Sensitivity Enhancement of Nonlinear Waveguide Sensors With Conducting Graphene Layer: TE Mode

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Abstract: We propose a three-layer waveguide sensor. The proposed sensor consists of a graphene thin layer with constant conductivity at the interface between air and dielectric media with thickness *d* sitting above a nonlinear layer. The sensitivity of the sensor is derived from the dispersion equation. The sensitivity is calculated for both TE_0 and TE_1. Results show that the sensitivity of the proposed sensor depends on the conductivity of the graphene layer, the angular frequency, and the thickness of the dielectric layer.

Keywords: Graphene; nonlinear Kerr like materials; optical sensor; waveguide sensor

1. Introduction

In waveguide sensors, the evanescent field can detect changes that occur in the neighboring region as a response for the change in refractive index. These sensors have various applications such as biosensing, chemical sensing, biochemical sensing, and humidity sensing [1–7]. For example, biosensor is used for biosensing, which is a process that detects the interactions of biological objects with a waveguide surface. The refractive index of the surface changes once an analytic particle fixes to the surface of the waveguide, resulting in changes in the evanescent field, which can be measured.

Previous attempts to improve the performance of sensors have been introduced [8–16]. In order to improve the performance of sensors in practical applications, the sensitivity should be maximized. Therefore, the appropriate sensor strata should be selected such that the refractive index change is extreme. The propagation of light in a media with intensity dependent refractive index is called nonlinear media. An intensively investigated nonlinear media, in which refractive index is directly proportional to the field intensity, is called Kerr-like media [11, 12, 14].

Graphene is added to the proposed waveguide structure to enhance its performance. Graphene is a novel two-dimensional crystalline material consisting of attached carbon atoms oriented in a hexagonal layer, with a carbon-carbon distance of 0.142 nm. It has marvelous properties. The electrons in graphene behave as if they are massless due to the interaction between electrons and hexagonal lattice. Therefore, electrons in graphene are governed by Dirac equation and named Dirac fermions [17]. Chen *et al.* reported that graphene conductance depended on frequency and resistivity became

linearly dependent on temperature at specific value of temperature [18]. Lee *et al.* measured the mechanical properties and the elastic properties of monolayer graphene by using nanoindentation in atomic force microscope. They concluded that deformation tests for nanoscale materials could be done outside the linear scale [19]. Wright *et al.* have observed that graphene has robust nonlinear optical reaction in the terahertz range [20]. Schedin *et al.* showed that micrometer graphene sensor could detect single actions [21]. The basic functioning idea of graphene sensor depends on the variation of its electrical conductivity because of adsorbed gas molecules on its surface. Bludov *et al.* investigated the electromagnetic waves travelling on the surface of a nonlinear dielectric material crowned by a graphene sheet [22]. The proposed structure is promising for stability of nonlinear plasmon-polaritons. In addition, dispersion characteristic and sensitivity of different layer waveguide structures including graphene and/or metamaterisls and metallic-plasmonic layer were studied in [23–25].

The focus of this study is on the sensitivity of a waveguide including a thin layer of graphene at the interface between air and dielectric media, which is supported by nonlinear medium. We only consider stabilized nonlinear transverse electric (TE) waves. Furthermore, we are seeking to optimize the sensitivity of the proposed sensor by changing variables such as the waveguide thickness and other physical parameters.

Next section will cover the theoretical background and introduction to the proposed structure. Section 3 will be dedicated to presenting the results and discussion followed by conclusion in Section 4.

2. Proposed structure and theory

A three-layer waveguide sensor is presented in Fig. 1. The proposed sensor consists of a graphene thin film laying at the interface between dielectric media and air. The dielectric is placed above a nonlinear media. The graphene is assumed to have conductivity (σ) and thickness in the order of 0.142 nm that will be considered very small and approximated to zero. The dielectric layer has permittivity ε_2 and thickness d. The nonlinear medium has permittivity expressed by a Kerr-like equation $\varepsilon_{nl} = \varepsilon_1 + \alpha E_y^2$, where ε_1 is the linear part of the permittivity, α is the strength of the nonlinear part, and E_y is the amplitude of the applied filed [11, 12, 14]. Air permittivity ε_3 equals to ε_0, the vacuum permittivity.
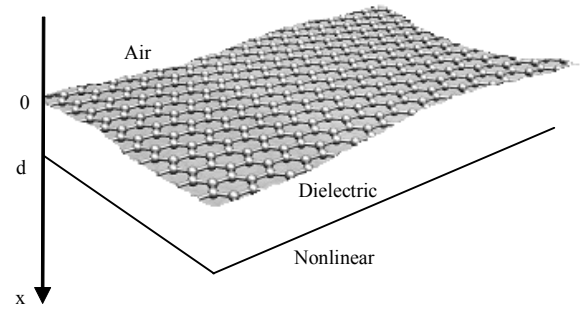

Fig. 1 Proposed structure. The field is assumed to travel along the *z* axis.

In Fig. 1, the wave is assumed to propagate in the *z*-direction. The electromagnetic field components are $\mathbf{E} = \{0, E_y, 0\}$ and $\mathbf{H} = \{H_x, 0, H_z\}$ which can be written in an oscillatory form as

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_y(x)e^{i(\omega t - \beta z)} \tag{1}$$

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}_y(x)e^{i(\omega t - \beta z)} \tag{2}$$

where ω is the wave radian frequency, $\beta = Nk_0$ is the propagation constant, N is the effective index of refraction, $k_0 = \omega/c$ is the propagation constant in free space, and c is the speed of light.

Using Maxwell equations and boundary conditions, we obtain the following dispersion equation

$$\tan(\gamma_2 d - m\pi) = \frac{(u+v)}{(1-uv)} \tag{3}$$

where

$$u = \frac{-\gamma_2}{i\sigma\omega - \gamma_3}, \quad v = -\frac{\gamma_2}{\gamma_1 \sqrt{1 - \frac{k_0^2 r}{\gamma_1^2}}}, \quad \gamma_1 = k_0^2 \sqrt{N^2 - \varepsilon_1},$$

$$\gamma_2 = k_0^2 \sqrt{\varepsilon_2 - N^2}, \quad n_3 = \sqrt{\varepsilon_3}, \quad \text{and} \quad \gamma_3 = k_0^2 \sqrt{N^2 - n_3^2},$$

and $r = \dfrac{\alpha E^2}{2}$ is the nonlinear factor.

The sensitivity S is defined by the variation of the effective refractive index N with respect to the changes in the cladding refractive index n_3 [9].

$$S = \frac{\partial N}{\partial n_3}. \tag{4}$$

To calculate S, (3) is differentiated with respect to n_3. Then, by rearranging the result, we get

$$S = \frac{(Y_2 X_2 + Y_2 v X_1)}{Z\, X_2^2 - (X_2 Y_1 + Y_3 X_2 + Y_1 v X_1 + X_1 Y_3 u)} \tag{5}$$

where

$$X_1 = u + v,\ X_2 = 1 - uv$$

$$Y_1 = \frac{(\gamma_3(i\sigma\omega - \gamma_3)\, k_0^2 N - \gamma_2^2\, k_0^2 N\,)}{\gamma_2 \gamma_3 (i\sigma\omega - \gamma_3)^2}$$

$$Y_2 = \frac{\gamma_2 k_0^2 n_3}{\gamma_3 (j\sigma\omega - \gamma_3)^2}$$

$$Y_3 = \left(\frac{k_0^2 N}{\gamma_2 \gamma_1 \sqrt{1 - \dfrac{k_0^2 r}{\gamma_1^2}}} + \frac{k_0^2 N\, \gamma_2}{\gamma_0^3 \sqrt{1 - \dfrac{k_0^2 r}{\gamma_1^2}}} + \frac{k_0^4 N\, \gamma_2 r}{\gamma_0^5 \left(1 - \dfrac{k_0^2 r}{\gamma_1^2}\right)^{1.5}} \right)$$

$$Z = -\frac{\sec^2(\gamma_2 d - m\pi) k_0^2 N d}{\gamma_2}.$$

3. Results and discussions

To calculate S for evanescent waves, the dispersion (3) is solved for the effective refractive index N, which is plugged into (5). In the calculation, the thickness of conducting graphene layer is assumed infinitesimally thin with constant conductivity σ.

The sensor layers have a nonmagnetic permeability that is $\mu_0 = \mu_2 = \mu_3 = 1$.

The real part of the sensitivity S for TE$_0$ ($m = 0$) is calculated as function of radian frequency ω at different values of d as displayed in Fig. 2. In the calculation, we consider $\varepsilon_1 = 2.25$, $\varepsilon_2 = 3$, $\varepsilon_3 = 1$, $\sigma = 6.089 \times 10^{-7}$ Siemens, $r = 0.5$ [26, 27], and d can take the following values: 75 nm, 65 nm, and 55 nm. We realize from Fig. 2 that as the value of d decreases the value of S increases. This can be

explained by checking (5). From (5), we notice that S has inverse proportionality with d through the Z term. The Z term for $m = 0$ is $-\sec^2(\gamma_2 d) k_0^2 N d / \gamma_2$. As d increases, the Z term increases, and S decreases. We can also see that as ω increases the sensitivity increases in case of $d = 75$ nm and $d = 65$ nm. However, in the case of $d = 55$ nm, it increases slightly, then it decreases.
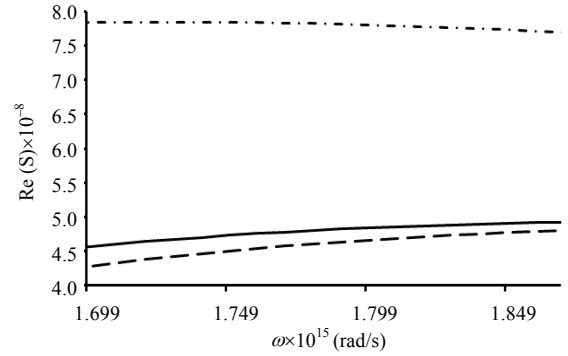

Fig. 2 Real part of the sensitivity for TE$_0$ versus frequency $\sigma = 6.089 \times 10^{-7}$ Siemens at different values of d. Dash line is when $d = 75$ nm, solid line is when $d = 65$ nm, and dash-dotted line is when $d = 55$ nm.
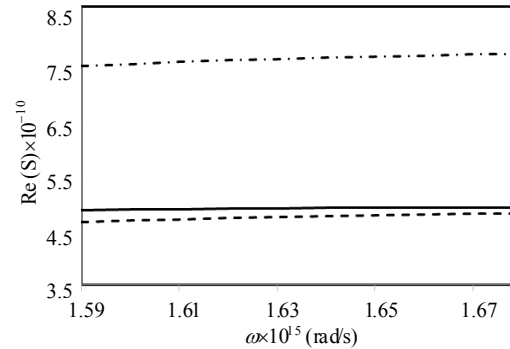

Fig. 3 Real part of the sensitivity for TE$_0$ versus frequency with $\sigma = 6.089 \times 10^{-6}$ Siemens at different values of d. Dash line is when $d = 85$ nm, solid line is when $d = 75$ nm, and dash-dotted line is when $d = 55$ nm.

To study the effect of the variation of graphene conductivity σ on S, we repeat the above calculations keeping the same variables and using $\sigma = 6.089 \times 10^{-6}$ Siemens. Figure 3 exhibits the real part of the sensitivity S for TE$_0$ ($m = 0$) in terms of radian frequency ω at different values of d. We consider the following values of d: 85 nm, 75 nm, and 55 nm. We obtain similar results as in Fig. 2 that is as d decreases the value of S increases. Also,

when ω increases, S increases. However, the values of S decrease by a factor of 100 as σ increases by a factor of 10. The same calculations are done at $\sigma=6.089\times10^{-5}$ Siemens. Result is displayed in Fig. 4. We obtain similar results with value of S smaller by a factor of 100.
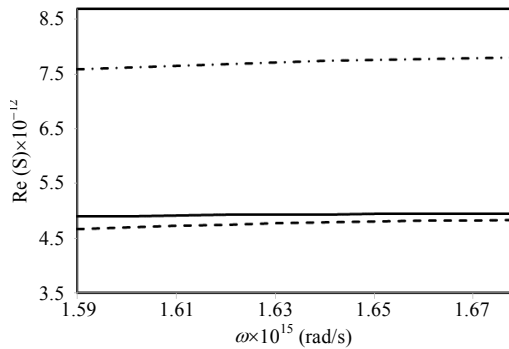

Fig. 4 Real part of the sensitivity for TE_0 versus frequency with $\sigma=6.089\times10^{-5}$ Siemens at different values of d. Dash line is when $d=85$ nm, solid line is when $d=75$ nm, and dash-dotted line is when $d=55$ nm.

The same calculations are performed for TE_1 ($m=1$). The real part of the sensitivity S is plotted versus frequency ω at different values of d as shown in Fig. 5. In the calculation, we consider $\varepsilon_1=2.25$, $\varepsilon_2=3$, $\varepsilon_3=1$, $\sigma=6.089\times10^{-7}$ Siemens, $r=0.5$, and d takes the following values: 75 nm, 65 nm, and 55 nm. We notice from Fig. 5 that as the value of d decreases the value of S increases. In addition, as ω increases, S increases. Comparing Fig. 2 for TE_0 with Fig. 5 for TE_1, we conclude that S is smaller for higher mode. That is, S is smaller for TE_1 than that for TE_0 which can be easily seen form (5).
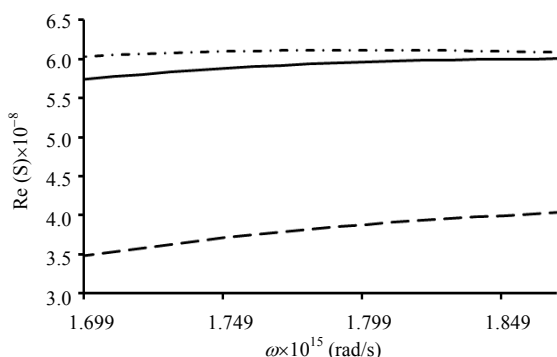

Fig. 5 Real part of the sensitivity for TE_1 versus frequency at different values of d. Dash line is when $d=75$ nm, solid line is when $d=65$ nm, and dash-dotted line is when $d=55$ nm.

4. Conclusions

Three-layer waveguide sensor is proposed in which a graphene thin film with constant conductivity lays at the interface between air and dielectric media. The dielectric medium is supported by nonlinear medium with Kerr-like permittivity. The sensitivity of the sensor is derived from the dispersion equation. The sensitivity has been numerically calculated using Maple as a function of frequency for both TE_0 and TE_1. Many interesting observations are obtained. The sensitivity is found to decrease for higher modes. It also increases as the thickness of the dielectric decreases. Moreover, the sensitivity decreases as the graphene conductivity increases. Furthermore, it is shown that the sensitivity is directly proportional to the angular frequency. Thus, the sensitivity of the proposed sensor can be controlled by changing the above mentioned variables.

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