-1/2}-\langle I_c^{-1/2}\rangle\right)\right]=-4\langle g\cos\theta\rangle, \tag{20}$$

respectively, where $\langle A\rangle=\frac{1}{2\pi}\oint A\mathrm{d}\theta$, f and g are rescaled by $g\to\xi_0^6g$, $f\to\xi_0^6f$. From Eqs. (19) and (20), the nonlinear displacement of the mode can be represented as

$$\frac{\xi_0}{r_s}=\left(\frac{6\sqrt{2}}{r_s^3q_s'''}\right)^{2/5}\left[-\delta\hat{W}_{\mathrm{MHD}}-\delta\hat{W}_{\mathrm{K}}(\omega)\right]^{1/5}\Pi_{\max}^{-1/5}, \tag{21}$$

with

$$\Pi=-\frac{1}{4}\oint g\mathrm{d}\theta\int_0^\infty\mathrm{d}f\left[\left(\langle I_c^{-1/2}\rangle^{-1}\right)^{-2/3}\langle I_c^{-3/2}\rangle\langle I_c^{-1/2}\rangle^{-3}\left(I_c^{-1/2}-\langle I_c^{-1/2}\rangle\right)\right]$$

and $\Pi_{\max}$ is the maximum value of Π.

3. Analysis and discussion

Neglecting the kinetic term in Eq. (21), the nonlinear radial displacement can be approximated to $\xi_0/r_s\propto(-\delta\hat{W}_{\mathrm{MHD}})^{1/5}$, which means a much weaker dependence of ξ_0/r_s on the potential energy than $\xi_0/r_s\propto(-\delta\hat{W}_{\mathrm{MHD}})$ of general internal kink mode. Thus, the nonlinear wave–wave interaction becomes more important by the slight flattening of q-profile in the early nonlinear stage of the internal kink modes.

Generally speaking, the background ion diamagnetic drift or precessional drift of trapped EPs produced during the external auxiliary heating can also drive the finite frequency branches of the modes, corresponding to the diamagnetic fishbone [8,11] or precessional fishbone [9,10], respectively. In the linear stage, the properties of the $m = 1$ mode excited by trapped fast particles have been discussed for locally flattened q-profile in previous works [12]. For a large fast ion beta β_h, the wave–particle interaction dominates the nonlinear dynamics of the mode, which is similar to the result of the trapped fast particle-excited modes [9]. On the other hand, however, the mode may saturate at a low level due to the stabilization of barely trapped fast particle or a smaller β_h with $\mathrm{Re}\delta\widehat{W}_\mathrm{K} \gtrsim 0$ [8,11].

In the previous linear theory, it was found that the growth rate of the mode could be determined by [12].

$$-\mathrm{i}\Lambda = \left(\frac{4}{3}\right)^{2/5}\left(\frac{m}{n}\right)^{1/5}\left(\frac{1}{r_\mathrm{s}^3 q_\mathrm{s}'''}\right)^{1/5}\left[-\delta\widehat{W}_\mathrm{MHD} - \delta\widehat{W}_\mathrm{K}(\omega)\right]^{3/5} \tag{22}$$

where $\Lambda = \sqrt{(1+\Delta)\omega(\omega-\omega_*)}/(m\omega_\mathrm{A})$, ω_* is ion diamagnetic frequency, the constant $(1+\Delta) = 3$ is the inertial enhancement factor [2]. By combining linear solution of Eq. (22) with nonlinear solution in the layer of Eq. (21), we find in the early nonlinear stage, the relation between radial displacement and mode frequency becomes

$$\frac{\xi_0}{r_\mathrm{s}} = C\left(\frac{1}{r_\mathrm{s}^3 q_\mathrm{s}'''}\right)^{1/3}\left[-\mathrm{i}\frac{\sqrt{3\omega(\omega-\omega_*)}}{\omega_\mathrm{A}}\right]^{1/3}, \tag{23}$$

where the constant parameter $C = (9/\sqrt{2})^{2/5}(4/3)^{4/15}\Pi_\mathrm{max}^{-1/5}$ and $m/n = 1$ is used. Neglecting ion diamagnetic drift effect, a weak dependence of ξ_0 on linear growth rate by the scaling of $\xi_0/r_\mathrm{s} \propto (\gamma/\omega_\mathrm{A})^{1/3}$ is explored and clearly different from the conventional kink/fishbone mode with a positive shear $\xi_0/r_\mathrm{s} \propto (\gamma/\omega_\mathrm{A})$ [2]. It is suggested that MHD nonlinearity rather than the linear fast particle driving in the layer dominates the mode saturation when the profile is locally flattened around the rational surface. The result of Eq. (22) is also valid to the pure growing mode with $\gamma >> \omega_\mathrm{r}$ in the absence of fast particles as $|\mathrm{Re}\delta\widehat{W}_\mathrm{K}| = 0$, the mode becomes general internal kink instability with $\mathrm{Re}\delta\widehat{W}_\mathrm{MHD} < 0$. On the other hand, for a marginal stable diamagnetic kink mode as $\gamma/\omega_\mathrm{A} \sim 0$ with $\omega_\mathrm{r} \sim \omega_*$, the mode amplitude depends on linear frequency with $\xi_0/r_\mathrm{s} \approx C(1/r_\mathrm{s}^3 q_\mathrm{s}''')^{1/3}(\omega_*/\omega_\mathrm{A})^{1/3}$, which indicates the relation of nonlinear mode displacement as $\xi_0/r_\mathrm{s} \propto (\omega_*/\omega_\mathrm{A})^{1/3}$. Moreover, we also find that ξ_0/r_s significantly depends on the zero magnetic shear width (δ/r_s with $q' = 0$) due to $\delta/r_\mathrm{s} \sim (\gamma/\omega_\mathrm{A})^{1/3}$, when $|\delta\widehat{W}_\mathrm{MHD}| << |\mathrm{Re}\delta\widehat{W}_\mathrm{K}|$. The flatness of q-profile determined by the parameter of $r_\mathrm{s}^3 q'''$ has a relation of $\xi_0/r_\mathrm{s} \propto (r_\mathrm{s}^3 q_\mathrm{s}''')^{-2/5}$. It has to be pointed out that, for simplicity, the modifications of larger ion Larmor radius and finite plasma resistivity may be significant in this case but not reported here [8].

4. Summary

In this paper, we have studied the nonlinear properties of internal kink mode with energetic particles in the flattened q-profile regimes of $q_\mathrm{s}' = q_\mathrm{s}'' \equiv 0$. Matching the solutions of the outer and inner regions to each other, we have also derived the analytical form of nonlinear displacement of the mode and represented it as a function of linear mode frequency. Since there is the weak current/pressure gradient driving in the current-flattening equilibrium, the fast particle properties play an important role in the nonlinear dynamic of mode saturation by changing the potential energy in the outer region. Neglecting ion diamagnetic drift effect, a weak dependence of ξ_0 on linear growth rate by the scaling of $\xi_0/r_\mathrm{s} \propto (\gamma/\omega_\mathrm{A})^{1/3}$ is explored and clearly different from conventional kink/fishbone mode with a positive shear $\xi_0/r_\mathrm{s} \propto (\gamma/\omega_\mathrm{A})$. The result also indicates that the mode saturation amplitude is dominated by the layer nonlinearity rather than the linear driving when the q-profile is flattened around $q = 1$.

Conflict of interest

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, "The theory of early nonlinear stage of m=1 instability with locally flattened q-profile".

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References

[1] H. Soltwisch, W. Stodiek, J. Manickam, J. Schluter, Current density profiles in the TEXTOR tokamak, Proc. 11th International Conf. On Plasma Physics and Controlled Nuclear Fusion Research, Kyoto, IAEA, Vienna, 1986, p. 263.
[2] R.J. Hastie, T.C. Hender, B.A. Carreras, L.A. Charlton, J.A. Holmes, Stability of ideal and resistive internal kink modes in toroidal geometry, Phys. Fluids 30 (1987) 1756.
[3] A. Bondeson, G. Vlad, H. Lütjens, Ideal MHD stability of internal kinks in circular and shaped tokamaks, Phys. Fluids B1 (1992) 1889.
[4] M.N. Bussac, R. Pellat, D. Edery, J.L. Soule, Internal kink modes in toroidal plasmas with circular cross sections, Phys. Rev. Lett. 35 (1975) 1638.
[5] J.W. Connor, R.J. Hastie, I. Pusztai, P.J. Catto, M. Barnes, High-*m* kink/tearing modes in cylindrical geometry, Plasma Phys. Contr. Fusion 56 (2014) 125006.
[6] M.N. Rosenbluth, R.Y. Dagazian, P.H. Rutherford, Nonlinear properties of the internal kink instability in the cylindrical tokamak, Phys. Fluids 16 (1973) 1894.
[7] A. Fasoli, C. Gormenzano, H.L. Berk, B. Breizman, S. Briguglio, et al., Physics of energetic ions, Nucl. Fusion 47 (2007) S264.
[8] B. Coppi, F. Porcelli, Theoretical model of fishbone oscillations in magnetically confined plasmas, Phys. Rev. Lett. 57 (1986) 2272.

[9] L. Chen, R.B. White, M.N. Rosenbluth, Excitation of internal kink modes by trapped energetic beam ions, Phys. Rev. Lett. 52 (1984) 1122.
[10] R.B. White, L. Chen, F. Romanelli, R. Hay, Trapped particle destabilization of the internal kink mode, Phys. Fluids 28 (1985) 278.
[11] F. Porcelli, Fast particle stabilization, Plasma Phys. Contr. Fusion 33 (1991) 1601.
[12] X.Q. Wang, R.B. Zhang, G. Meng, Trapped fast particle destabilization of internal kink mode for the locally flattened q-profile with an inflection point, Phys. Plasmas 23 (2016) 074506.