Fiber Optic Gyroscope Dynamic North-Finder Algorithm Modeling and Analysis Based on Simulink

Zhengyi ZHANG[*] and Chuntong LIU

Department Two, Rocket Force University of Engineering , Xi'an, 710025, China

[*]Corresponding author: Zhengyi ZHANG   E-mail: 18809231139@163.com

Abstract: In view of the problems such as the lower automation level and the insufficient precision of the traditional fiber optic gyroscope (FOG) static north-finder, this paper focuses on the in-depth analysis of the FOG dynamic north-finder principle and algorithm. The simulation model of the FOG dynamic north found algorithm with the least square method by points is established using Simulink toolbox, and then the platform rotation speed and sampling frequency, which affect FOG dynamic north found precision obviously, are simulated and calculated, and the optimization analysis is carried out as a key consideration. The simulation results show that, when the platform rotation speed is between 4.5°/s and 8.5°/s and the sampling frequency is at about 50 Hz in the case of using the parameters of this paper, the FOG dynamic north finding system can reach the higher precision. And the conclusions can provide the reference and validation for the engineering and practical of FOG dynamic north-finder.

Keywords: Fiber optics; optical fiber gyroscope; dynamic north found; modeling

1. Introduction

With the in-depth development of equipment information, modern warfare has been gradually changed to all-weather, all-direction, rapid maneuver, and precision strike direction, and this will require the weapon systems having the abilities of fast and precise positioning and directing [1, 2]. For the inertial north finding technology is not dependent on the outside world, it has become one of the important guarantees for the weapon system realizing autonomic, fast, mobile, and accurate targeting goals [3, 4]. Fiber optic gyroscope (FOG) north-finder is one of new inertial north-finding technologies, which is increasingly used in defense and civil applications. It not only provides ideal directional information for the weapon system and equipment, but also provides precise orientation and attitude control reference for oil drilling, robots, and other civilian areas [5, 6].

The principle of the FOG north-finding mainly relies on its sensitiveness of the horizontal component of the earth angular rate. For this value is very small, the north-finder resolution mainly depends on the precision of the FOG. However, it will pay a higher cost for improving the level of hardware resources on the basis of manufacturing processes and technology at present. On the other hand, the north-finding algorithm affects the FOG north-finding precision and rapidity obviously, so the north-finding algorithm can be considered to be improved in order to better meet the practical

requirements for the gyroscope north-finding system. Among the FOG north-finding algorithm, static north-finding is a traditional method and has been widely used, but the operation is complex, and north-finding precision is limited [7, 8]. The FOG dynamic north-finding plan is a new inertial positioning method, which is being explored and taken much of the focus of researchers at home and abroad. It refers to a new method that the FOG north seeker rotates with the turntable around its central axis in continuous constant-speed in the process, and it can calculate the initial orientation according to the FOG output signal [9]. Compared with the FOG static north finding plan, the FOG output signal can be modulated periodically by continuous constant-speed mechanical rotation. By using the appropriate solution, it can inhibit the constant drift and random drift of the FOG effectively, shorten the north finding time, and improve the accuracy of north finder solution. However, for the reason of structure and limited process level, there are few examples of the FOG dynamic north finding used in engineering [10].

According to the above background, this paper studies mainly on the FOG dynamic north finding principle and algorithm by using Simulink simulation toolbox of MATLAB, and focuses on the simulation model building and analysis of the least square method by points of the FOG dynamic north finding algorithm, which is used to realize the parameters optimization design of the rotation speed and sampling frequency for the FOG north finder system, and provide reference for engineering and applications of the FOG dynamic north finder.
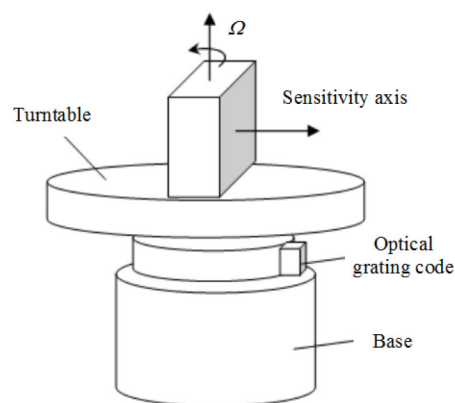
2. FOG and its dynamic north finding principle

The FOG is a new all-solid-state angular rate sensor based on the Sagnac effect. Compared with the conventional electro-mechanical gyroscope, it not only has the advantages of quick start, wide dynamic range, and high overload, but also has better zero deviation repeatability, and it is not sensitive to the movement and noise of the intersecting axes [11, 12]. In the practical engineering application, although the north finding ways are different, its basic principle is to measure the horizontal component (ω^b_{ie}) of the earth rotation angular rate (ω_{ie}) in the geographic coordinate system and calculate the included angle between the carrier ordinate axis direction and geographic north direction or give the azimuth of true north to realize autonomy orientation. Its principle is shown in Fig. 1(a).
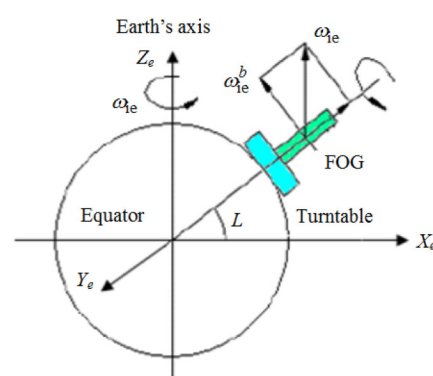
(a)

(b)

Fig. 1 Fiber optic gyro and its dynamic north-seeking principle: (a) principle of the fiber-optic gyro north-finding and (b) principle of the fiber optical gyro dynamic north seeker system.

The FOG dynamic north finding algorithm refers to a way in the north finding process where gyro rotates with the turntable from the initial location around the vertical axis at a constant angular rate Ω

continuously, and real-time sampling of gyro output value is carried out, then we can calculate the azimuth directly or included angle between the carrier ordinate axis direction and geographic north direction according to different algorithms. Its principle is shown in Fig. 1(b).

Ideally, when FOG rotates with the turntable at a constant angular rate ω, the projection of the horizontal component of the earth rotation angular rate on the carrier ordinate axis direction can be expressed as [13]

$$\omega_{ie}^{b} = \omega_{ie}\left[\cos L \cos\theta \cos\left(2\pi f t_i + \Psi_0\right) + \sin\theta\sin L\right] \tag{1}$$

where ω_{ie} is the earth rotation angular rate and its value is $15.04108°$ per hour, L is the latitude value of the carrier, Ψ_0 is the included angle between the carrier ordinate axis direction and geographic north direction, θ is the pitch angle, f is the rotation frequency of the turntable, and obviously ω is $2\pi f$.

Theoretically, when the input state of the fiber-optic gyroscope is constant, its output is a constant proportional to ω_{ie}^{b}. But the actual measurement of output includes several random error terms because of the existence of the gyro drift of itself, thermal noise of data acquisition circuit, variation of the rotation rate of the turntable, and low frequency noise (ground shaking and wind) in the ambient environment. Therefore, the practical dynamic output model of the fiber optic gyroscope can be expressed as [14]

$$\omega_{outi}\left(t\right) = K\omega_{ie} \times$$
$$\left[\cos L \cos\theta \sin\left(\frac{\pi}{2} - \left(2\pi f t_i + \Psi_0\right)\right) + \sin\theta\sin L\right] + \varepsilon_0 + \varepsilon_i \tag{2}$$

where K is the scale factor of the FOG, ε_0 is the constant zero deviation of the FOG, and ε_i is the random drift of the gyro including white noise.

In ideal conditions, the FOG can be fixed on the leveling turntable, then θ is zero. When the turntable rotates at a constant angular rate (Ω), the output model of the fiber optic gyro can be simplified as

$$\omega_{outi}\left(t_i\right) = K\omega_{ie}\cos L \cos\left(\Omega t_i + \Psi_0\right) + \varepsilon_0 + \varepsilon_i \tag{3}$$

where Ψ_0 can be calculated according to the above equation, then the azimuth of the true north direction is known, and the task of north finding is completed. Overall, the dynamic north-seeking scheme has higher requirements on the overall performance of the system, and its outstanding advantages include better precision and short north-finding time, which meet the requirements of the development direction toward integration, automation, high precision, and short time.

3. Modeling and simulation analysis of dynamic north-finding algorithm

3.1 Modeling and simulation analysis of dynamic north-finding algorithm

The above parts are basic principles of the FOG dynamic north finding. When FOG components rotate with the turntable at a constant speed, the dynamic output at a particular sampling frequency in real time can be measured. Then according to total sampling points of a full period or several full periods, least squares parameter estimation is used to calculate the initial included angle (Ψ_0) between the carrier ordinate axis direction and geographic north direction.

According to the fiber optic gyroscope dynamic output model in (3), it can be broken down into

$$\omega_{outi}\left(t_i\right) = K\omega_{ie}\cos L \cos\Omega t_i \cos\Psi_0 - K\omega_{ie}\cos L \sin\Omega t_i \sin\Psi_0 + \varepsilon_0 + \varepsilon_i \tag{4}$$

Let $A = K\omega_{ie}\cos L \cos\Psi_0$, $B = -K\omega_{ie}\cos L \cos\Psi_0$, $\alpha_i = \Omega t_i$ and the effects of error are overlooked then as follows:

$$\omega_{outi}\left(t_i\right) = A\cos\alpha_i + B\sin\alpha_i. \tag{5}$$

By using least squares parameter estimation, estimates of A and B can be derived as follows:

$$
\begin{cases}
\hat{A} = \dfrac{\sum\limits_{i=1}^{n}\sin^2\alpha_i \sum\limits_{i=1}^{n}\omega_{\text{out}i}\cos\alpha_i - \sum\limits_{i=1}^{n}\omega_{\text{out}i}\sin\alpha_i \sum\limits_{i=1}^{n}\sin\alpha_i\cos\alpha_i}{\sum\limits_{i=1}^{n}\sin^2\alpha_i \sum\limits_{i=1}^{n}\cos^2\alpha_i - \left(\sum\limits_{i=1}^{n}\sin\alpha_i\cos\alpha_i\right)^2} \\[4mm]
\hat{B} = \dfrac{\sum\limits_{i=1}^{n}\cos^2\alpha_i \sum\limits_{i=1}^{n}\omega_{\text{out}i}\sin\alpha_i - \sum\limits_{i=1}^{n}\omega_{\text{out}i}\cos\alpha_i \sum\limits_{i=1}^{n}\sin\alpha_i\cos\alpha_i}{\sum\limits_{i=1}^{n}\sin^2\alpha_i \sum\limits_{i=1}^{n}\cos^2\alpha_i - \left(\sum\limits_{i=1}^{n}\sin\alpha_i\cos\alpha_i\right)^2}
\end{cases}
. \tag{6}
$$

At this point, the estimate of the initial azimuth is as follows:

$$
\varPsi_0 = \text{ac}\tan\left(-\hat{B}/\hat{A}\right). \tag{7}
$$

When the dynamic north finding is carried out by using this method, relevant parameters can be adjusted flexibly according to different requirements of the precision and real-time requirements, which has advantages of simple operation and system stability.

3.2 Modeling and analysis of algorithms

According to the principle of the least squares estimation algorithm used in the dynamic north-seeking of the fiber optic gyroscope, models of the algorithm are built by using the MATLAB Simulink simulation tool. Specific parameters of the models are set as follows: the scale factor of the FOG $K_s = 0.81$, the latitude is $34°16'$, the earth rotation angular rate $\omega_{\text{e}} = 15.04\,108\,°/h$, and the initial included angle between the fiber optic gyroscope sensing axis and true north is $10°$. For the complexity of the specific model of algorithm and space limitations, this paper focuses on the analysis and discussion of simulation results. In the dynamic north-seeking system of the FOG, when hardware resources are determined, the rotary speed and sampling frequency are two important parameters affecting the north-seeking precision. Simulation analysis is carried out aiming at the effect of the variety of both parameters on the north-seeking precision by using the model above.

Firstly, under the condition that the sampling frequency and other parameters remain constant, the speeds of the rotating platform are taken respectively as $1\,°/s$, $2\,°/s$, $5\,°/s$, and $10\,°/s$. The calculated outputs of dynamic north-finding system of the fiber-optic gyro are shown in Fig. 2.
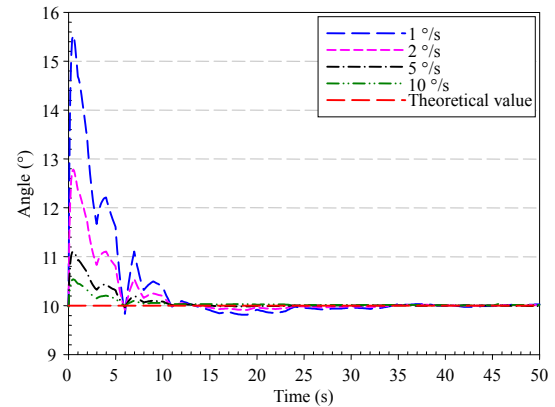

Fig. 2 Speed of the simulation results.

It can be seen from the simulation results in Fig. 2 that in the initial stage of the north-finding system, the deviation between the output value and the theoretical value $(10°)$ is large, and $20\,s$ later, the output is close to the theoretical value and tends to be steady. In addition, according to the simulation results, it can be seen that when the platform rotates at different speeds, steady-state errors between the final steady-state output of the north-seeking system and the theoretical value are different. Different steady-state errors are calculated when the rotary speed of the turntable varies from $1\,°/s$ to $10\,°/s$, which are shown in Fig. 3.

The results in Fig. 3 show that when other parameters of the FOG remain unchanged, the rotary speed of the platform has an obvious effect on the precision of the north-seeking. It will result in a large error when the speed is too high or too low. According to the parameters of the FOG used in the experiment, when the speed of the rotating platform

is approximately between 4.5 °/s and 8.5 °/s, the theoretical precision of the north-seeking can be limited within a range of about ± 10″. Therefore, there is necessity to determine an appropriate speed in the practical application of the FOG dynamic north-seeking system.
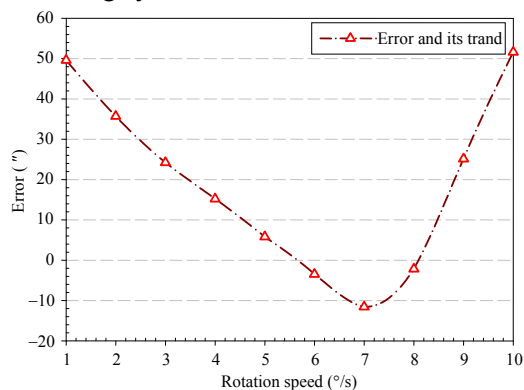

Fig. 3 Speed north errors.

The above results in Fig. 3 show that when other parameters of the FOG remain unchanged, the rotary speed of the platform has an obvious effect on the precision of the north-seeking. It will result in a large error when the speed is too high or too low. According to the parameters of the FOG used in the experiment, when the speed of the rotating platform is approximately between 4.5 °/s and 8.5 °/s, the theoretical precision of the north-seeking can be limited within a range of about ± 10″. Therefore, there is necessity to determine an appropriate speed in the practical application of the FOG dynamic north-seeking system.

In addition, the sampling frequency of the FOG output has a certain effect on the north-seeking precision. When all other parameters are constant, the sampling frequency of the models is set as 10 Hz, 20 Hz, 50 Hz, and 100 Hz for simulation, and the results of different sampling frequencies are shown in Fig. 4.

According to the simulation results, it can be seen that when the sampling frequency is low, the output error of the north-seeking system is large. As the sampling frequency increases step by step, the steady-state error gradually decreases to a steady state. It is shown in Fig. 5 that how error trend between the output of north-seeking system and the theoretical
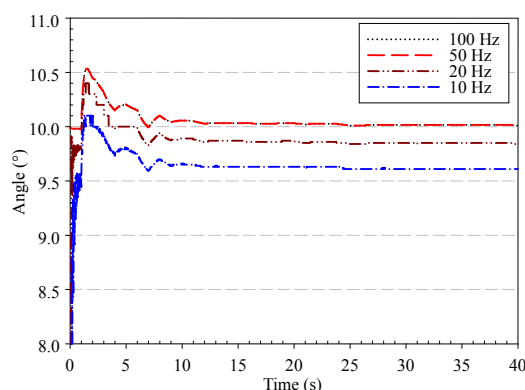
value varies as the sampling frequency varies.

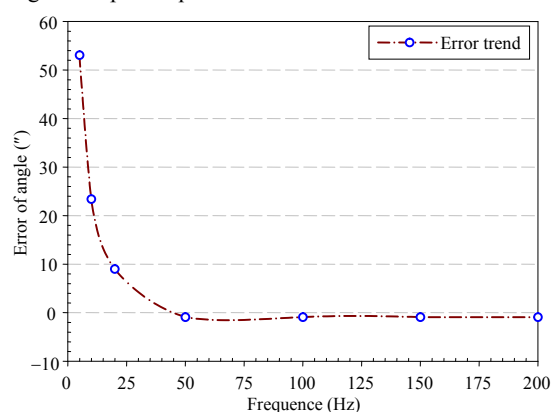Fig. 4 Sample frequencies of the simulation results.

Fig. 5 Error with sampling frequencies of north trend.

According to the above calculations, it is shown in Fig. 5 that when the sampling frequency is lower than 50 Hz, the north-finding error is large, but the steady-state error of the FOG north finding system is stabilized at about –1″ when the sampling frequency is higher than 50 Hz. So the high sampling frequency will not improve the precision, instead it will increase the cost. Therefore, the sampling frequency of the FOG north finding system is usually determined by the maximum sampling frequency of the FOG output.

Therefore, in summary, when the FOG is used in the dynamic north finding, the best turntable rotation speed and output sampling frequency should be determined by the precision and performance parameters of the FOG, in order to obtain the higher precision of the north finding with less time. Under the FOG parameters in this paper, when the angular rate of the rotating platform is approximately between 4.5°/s and 8.5°/s and the sampling frequency is 50 Hz, it will get higher north-seeking precision.

As a new type of all solid-state angular rate sensor, the fiber optic gyroscope is very suitable to be the main inertial measurement unit of the north-seeking orientating system. Compared with the traditional static north-finding method, the dynamic north-seeking scheme of the FOG can effectively control the gyro constant drift and random drift, reduce the search time, and improve the precision of the north-finding solution. Based on the analysis of the principle of the dynamic north-seeking with the continuous rotary FOG, this paper focuses on the analysis of the principle of the least squares parameter estimation algorithm, and the model of the dynamic north-seeking system is built by using the Simulink simulation tool. The simulation and optimization analysis of the sampling frequency and rotary speed of the turntable are carried out, which are the two key factors affecting the precision of the north finding. The simulation results show that it will affect the north-seeking precision when the speed of the rotating platform is too low or too high, so there is a speed range of high precision. As the sampling frequency of the system gradually increases, the steady-state error of the north-seeking gradually decreases to a constant. But too high sampling frequency will not improve the precision, instead, it will increase the cost. Therefore, the conclusions of this paper can provide the theoretical reference and simulation basis for achieving high precision and practical application of the dynamic north finding of the FOG.

Acknowledgment

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