

PHOTONICS Research

Wave and particle properties can be spatially separated in a quantum entity

PRATYUSHA CHOWDHURY,^{1,4} ARUN KUMAR PATI,^{2,5} AND JING-LING CHEN^{3,*} 

¹Indian Institute of Technology Guwahati, Guwahati, Assam 781039, India

²Quantum Information and Computation Group, Harish-Chandra Research Institute, Jhansi, Allahabad 211019, India

³Theoretical Physics Division, Chern Institute of Mathematics, Nankai University, Tianjin 300071, China

⁴e-mail: pchowdhury@iitg.ac.in

⁵e-mail: akpati@hri.res.in

*Corresponding author: chenjl@nankai.edu.cn

Received 16 March 2021; revised 27 April 2021; accepted 8 May 2021; posted 10 May 2021 (Doc. ID 425101); published 1 July 2021

Wave and particle are two fundamental properties of nature. The wave–particle duality has indicated that a quantum object may exhibit the behaviors of both wave and particle, depending upon the circumstances of the experiment. The major significance of wave–particle duality has led to a fundamental equation in quantum mechanics: the Schrödinger equation. At present, the principle of wave–particle duality has been deeply rooted in people’s hearts. This leads to a common-sense perception that wave property and particle property coexist simultaneously in a quantum entity, and these two physical attributes cannot be completely separated from each other. In classical physics, a similar common-sense thought is that a physical system is inseparable from its physical properties. However, this has been recently challenged and beaten by a quantum phenomenon called the “quantum Cheshire cat,” in which a cat and its grin can be spatially separated. In this work, we propose a thought experiment based on the technology similar to the quantum Cheshire cat. We find that wave and particle attributes of a quantum entity can be completely separated, thus successfully dismantling the wave–particle duality for a quantum entity. Our result is still consistent with the complementarity principle and deepens the understanding of quantum foundations. © 2021 Chinese Laser Press

<https://doi.org/10.1364/PRJ.425101>

1. INTRODUCTION

Whether light is a wave or a particle has been a long-term debate that can be traced back to Newton’s corpuscular theory and Huygens’ wave theory in the 17th century [1,2]. The phenomena of interference, diffraction, and polarization have convinced people that light could be fully described by a wave, but the appearance of the photoelectric effect has introduced indisputable evidence that light exhibited particle property in the microscopic world [3]. As a compromise, the wave–particle duality of light was eventually and widely accepted [4]. In 1923, the French physicist Louis de Broglie generalized the viewpoint of wave–particle duality from light to electron, and also to all other matters [5,6]. He boldly proposed that electrons with momentum p should exhibit the wave properties with an associated wavelength $\lambda = h/p$, with h being Planck’s constant. Later on, Davisson and Germer experimentally confirmed de Broglie’s hypothesis about the wave–particle duality of matters by observing the electron diffraction effects [7]. Subsequently, the wave–particle duality has laid the foundation for the development of a new quantum theory; e.g., it has stimulated the establishment of Schrödinger’s equation, a fundamental equation in quantum mechanics.

Even so many years after the development of quantum mechanics, the wave–particle duality is still one of the most intriguing features of the theory. Such a duality supposes that a quantum particle is accompanied by a wave; i.e., both the particle and the wave are assumed to exist objectively. The duality has its own roots in the complementarity principle [8]. It has been studied extensively in the past, but still continues to amaze researchers with its profound implications. The most dramatic consequence of the wave–particle duality is the quantum interference that is displayed on a screen when we send photons or particles in a double-slit setup. The remarkable thing is that this quantum interference occurs even if only one particle is sent at a time and the particle seems somehow to pass through both slits at once, thus leading to interference. How each particle passes through both slits is still a mystery. It may be noted that to explain the quantum interference, it has been postulated that when the quantum entity impinges on the beam splitter, the particle may be going along one path but the wave is divided and travels along both the paths. The wave that goes along the arm where the particle is not present is called an empty wave [9]. Although there have been long-drawn-out debates on empty waves (i.e., waves that do not contain the associated

particle properties), this proposal still seems to confuse many people, and is accepted by some and disregarded by others (see, for example, Refs. [9,10]). It may be the case that the nature of quantum entity may be different than what the wave-particle duality has actually depicted [11]. For example, the wave and the particle we associate with a quantum entity are not same as the wave and the particle that we see in the classical world. Recently, there has been an attempt to quantify the nature of particle using a resource theoretic framework [12] where it was proposed that for each quantum entity there are myriads of waves and particles.

Another intriguing aspect of quantum mechanics is the concept of weak measurement [13–17] with suitable pre- and post-selections. Using the weak measurement formalism, it has been suggested that the quantum Cheshire cat [18] can be a possibility where a cat and its grin can be spatially separated. In quantum mechanics, this essentially means that with suitable pre- and post-selected states one can spatially separate the spin of a particle and the particle itself. In recent years, this work has raised lot of questions about separating an attribute of a physical system from the system itself; it is a concept that seems only possible in fiction [19]. However, when this becomes a scientific result, then it is bound to attract the attention of scientists from all over the world. Over last few years, a lot of work has been done in this area to unravel the mysteries of nature [20–34]. It should be further noted that this phenomenon has not only become a theoretical construct, but also been experimentally verified [26–28,34].

The enduring view about the wave-particle duality has suggested that a quantum entity behaves like both a wave and a particle. Suppose one can spatially separate the wave property and the particle property of lights or electrons. This immediately gives rise to some fundamental questions. Can one still observe the interference fringes on the screen when he/she adopts lights with a solely particle property to perform Young-type double-slit experiments? Can one still observe the photoelectric effect when he/she adopts lights with only wave property? Can one still observe the diffraction effects when he/she adopts electrons with a solely particle property to perform the corresponding experiments? Undoubtedly, to answer the above questions and some others, a crucial step is to develop a technology to completely separate the wave property and the particle property for a single physical entity.

In this work, we intend to investigate whether any profound implication can be drawn by linking the wave-particle duality and the quantum Cheshire cat. We shall propose a thought experiment with the help of the quantum Cheshire cat, such that it is possible to spatially separate the particle aspects from the wave aspects for a quantum entity using suitable pre- and post-selections. We will show that the particle attribute is not displayed in one arm of the interferometer, and the wave attribute is not displayed in another arm of the interferometer. Nevertheless, we will show that the quantum entity respects a new complementarity. A conclusion and discussions will be made at the end.

2. THEORETICAL FRAMEWORK

In a recent paper [35], Rab *et al.* made outstanding progress by presenting an experimental setup called the wave-particle (WP)

toolbox. A schematic illustration of the toolbox can be found in Fig. 1. Conversion from the coherence superposition of the polarization states to the coherence superposition of the wave and particle entities exploits the wave-particle box.

The mode conversion reads $|\psi_{in}\rangle = (\cos \alpha|H\rangle + \sin \alpha|V\rangle) \xrightarrow{\text{PBS}} |\psi_1\rangle = \cos \alpha|H\rangle|1\rangle + \sin \alpha|V\rangle|2\rangle$, where PBS denotes the polarizing beam splitter, parameter α can be adjusted by a half-wave plate, and $|H\rangle$ and $|V\rangle$ denote the horizontal and vertical polarization states, respectively. Now a half-wave plate (HWP) acts on the first path; hence, $|H\rangle \rightarrow |V\rangle$, then the state $|\psi_1\rangle$ becomes the state $|\psi_2\rangle = |V\rangle(\cos \alpha|1\rangle + \sin \alpha|2\rangle)$, where $|n\rangle$ represents the state of a photon traveling along the n -th path. We can ignore the polarization degree of freedom in $|\psi_2\rangle$. Then, each path further bifurcates at a balanced beam splitter (BS).

Hence $|\psi_2\rangle \xrightarrow{\text{BS}} |\psi_3\rangle$, where $|\psi_3\rangle = \cos \alpha[\frac{1}{\sqrt{2}}(|1\rangle + e^{i\phi_1}|3\rangle)] + \sin \alpha[\frac{1}{\sqrt{2}}(|2\rangle + e^{i\phi_2}|4\rangle)]$, and ϕ_1, ϕ_2 are the relative phases introduced by the phase shifters placed in paths 3 and 4. Now paths 1 and 3 are recombined by a beam splitter again. The phase shift between paths 1 and 3 is chosen to induce the following mode conversion: $|1\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|1\rangle + |3\rangle)$, $|3\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|1\rangle - |3\rangle)$. This leads to $\frac{1}{\sqrt{2}}(|1\rangle + e^{i\phi_1}|3\rangle) \xrightarrow{\text{BS}} e^{i\frac{\phi_1}{2}}(\cos \frac{\phi_1}{2}|1\rangle - i \sin \frac{\phi_1}{2}|3\rangle) = |W\rangle$. Then, one has $|\psi_3\rangle \rightarrow |\psi_{out}\rangle = \cos \alpha|W\rangle + \sin \alpha|P\rangle$, with $|P\rangle = \frac{1}{\sqrt{2}}(|2\rangle + e^{i\phi_2}|4\rangle)$.

Thus, due to such a toolbox, for a single photon prepared initially in a polarization state $|\psi_{in}\rangle = \cos \alpha|H\rangle + \sin \alpha|V\rangle$, one finally obtains the output state as $|\psi_{out}\rangle = \cos \alpha|W\rangle + \sin \alpha|P\rangle$, which is a wave-particle superposition state of a single photon. Here, $|W\rangle$ and $|P\rangle$ denote, respectively, the wave and particle states as

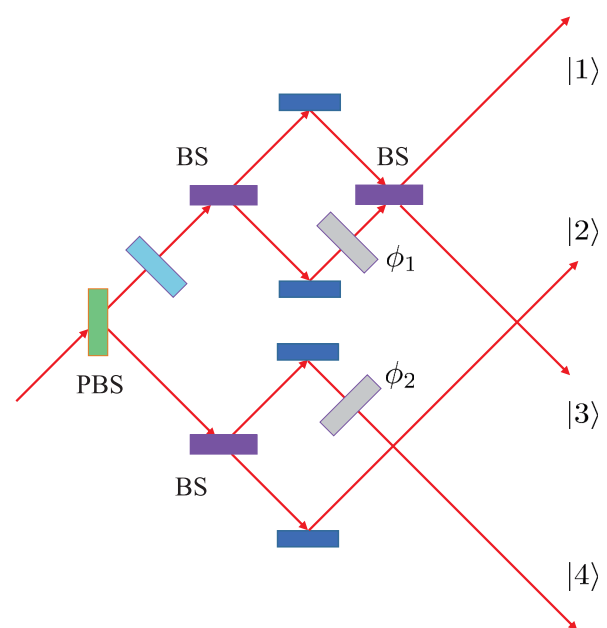


Fig. 1. Schematic illustration of the wave-particle toolbox.

$$\begin{aligned}
 |W\rangle &\equiv |\text{Wave}\rangle = e^{i\frac{\phi_1}{2}} \left(\cos \frac{\phi_1}{2} |1\rangle - i \sin \frac{\phi_1}{2} |3\rangle \right), \\
 |P\rangle &\equiv |\text{Particle}\rangle = \frac{1}{\sqrt{2}} \left(|2\rangle + e^{i\phi_2} |4\rangle \right).
 \end{aligned}
 \tag{1}$$

Operationally, these states represent the capability ($|W\rangle$) and incapability ($|P\rangle$) of the photon to produce interference. Here, $|n\rangle$, $n \in \{1, \dots, 4\}$ denotes the n -th output mode from the wave-particle toolbox, and ϕ_1 and ϕ_2 are two controllable phase shifts in the toolbox. If we represent the state of the photon as $|W\rangle$, then the probability to detect the photon in the path ($n = 1, 3$) depends on the phase ϕ_1 . In this case, the photon must have traveled along both paths simultaneously, thus revealing its wave behavior. If we represent the state of the photon as $|P\rangle$, then the probability to detect the photon in the path ($n = 2, 4$) is $\frac{1}{2}$ and does not depend on the phase ϕ_2 . In this case, the photon must have traveled only one of the two paths, showing its particle behavior. In our setup, for simplicity, the phase shifters in the paths are the same; i.e., $\phi_1 = \phi_2$.

Following Ref. [35], the illustration of spatially separating the wave and particle properties of a single photon is given in Fig. 2. To separate the wave and particle properties, we first need to choose the preselected state as

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle) (\cos \alpha |W\rangle + \sin \alpha |P\rangle),
 \tag{2}$$

with $|L\rangle$ and $|R\rangle$ denoting the left and the right arms, respectively. To prepare such a preselected state, in Fig. 2 the initial state $|\psi_{in}\rangle$ is put into the wave-particle toolbox. The action of the toolbox is to convert it to the state $|\psi_{out}\rangle$. We then send it toward a 50:50 beam splitter (i.e., BS1) and this will produce the desired preselected state $|\Psi_i\rangle$.

Second, we choose the post-selected state as

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}} (|L\rangle |W\rangle + |R\rangle |P\rangle).
 \tag{3}$$

Essentially, we want to perform a measurement that gives the answer, “yes,” whenever the state is $|\Psi_f\rangle$, and answer, “no,”

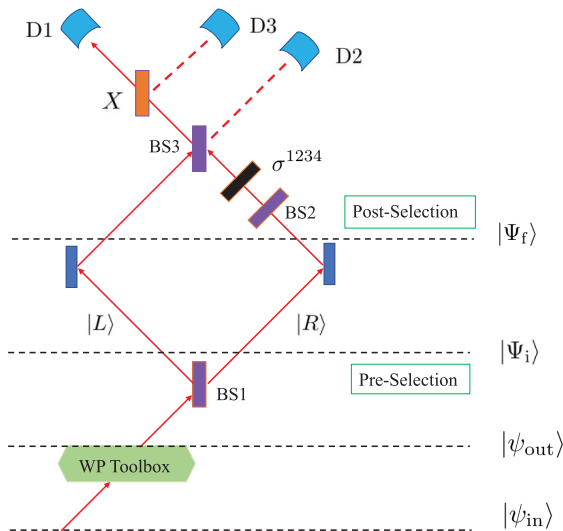


Fig. 2. Illustration of spatially separating the wave property and the particle property of a single photon.

when the state is orthogonal to $|\Psi_f\rangle$. We consider only the cases where the answer, “yes,” is obtained. Such a measurement setup can be realized by the optical setup as shown in Fig. 2. The post-selection consists of a beam splitter BS2 followed by the σ^{1234} operator on the right arm, with

$$\sigma^{1234} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
 \tag{4}$$

On the right arm, the particle state can be converted to the wave state after the actions of BS2 and σ^{1234} ; i.e.,

$$\begin{aligned}
 |R\rangle |P\rangle &\xrightarrow{\text{BS}} |R\rangle e^{i\frac{\phi_1}{2}} \left(\cos \frac{\phi_1}{2} |2\rangle - i \sin \frac{\phi_1}{2} |4\rangle \right) \\
 &\xrightarrow{\sigma^{1234}} |R\rangle e^{i\frac{\phi_1}{2}} \left(\cos \frac{\phi_1}{2} |1\rangle - i \sin \frac{\phi_1}{2} |3\rangle \right) \\
 &\rightarrow |R\rangle |W\rangle.
 \end{aligned}
 \tag{5}$$

We can now verify the result of our post-selection setup. By substituting Eq. (5) into the post-selected state, we have

$$\begin{aligned}
 |\Psi_f\rangle &= \frac{1}{\sqrt{2}} (|L\rangle |W\rangle + |R\rangle |P\rangle) \\
 \xrightarrow{\text{BS}, \sigma^{1234}} |\Psi_{f_1}\rangle &= \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle) |W\rangle.
 \end{aligned}
 \tag{6}$$

The beam splitter BS3 is chosen as $|L\rangle \xrightarrow{\text{BS3}} \frac{1}{\sqrt{2}} (|R\rangle - |L\rangle)$, $|R\rangle \xrightarrow{\text{BS3}} \frac{1}{\sqrt{2}} (|R\rangle + |L\rangle)$, such that the state $|\Psi_{f_1}\rangle$ turns to $|\Psi_{f_2}\rangle = |R\rangle |W\rangle$, and the detector D2 does not click. Finally, the action of the operator $X = |W\rangle \langle W|$ is such that only the wave state $|W\rangle$ is transmitted and the particle state $|P\rangle$ is reflected. Hence, the detector D3 does not click, and the detector D1 clicks with certainty if the post-selected state is indeed $|\Psi_f\rangle$.

This is the explicit calculation to show that, with the proposed setup, the detector D1 always clicks if $|\Psi_f\rangle$ is the post-selected state. If any other state is chosen, there will be finite probability that the detectors D2 and D3 will click. A photon starting in any state orthogonal to $|\Psi_f\rangle$ will either end up at detector D2 or D3 and certainly will not fire D1. So we can conclude that we have been able to design the setup by introducing certain operators such that only the particular post-selected state gives a D1 click with 100% probability. With this measurement setup the state $|\Psi_f\rangle$ will certainly end up in detector D1, and any state orthogonal to $|\Psi_f\rangle$ will end up in detector D2 or D3. We only focus on the cases when the detector D1 clicks.

Because we know that in the context of pre- and post-selections, the measurement strategy used is the weak measurement so we try to perform suitable weak measurements and extract information about the wave and particle aspects of the photon through these weak values. Following the quantum Cheshire cat proposal [35], which has allowed the separation of the properties of a particle from the particle itself, here we shall separate the wave and particle attributes of a quantum entity. We now move on to define various operators that measure

whether the wave and particle attributes are present in the left and right arms. Explicitly, we have the operators

$$\begin{aligned}\Pi_p^R &= \Pi^R \otimes \Pi_p = |R\rangle\langle R| \otimes |P\rangle\langle P|, \\ \Pi_p^L &= \Pi^L \otimes \Pi_p = |L\rangle\langle L| \otimes |P\rangle\langle P|,\end{aligned}\quad (7)$$

which determine if particle attributes are there in the right and left arms, respectively. Similarly, the operators

$$\begin{aligned}\Pi_W^R &= \Pi^R \otimes \Pi_W = |R\rangle\langle R| \otimes |W\rangle\langle W|, \\ \Pi_W^L &= \Pi^L \otimes \Pi_W = |L\rangle\langle L| \otimes |W\rangle\langle W|,\end{aligned}\quad (8)$$

determine if the wave attributes are there in the right and left arms, respectively.

Now the weak value of any observable \hat{A} is given by

$$\langle \hat{A} \rangle_w = \frac{\langle \Psi_f | \hat{A} | \Psi_i \rangle}{\langle \Psi_f | \Psi_i \rangle}, \quad (9)$$

where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the preselected and the post-selected states, respectively. We find that the weak values of these observables in our setup are

$$\begin{aligned}\langle \Pi_p^L \rangle_w &= 0, & \langle \Pi_p^R \rangle_w &= \frac{\sin \alpha}{\cos \alpha + \sin \alpha}, \\ \langle \Pi_W^R \rangle_w &= 0, & \langle \Pi_W^L \rangle_w &= \frac{\cos \alpha}{\cos \alpha + \sin \alpha}.\end{aligned}\quad (10)$$

It may be emphasized that a nonvanishing weak value of a projector indicates whether the system has been in the particular state represented by that projector between the pre- and post-selections. Similarly, if the weak value of the projector is null, then the system has not been in that state between the pre- and post-selections. Based on the above result, we see that the particle property is zero in the left arm, and the wave property is zero in the right arm. Therefore, we can safely conclude that the particle property of the photon is constrained to the right arm, and the wave property of the photon is constrained to the left arm in such a pre- and post-selected setup. This indicates that the wave and particle properties of the single photon have indeed been spatially separated. Thus, with the help of suitable pre- and post-selections, we can dismantle the wave and particle nature of a single photon. For $\alpha = \frac{\pi}{4}$, we have equal superposition of the wave and the particle states in the pre-selection; i.e., $|\Psi_i\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle) \otimes \frac{1}{\sqrt{2}}(|W\rangle + |P\rangle)$, and the weak values for the particle and wave attributes are given by $\langle \Pi_p^R \rangle_w = \frac{1}{2}$ and $\langle \Pi_W^L \rangle_w = \frac{1}{2}$. In this case, half of the particle attribute is present in the right arm and half of the wave attribute is present in the left arm of the interferometer.

Complementarity. The above scheme is also applicable for any quantum entity such as an electron or a neutron. One interesting point is that even though we have been able to dismantle the wave and particle properties of a single photon, this is actually consistent with the complementarity principle that we will discuss here. The quantum entity respects “unity in diversity.” Note that $\{|L\rangle, |R\rangle\} \in \mathcal{H}^2$ with $\Pi_{\mathcal{H}^2} = \Pi_L + \Pi_R = 1$ (here, 1 being the 2×2 unit matrix), and the modes in the wave–particle toolbox $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\} \in \mathcal{H}^4$ with $\sum_{i=1}^4 \Pi_i = \sum_{i=1}^4 |i\rangle\langle i| = 1 \otimes 1$. We can define another orthonormal basis $\{|W\rangle, |\bar{W}\rangle, |P\rangle, |\bar{P}\rangle\} \in \mathcal{H}^4$ and with the resolution of identity as given by $\Pi_{\mathcal{H}^4} = \Pi_W + \Pi_{\bar{W}} + \Pi_P + \Pi_{\bar{P}} = 1 \otimes 1$. With the pre- and post-selected states, as given in Eqs. (2) and (3), we have $\langle \Pi_{\mathcal{H}^2} \otimes \Pi_{\mathcal{H}^4} \rangle = 1$; i.e.,

$$\begin{aligned}\langle \Pi_W^L \rangle_w + \langle \Pi_p^L \rangle_w + \langle \Pi_{\bar{P}}^L \rangle_w + \langle \Pi_{\bar{W}}^L \rangle_w \\ + \langle \Pi_W^R \rangle_w + \langle \Pi_p^R \rangle_w + \langle \Pi_P^R \rangle_w + \langle \Pi_{\bar{P}}^R \rangle_w = 1.\end{aligned}\quad (11)$$

Further, we note that the weak values for various projectors satisfy these conditions:

$$\begin{aligned}\langle \Pi_p^L \rangle_w &= \langle \Pi_p^L \rangle_w = \langle \Pi_W^R \rangle_w = \langle \Pi_{\bar{W}}^R \rangle_w = 0, \\ \langle \Pi_p^R \rangle_w &= \frac{\sin \alpha}{\cos \alpha + \sin \alpha}, & \langle \Pi_P^R \rangle_w &= 0, \\ \langle \Pi_W^L \rangle_w &= \frac{\cos \alpha}{\cos \alpha + \sin \alpha}, & \langle \Pi_{\bar{W}}^L \rangle_w &= 0.\end{aligned}\quad (12)$$

Therefore, we have

$$\langle \Pi_p^R \rangle_w + \langle \Pi_W^L \rangle_w = 1. \quad (13)$$

This is a new complementarity relation between the wave and particle attributes in the weak measurement setting; i.e., the sum of the wave attribute in the left path and the particle attribute in the right path cannot be arbitrarily large. Interestingly, even though the wave and particle attributes have been dismantled, the prediction is consistent with the complementarity principle.

3. CONCLUSION AND DISCUSSION

The wave–particle duality is a fundamental concept of quantum mechanics, which implies that a physical entity is both a wave and a particle. There have been a lot of debates regarding the wave–particle duality in the past, and it has been an interesting topic of research as well as one of the least understood aspects in quantum mechanics. Although this duality has worked well in physics to produce experimental confirmations, its interpretation is still being discussed. Although physicist Niels Bohr viewed such a duality as one aspect of the concept of the complementarity principle, there may be more to it. In this work, by exploiting the advantages of weak measurement and a pre- and post-measurement setup, we have spatially separated the so-called wave and particle attributes of a quantum entity. Even though they are dismantled, they still respect a new complementarity relation. This also brings up some further fundamental questions. What is the wave aspect in the left arm of the interferometer like? How is it different from the general wave properties exhibited by an entity? Similarly, we may also ask: what is the “solely particle” aspect like in the right arm like? It would be interesting to find out if the interference fringes on the screen vanish when one adopts the lights with solely particle property to perform the Young-type double-slit experiments, and also to see if the electron diffraction effects disappear when one adopts the electrons with solely particle property to perform the corresponding experiments.

In our work, we have realized the possibility of completely separating wave property and particle property for a quantum object. The proposal in our work is related to the quantum Cheshire cat, for which some physical attributes can be separated from the particle itself. In the next stage, we would like to further consider a tripartite separation; i.e., the separation among the quantum object itself, the wave attribute, and the particle attribute. Once such a separation is achieved, then

one will obtain a quantum Cheshire “supercat.” We anticipate further experimental progress in this direction in the near future.

Funding. National Natural Science Foundation of China (11875167, 12075001).

Acknowledgment. Jing-Ling Chen is supported by the National Natural Science Foundation of China.

Disclosures. The authors declare no conflicts of interest.

Data Availability. No data were generated or analyzed in the presented research.

REFERENCES

1. M. Tegmark and J. A. Wheeler, “100 years of quantum mysteries,” *Sci. Am.* **284**, 68–75 (2001).
2. M. Sands, R. Feynman, and R. Leighton, *The Feynman Lectures on Physics III, Quantum Mechanics* (Addison-Wesley, 1965).
3. A. Einstein, “Über ein die Erzeugung und Verwandlung des Lichtes betreffendes heuristisches Gesichtspunkt,” *Ann. Phys.* **322**, 132–148 (1905).
4. J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement* (Princeton University, 1984).
5. L. de Broglie, “Waves and quanta,” *Nature* **112**, 540 (1923).
6. L. de Broglie, “Radiation–waves and quanta,” *C. R.* **177**, 507–510 (1923).
7. G. J. Davisson and L. H. Germer, “Diffraction of electrons by a crystal of nickel,” *Phys. Rev.* **30**, 705–740 (1927).
8. N. Bohr, “The quantum postulate and the recent development of atomic theory,” *Nature* **121**, 580–590 (1928).
9. F. Selleri, “On the wave function of quantum mechanics,” *Lett. Nuovo Cimento* **1**, 908–910 (1969).
10. L. Hardy, “On the existence of empty waves in quantum theory,” *Phys. Lett. A* **167**, 11–16 (1992).
11. J. Sperling, S. De, T. Nitsche, J. Tiedau, S. Barkhofen, B. Brecht, and C. Silberhorn, “Wave-particle duality revisited: neither wave nor particle,” arXiv:1907.09836 (2019).
12. S. Das, I. Chakrabarty, A. K. Pati, A. S. De, and U. Sen, “Quantifying the particle aspect of quantum systems,” arXiv:1812.08656 (2018).
13. Y. Aharonov, D. Z. Albert, and L. Vaidman, “How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100,” *Phys. Rev. Lett.* **60**, 1351–1354 (1988).
14. Y. Aharonov and L. Vaidman, “Properties of a quantum system during the time interval between two measurements,” *Phys. Rev. A* **41**, 11–20 (1990).
15. R. Jozsa, “Complex weak values in quantum measurement,” *Phys. Rev. A* **76**, 044103 (2007).
16. O. Hosten and P. Kwiat, “Observation of the spin Hall effect of light via weak measurements,” *Science* **319**, 787–790 (2008).
17. J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart, and C. Bamber, “Direct measurement of the quantum wavefunction,” *Nature* **474**, 188–191 (2011).
18. Y. Aharonov, S. Popescu, D. Rohrlich, and P. Skrzypczyk, “Quantum Cheshire cats,” *New J. Phys.* **15**, 113015 (2013).
19. L. Carroll, *Alice’s Adventures in Wonderland* (Broadview, 1865).
20. I. Ibnouhsein and A. Grinbaum, “Twin quantum Cheshire photons,” arXiv:1202.4894 (2012).
21. Y. Guryanova, N. Brunner, and S. Popescu, “The complete quantum Cheshire cat,” arXiv:1203.4215 (2012).
22. A. D. Lorenzo, “Hunting for the quantum Cheshire cat,” arXiv:1205.3755 (2012).
23. A. Matzkin and A. K. Pati, “Three-box paradox and “Cheshire cat grin”: the case of spin-1 atoms,” *J. Phys. A* **46**, 315307 (2013).
24. Y. Aharonov, E. Cohen, and S. Popescu, “A current of the Cheshire Cat’s smile: dynamical analysis of weak values,” arXiv:1510.03087 (2015).
25. R. Correa, M. F. Santos, C. H. Monken, and P. L. Saldanha, “Quantum Cheshire cat as simple quantum interference,” *New J. Phys.* **17**, 053042 (2015).
26. D. P. Atherton, G. Ranjit, A. A. Geraci, and J. D. Weinstein, “Observation of a classical Cheshire cat in an optical interferometer,” *Opt. Lett.* **40**, 879–881 (2015).
27. T. Denkmayr, H. Geppert, S. Sponar, H. Lemmel, A. Matzkin, J. Tollaksen, and Y. Hasegawa, “Observation of a quantum Cheshire cat in a matter-wave interferometer experiment,” *Nat. Commun.* **5**, 4492 (2014).
28. J. M. Ashby, P. D. Schwarz, and M. Schlosshauer, “Observation of the quantum paradox of separation of a single photon from one of its properties,” *Phys. Rev. A* **94**, 012102 (2016).
29. J. D. Bancal, “Quantum physics: isolate the subject,” *Nat. Phys.* **10**, 11–12 (2014).
30. Q. Duprey, S. Kanjilal, U. Sinha, D. Home, and A. Matzkin, “The quantum Cheshire cat effect: theoretical basis and observational implications,” *Ann. Phys.* **391**, 1–15 (2018).
31. D. Das and A. K. Pati, “Can two quantum Cheshire cats exchange grins?” *New J. Phys.* **22**, 063032 (2020).
32. D. Das and A. K. Pati, “Teleporting grin of a quantum Cheshire cat without cat,” arXiv:1903.04152 (2019).
33. D. Das and U. Sen, “Delayed choice of paths selected by grin and snarl of quantum Cheshire cat,” arXiv:2001.00669 (2020).
34. Z. H. Liu, W. W. Pan, X. Y. Xu, M. Yang, J. Zhou, Z. Y. Luo, K. Sun, J. L. Chen, J. S. Xu, C. F. Li, and G. C. Guo, “Experimental exchange of grins between quantum Cheshire cats,” *Nat. Commun.* **11**, 3006 (2020).
35. A. S. Rab, E. Polino, Z. X. Man, N. B. An, Y. J. Xia, N. Spagnolo, R. L. Franco, and F. Sciarrino, “Entanglement of photons in their dual wave-particle nature,” *Nat. Commun.* **8**, 915 (2017).