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Research Article

Theoretical simulation of high-voltage discharge with runaway electrons in sulfur hexafluoride at atmospheric pressure

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Abstract

The results of theoretical simulation of runaway electron generation in high-pressure pulsed gas discharge with inhomogeneous electric field are presented. Hydrodynamic and kinetic approaches are used simultaneously to describe the dynamics of different components of low-temperature discharge plasma. Breakdown of coaxial diode occurs in the form of a dense plasma region expanding from the cathode. On this background there is a formation of runaway electrons that are initiated by the ensemble of plasma electrons generated in the place locally enhanced electric field in front of dense plasma. It is shown that the power spectrum of fast electrons in the discharge contains electron group with the so-called "anomalous" energy.

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Keywords: Pulsed gas discharge; High-pressure gas breakdown; Fast electron in gas discharge; Runaway electron in plasma

1. Introduction

The phenomenon of runaway electrons generation in high-pressure gas discharges is widely studied in recent years [1−4]. Mostly it is connected to the progress in the field of high-voltage pulse generation with a short rise time of the voltage amplitude and the appearance of experimental equipment with picosecond time resolution [1,2].

The main factor affecting the amount of fast electrons is the possibility of creating a strong overvoltage of a discharge gap at the initial stage of the current growth in the gas diode. Multiple overvoltage (compared to static breakdown voltage) is achieved for a short time during the application of a large amplitude voltage pulse with a sub-nanosecond duration of the leading edge to the discharge gap. At present the fact of fast (runaway) electrons detection can be firmly established at the initial stage of high pressure gas breakdown in gaps with strongly non-uniform distribution of the electric field. At the same time, researchers have obtained fast electron current pulses with largely spread parameters: amplitudes from 0.1 up to tens of amperes with durations from tens of picoseconds to nanoseconds [1,2]. As the number of runaway electrons depends strongly on several critical parameters, i.e., gas type, geometric enhancement of the electric field near the sharp edges of electrodes, scales of field-enhancement regions, time resolution of the experimental equipment, the experimental results of fast electrons detection will also differ considerably.

Essential non-stationarity and spatial three-dimensionality in real experiments pose a great challenge for theoretical modeling. Simple zero-dimensional and one-dimensional theoretical models [4,5] could demonstrate the mechanism

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of runaway electron beam formation, but do not allow a proper comparison with the existing experiments.

The simplest geometry was chosen for theoretical analysis of the discharge having strongly inhomogeneous electric field. We have applied a previously developed hybrid theoretical approach (refer to Ref. [4]) to coaxial geometry of sub-nanosecond discharge gap in sulfur hexafluoride (SF_6) at atmospheric pressure.

This gas has a molecule with a high energy of electron affinity that facilitates rapid attachment of free electrons to the molecule forming stable negative ions. Attachment of free electrons leads to a significant increase of static breakdown value of the reduced electric field strength. For atmospheric pressure, it equals to 89 kV•cm^{-1}•atm, which is more than twice higher of breakdown fields in pure nitrogen (35 kV•cm^{-1}•atm). Moreover, the conversion of a free-electron conductivity of plasma into being ionic will drastically reduce plasma column conductivity, beneficial to maintain a relatively high electric field strength. All of the above improves the probability of transition electrons entering into the continuous acceleration regime. Although complex molecule SF_6 has a relatively high elastic cross-section, the high field strength allows us to observe a certain number of fast electrons [6,7].

2. Model of gas discharge with runaway electrons flow

2.1. Description of discharge dynamics

Main physical approximations of the discharge model and the corresponding computational method of runaway electrons distribution function have been previously described with respect to one-dimensional planar discharge geometry in nitrogen in our paper [4]. Here we have applied this model to one-dimensional axisymmetric geometry of the discharge gap filled with electronegative gas.

Since in all cases of calculations all significant processes occurred at characteristic sub-nanosecond time scales, we neglected the motion of ions in the mathematical description of current transporting (special calculations have shown that taking into account the ion motion does not lead to any significant changes in results).

In discharge column production and attachment of electrons are described by the continuity equation for electrons density n_e under the drift-diffusion approximation:

$$\frac{\partial n_e}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(w_e n_e - D_e\frac{\partial n_e}{\partial r}\right)\right] + (\alpha - \eta)w_e n_e. \quad (1)$$

Here $w_e(E)$ and D_e are the electrons drift velocity and scalar diffusivity coefficient respectively, $\alpha(E)$ and $\eta(E)$ are Townsend and attachment coefficients as functions of local electric field strength $E(r)$. Graphs of these functions are shown in Fig. 1. All of these parameters for electrons kinetics calculations in SF_6 were taken from Ref. [8].

Evolution of the electric field strength was calculated from 1D-axisymmetric total current ($J(t)$) conservation law

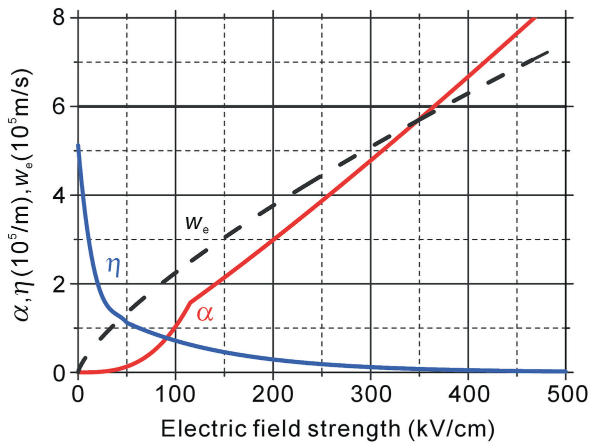

Fig. 1. Electric field dependences of Townsend and attachment coefficients α, η and electron drift velocity w_e for atmospheric pressure.

$$\varepsilon_0\frac{\partial E}{\partial t} = \frac{J(t)}{2\pi r} + e\left(w_e n_e - D_e\frac{\partial n_e}{\partial r}\right), \quad (2)$$

here, e is elementary charge, ε_0 is vacuum absolute permittivity. The equivalent electric discharge circuit is made of a connected in-series pulse voltage source $U_0(t)$, a ballast resistor R, and a coaxial discharge gap (the cathode and anode radii are r_c and r_a respectively, the tube length is L) filled with gas at atmospheric pressure (101 kPa). Running current $J(t)$ per unit length has been calculated from the Kirchhoff's equivalent equation for discharge circuits:

$$J(t) = \frac{1}{LR}\left[U_0(t) - \int_{r_c}^{r_a} E(r,t)\mathrm{d}r\right]. \quad (3)$$

The system of Eqs. (1)–(3) describes the evolution of the initial distribution for a given shape of external voltage pulse source $U_0(t)$. We set the initial low level of electron density equal to $n_e(r, 0) = n_0(r)$ and zero initial electric field. We also applied zero Neumann boundary conditions to the electron number density ($\partial n_e(0, t)/\partial r = 0$) in order to omit detailed description of electrons emission.

Numerical solution of Eqs. (1)–(3) is based on specific discretization procedures. The purpose of this method is to reduce the partial differential equation system into the system of ordinary differential equations (Method of Lines) [9] and to solve it using common ODE solvers (i.e. RKF45). As for the fluxes discretization in the continuity Eq. (1), we applied WENO-3-LF [10] scheme in the framework of finite volume method [11] for convective terms.

2.2. Description of runaway electrons

Runaway electron dynamics is described as collisionless transport in previously computed electric field $E(r, t)$, and any possible collision leads to the loss of the runaway electron. For a high-voltage discharge in atmospheric SF_6, the relativistic Boltzmann kinetic equation was used:

$$\gamma\left[\frac{\partial f}{\partial t}+\frac{p}{m\gamma}\frac{\partial f}{\partial r}-eE(r,t)\frac{\partial f}{\partial p}\right] = \alpha(E)w_e(E)n_e(r,t)f_0(p)-n_g v\sigma^*(p)f(p). \quad (4)$$

Here, $f(r, p, t)$ is the spatial non-uniform runaway electron distribution function, m is the electron mass, $p = mv\gamma$ is an electron momentum (where v is the runaway electron velocity), $\gamma = \sqrt{1+(p/mc)^2}$ is the relativistic factor, $f_0(p)$ is the Maxwell distribution supposed to be for new electrons initially, n_g is the gas molecule density, $\sigma^*(p)$ is the momentum-dependent transport electron cross-section [12] with asymptotic extrapolation for high energies.

Numerical solution of Eq. (4) is based upon Ref. [13]. Numerical code was implemented in Mathworks MATLAB environment.

3. Simulation results

Typical results of calculations of a discharge in SF_6 at atmospheric pressure are given in Figs. 2 and 3. The applied voltage pulse from an external source has an amplitude of 200 kV with 1 ns duration at the pulse edge, as shown in Fig. 2 by the black dashed line. These voltage pulse parameters are the most illustrative for analyzing the breakdown phenomenon.

In order to simulate how the transmission line affects the discharge process in actual experiments, we included the ballast resistor $R = 75\ \Omega$ in the equivalent circuit between the diode and the voltage source. Here, the total current in the discharge circuit (see Eq. (3)) is taken per unit length of the diode with $L = 1$ cm.

The anode in the coaxial diode has an external radius $r_a = 10$ mm, and an inner radius (cathode) $r_c = 1$ mm. In Fig. 2, the blue solid line shows the calculated time dependence of diode voltage.

We usually study the process of multi-electron initiation of gas breakdown, assuming an initial concentration of electrons at 10^3 cm^{-3}.

As well as in our formulation where runaway electrons are those electrons that do not collide, in each section (including the anode plane) we have a wide range of energy, most of which contains the plasma electrons with low energy. Of course, we are interested only in fast electrons, which are usually observed by the experimenters [1,2]. Therefore, in order to calculate the fast electron spectrum at the anode, we added a foil filter that cuts off the low-energetic part (<10 keV) of the electron beam. The attenuation factor of the filter corresponds to a 10-μm-thickness aluminum foil [14].

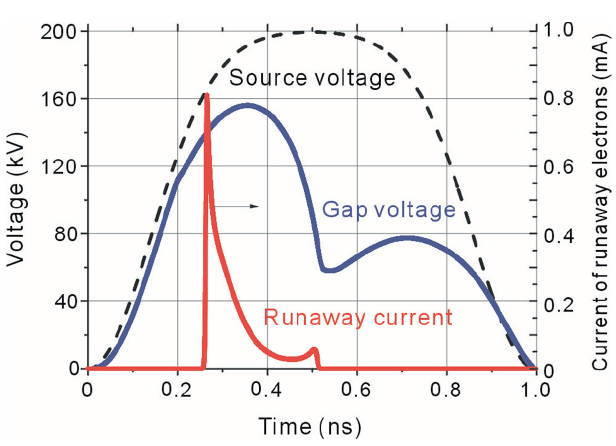

Fig. 2. Voltage at the power source (the black dashed line), diode voltage drop (the blue solid line) and fast electron current after passing the anode foil (the red solid line) v.s. time.

Fig. 3 shows the spatial-temporal electric field distribution at the breakdown stage for the diode with a high degree of electric field inhomogeneity $E(r_c)/E(r_a) = 10$.

At breakdown, the filling of the gap with plasma in the coaxial diode elapses spatially and inhomogeneously. One can talk about the propagation of the ionization wave during gap breakdown in the non-uniform geometry. It is clearly seen from Fig. 3 that the movement of the wave front is reflected in the movement of the point of maximum electric field. Directly behind the front of the ionization wave, the electric field strength almost falls to zero, as the electronic

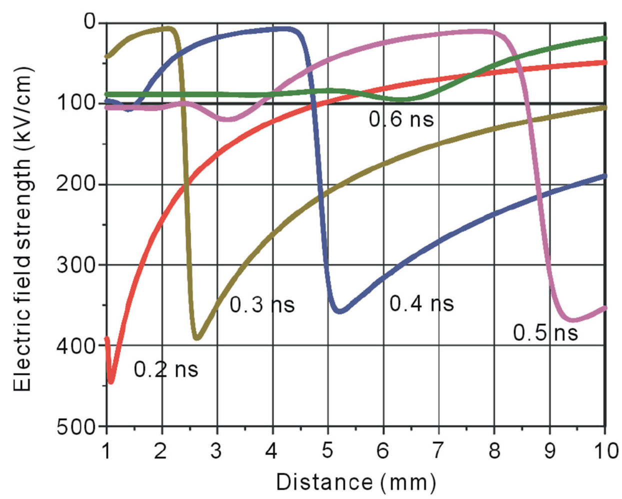

Fig. 3. Spatial distribution of the electric field in the discharge gap for different times after the arrival of the voltage pulse at the coaxial diode.

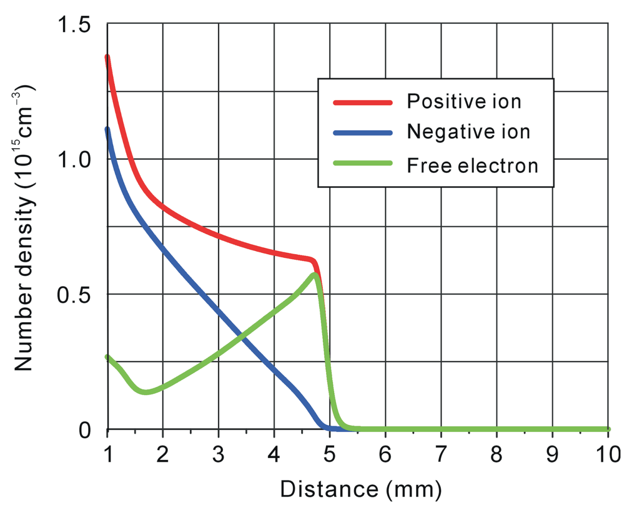

Fig. 4. Spatial distribution of the plasma species densities at the time point of 0.4 ns.

conductivity of plasma is very high. Fig. 4 shows the distribution of plasma species densities at the time of 0.4 ns. A similar structure also holds for other time points. Closer to the cathode the concentration of free electrons is greatly decreased due to the attachment to SF_6 molecules, and the conductivity becomes partially ionic. Therefore, the field strength increases to 100 kV/cm level, which value is the balance point between ionization and attachment rates (also see Fig. 1).

As can be seen from Figs. 2 and 3, the main pulse of fast-electron current starts and almost ends before the ionization front reaches the anode. This indicates the complex mechanism of electric current transfer in the diode. In order to understand the current transfer structure, the time dependences of anode currents in a logarithmic scale are shown in Fig. 5.

The electric current is mainly capacitive until the dense plasma fills the diode. Only a small portion of the total current is carried by fast electrons at this discharge phase. As the ionization wave front approaches the anode, the kinetic energy of non-scattered electrons decreases sharply. They can no longer pass through the 10 μm Al filter, but still constitute a significant part of the convective electron flow at the anode. Only after complete diode filling by plasma the current becomes completely drifted since the electric field is no longer able to generate runaway (non-scattered) electrons flow.

Among possible applications of high-voltage discharges, the power spectrum of fast electrons is the most important. Integral of fast electron spectra per pulse is given in Fig. 6.

First thing that attracts attention is the existence of electron group whose energy exceeds the “amplitude” value eU_{max}. Such electrons are known as electrons having “anomalous” energy [15]. In our conditions, the fraction of “anomalous” electrons is equal to about 30%.

According to our calculations, the nature of “anomalous” energy, observed at some fraction of runaway electrons, is fully consistent with the mechanism of polarization acceleration, previously described in Ref. [16]. An acceleration of charged particles by “running electric field” is not a potential mechanism. Therefore, the running impulse of the electric field can accelerate particles to energies greater than the voltage drop in the pulse.

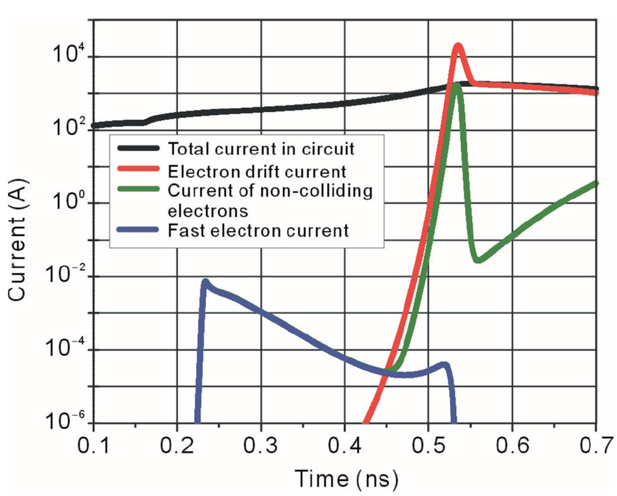

Fig. 5. Time dependence of the anode currents.

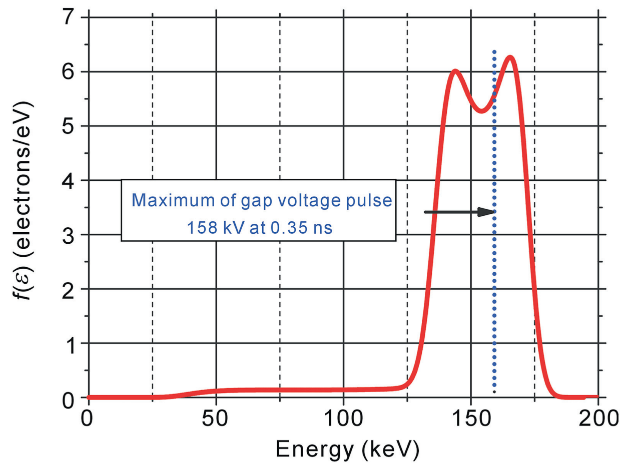

Fig. 6. Power spectrum of the fast electron current pulse.

Fig. 7 shows the instantaneous phase portraits of runaway electrons $f(r, p, t)$. Near the color legend numbers indicate the logarithm of the distribution function in the unit of a number density ($\lg[f(r, p/mc)]$ $[m^{-3}]$).

Delicate details of runaway electron flow generation in the breakdown of the gas-filled diode are clearly seen.

At time points before 0.2 ns, the flow of runaway electron starts to form in close proximity to the cathode wire. Here the most dense plasma and strengthened field are concentrated. At this point, the fast electrons have not yet reached the anode, therefore the fast electron current is still zero, as shown in Fig. 5.

By the next time of 0.3 ns, the fast electron flux has already fallen on the anode, but the current of plasma electrons (discharge current) is still very low. Since in this time interval runaway electrons are accelerated in an increasing electric field, they come with an “anomalously” high energy to the anode. It is at this stage that the high-energy peak of the spectrum has been formed as shown in Fig. 6.

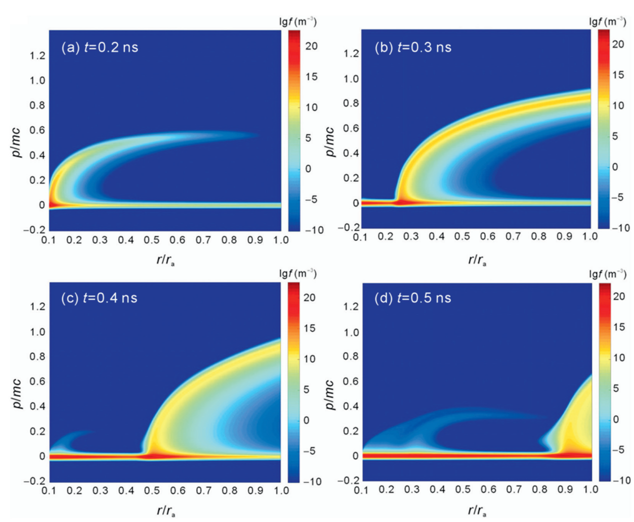

Fig. 7. Runaway electrons distribution function $f(r, p)$ corresponding to four different time points: (a) 0.2 ns, (b) 0.3 ns, (c) 0.4 ns and (d) 0.5 ns.

At this time point the front of the ionization wave is at a distance of 1.5 mm from the cathode. At the wave front the plasma number density reaches 6×10^{14} cm^{-3}. Further ionization wave front moves towards the anode at a speed equal to $(1-3) \times 10^9$ cm/s.

At the time of 0.4 ns, the fast electron current is still quite significant, but its kinetic energy is slightly reduced. At this time, the second peak of spectrum is formed (low energy) as shown in Fig. 6. The average electric field strength reaches a maximum value almost as the entire voltage drop (~140 kV) concentrated on the length of ~5 mm (also see Fig. 3).

Finally, at the time of 0.5 ns the ionization wave almost reaches the anode. The spectrum of fast electrons becomes nearly flat in a range from 50 to 120 keV as shown in Fig. 6.

It is interesting to note that the simulation indicates the presence of another group of runaway electrons in dense plasma near the cathode. However, the absolute number of electrons in this group is very small. Thus, in this specific case less than one particle is represented in this group. The ability to describe such small effects is one of the advantages of our hybrid model as compared to Monte Carlo or Particle-In-Cell methods.

4. Conclusions

Using the hybrid mathematical model of a high-voltage discharge in sulfur hexafluoride enabled us to investigate the mechanism of runaway electrons flow generation and its energy spectrum in details. In particular, it was shown that generation of runaway electrons is initiated by the ensemble of plasma electrons generated in high electric field regions. Acceleration of runaway electrons in traveling ionization wave inevitably leads to the formation of electron group with "anomalous" energy. Runaway electrons are ahead of the ionization wave, therefore can produce new secondary electrons at its front. Main quantitative parameters of the fast electron beam agree well with the experimental data [17].

Acknowledgments

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