PHOTONICS Research

Nonreciprocal transition between two nondegenerate energy levels

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Stimulated emission and absorption are two fundamental processes of light–matter interaction, and the coefficients of the two processes should be equal. However, we will describe a generic method to realize the significant difference between the stimulated emission and absorption coefficients of two nondegenerate energy levels, which we refer to as a nonreciprocal transition. As a simple implementation, a cyclic three-level atom system, comprising two nondegenerate energy levels and one auxiliary energy level, is employed to show a nonreciprocal transition via a combination of synthetic magnetism and reservoir engineering. Moreover, a single-photon nonreciprocal transporter is proposed using two one-dimensional semi-infinite coupled-resonator waveguides connected by an atom with nonreciprocal transition effect. Our work opens up a route to design atom-mediated nonreciprocal devices in a wide range of physical systems.

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1. INTRODUCTION

According to Einstein's phenomenological radiation theory [1], the absorption coefficient should be equal to the stimulated emission coefficient between two nondegenerate energy levels. When the spontaneous emission can be neglected, a two-level system undergoes optical Rabi oscillations under the action of a coherent driving electromagnetic field [2]. However, can we make the absorption coefficient different from the stimulated emission coefficient for the transition between two energy levels with different eigenvalues, i.e., nonreciprocal transition between two nondegenerate energy levels? The answer is yes. In this paper, we describe a generic method to realize a nonreciprocal transition between two nondegenerate energy levels, and we show that the absorption and stimulated emission coefficients can be controlled via a combination of synthetic magnetism and reservoir engineering.

Theoretical research has shown that [3] a combination of synthetic magnetism and reservoir engineering can be used to implement nonreciprocal photon transmission and amplification in coupled photonic systems, and this has been confirmed by a recent experiment [4]. Based on a similar mechanism, many different schemes for nonreciprocal photon transport are proposed theoretically [5–8] and implemented experimentally [9–12]. Synthetic magnetism is an effective approach to achieve nonreciprocal transport of uncharged particles, such as photons [13–16] or phonons [17,18], for potential applications in simulating quantum many-body phenomena [19–25] and creating devices robust against disorder and backscattering [26–30]. Reservoir engineering [31] has been a significant subject for generating useful quantum behavior by specially designing the couplings between a system of interest and a structured dissipative environment, such as cooling mechanical harmonic oscillators [32]; synthesizing quantum harmonic oscillator states [33]; and generating state-dependent photon blockades [34], stable entanglement between two nanomechanical resonators [35,36], and squeezed states of nanomechanical resonators [37–39].

In this paper, we introduce the concept of nonreciprocity to investigate the transitions between different energy levels and generalize the general strategy for nonreciprocal photon transmission [3] to atomic systems to achieve a nonreciprocal

transition between two nondegenerate energy levels. As a simple implementation, a cyclic three-level atom system, comprising two nondegenerate energy levels and one auxiliary energy level, is employed to show a nonreciprocal transition via a combination of synthetic magnetism and reservoir engineering.

In application, the atomic systems with nonreciprocal transitions allow one to generate nonreciprocal devices. In this paper, a single-photon nonreciprocal transporter is proposed in a system of two one-dimensional (1D) semi-infinite coupled-resonator waveguides (CRWs) connected by an atom based on the nonreciprocal transition effect. The nonreciprocal transition effect provides a new routine to design atom-mediated nonreciprocal devices in a variety of physical systems.

2. GENERAL METHOD FOR NONRECIPROCAL TRANSITION

A general model of two nondegenerate energy levels $|a\rangle$ and $|b\rangle$ for nonreciprocal transition is shown in Fig. 1(a). The effective couplings between the two levels come from two different physical interactions. The first method is through a coherent interaction H_{coh}, which is described by $H_{\text{coh}} = \Omega|a\rangle\langle b| + \Omega^*|b\rangle\langle a|$ with complex coupling strength Ω. The simplest implementation of the coherent interaction H_{coh} is driving the two levels with a coherent laser field.

The second method is through coupling to a common engineered reservoir. A dissipative interaction H_{dis} between the two levels can be obtained by adiabatically eliminating the engineered reservoir. The effective Hamiltonian for the dissipative interaction H_{dis} can be written in a non-Hermitian form as $H_{\text{dis}} = -i\gamma(|a\rangle\langle b| + |b\rangle\langle a|)$ with positive real strength γ. This dissipative version of interaction can be implemented by an auxiliary energy level, which is damping much faster than the two levels. The details of the realization will be shown in the next section.

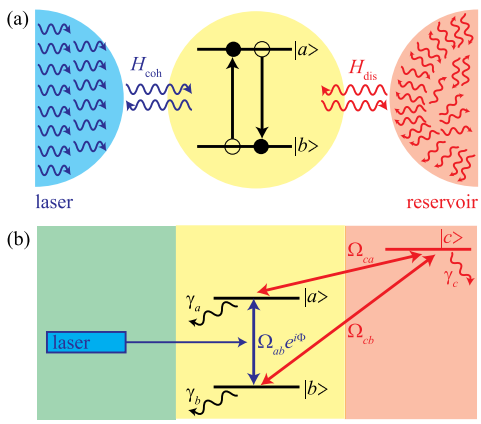

Fig. 1. (a) Schematic diagram for generating nonreciprocal transition: two nondegenerate energy levels $|a\rangle$ and $|b\rangle$ are coupled to one another via a coherent interaction H_{coh}, and they are also coupled to the same engineered reservoir. (b) Schematic diagram for implementation of a nonreciprocal transition in a cyclic three-level atom (characterized by $|a\rangle$, $|b\rangle$, and $|c\rangle$). A laser field ($\Omega_{ab}e^{i\Phi}$) is applied to drive the direct transition between the two levels $|a\rangle$ and $|b\rangle$, and they are also coupled indirectly by the auxiliary level $|c\rangle$ through two laser fields (Ω_{ca} and Ω_{cb}), where the decay of level $|c\rangle$ is much faster than that of the other two levels, i.e., $\gamma_c \gg \max\{\gamma_a, \gamma_b\}$, so the auxiliary level $|c\rangle$ serves as an engineered reservoir.

Based on the two distinct methods, the total Hamiltonian for the interaction between the two levels is

$$H_{\text{coh+dis}} = (\Omega - i\gamma)|a\rangle\langle b| + (\Omega^* - i\gamma)|b\rangle\langle a|. \tag{1}$$

When $\Omega = i\gamma$ and $\Omega^* \neq \Omega$, there is only transition $|a\rangle \to |b\rangle$ but $|b\rangle \nrightarrow |a\rangle$. Instead, when $\Omega^* = i\gamma$ and $\Omega^* \neq \Omega$, there is only transition $|b\rangle \to |a\rangle$ but $|a\rangle \nrightarrow |b\rangle$.

3. NONRECIPROCAL TRANSITION WITH CYCLIC THREE-LEVEL TRANSITION

To make the method more concrete, we show how to implement nonreciprocal transition in a cyclic three-level atom as depicted in Fig. 1(b). We consider a cyclic three-level atom ($|a\rangle$, $|b\rangle$, and $|c\rangle$) driven by three classical coherent fields (at rates Ω_{ij}, frequencies ν_{ij}, phases ϕ_{ij}, with $i, j = a, b, c$) that is described by a Hamiltonian (see Appendix A):

$$\begin{aligned} H = {} & (\Delta_{ab} - i\gamma_a)|a\rangle\langle a| - i\gamma_b|b\rangle\langle b| + (\Delta_{cb} - i\gamma_c)|c\rangle\langle c| \\ & + (\Omega_{ab}e^{i\Phi}|a\rangle\langle b| + \Omega_{cb}|c\rangle\langle b| + \Omega_{ca}|c\rangle\langle a| + \text{H.c.}), \end{aligned} \tag{2}$$

where $\Delta_{ij} = \omega_{ij} - \nu_{ij}$ $(i, j = a, b, c)$, ω_{ij} is the frequency difference between levels $|i\rangle$ and $|j\rangle$; γ_i $(i = a, b, c)$ are the decay rates. We assume that $\nu_{ab} = \nu_{cb} - \nu_{ca}$, so the detuning $\Delta_{ab} = \Delta_{cb} - \Delta_{ca}$. The synthetic magnetic flux $\Phi \equiv \phi_{ab} - \phi_{cb} + \phi_{ca}$ is the total phase of the three driving fields around the cyclic three-level atom and independent of the local redefinition of states $|i\rangle$. The time-reversal symmetry of the system is broken when we choose the phase $\Phi \neq n\pi$ (n is an integer) even without spontaneous emissions ($\gamma_a = \gamma_b = \gamma_c = 0$), and this is one of the key ingredients for nonreciprocal transition. In addition, we assume that the decays satisfy the conditions $\min\{\omega_{ca}, \omega_{cb}\} \gg \gamma_c \gg \max\{\Omega_{ca}, \Omega_{cb}, \gamma_a, \gamma_b\}$, so that level $|c\rangle$ serves as an engineered reservoir.

In order to show the nonreciprocal transition between levels $|a\rangle$ and $|b\rangle$ intuitively, we can derive an effective Hamiltonian by eliminating level $|c\rangle$ (the engineered reservoir) adiabatically (see Appendix B) under the assumption that $\gamma_c \gg \max\{\gamma_a, \gamma_b\}$. Then an effective Hamiltonian only including levels $|a\rangle$ and $|b\rangle$ is given by

$$\begin{aligned} H_{\text{eff}} = {} & (\Delta_a - i\Gamma_a)|a\rangle\langle a| + (\Delta_b - i\Gamma_b)|b\rangle\langle b| \\ & + J_{ab}|a\rangle\langle b| + J_{ba}|b\rangle\langle a|, \end{aligned} \tag{3}$$

with the detunings $\Delta_a \equiv \Delta_{ab} - \Omega_{ca}^2\Delta_{cb}/(\gamma_c^2 + \Delta_{cb}^2)$ and $\Delta_b \equiv -\Omega_{cb}^2\Delta_{cb}/(\gamma_c^2 + \Delta_{cb}^2)$, effective decay rates $\Gamma_a \equiv \gamma_a + \Omega_{ca}^2\gamma_c/(\gamma_c^2 + \Delta_{cb}^2)$ and $\Gamma_b \equiv \gamma_b + \Omega_{cb}^2\gamma_c/(\gamma_c^2 + \Delta_{cb}^2)$, and effective coupling coefficients

$$J_{ab} \equiv \Omega_{ab}e^{i\Phi} - i\frac{\Omega_{ca}\Omega_{cb}(\gamma_c - i\Delta_{cb})}{\gamma_c^2 + \Delta_{cb}^2}, \tag{4}$$

$$J_{ba} \equiv \Omega_{ab}e^{-i\Phi} - i\frac{\Omega_{ca}\Omega_{cb}(\gamma_c - i\Delta_{cb})}{\gamma_c^2 + \Delta_{cb}^2}. \tag{5}$$

The effective coupling coefficients J_{ab} and J_{ba} include two terms: the first term comes from the coherent driving field and is dependent on the synthetic magnetic flux Φ, and the second term is induced by the auxiliary level $|c\rangle$. Under the resonant condition $\Delta_{ab} = \Delta_{ca} = \Delta_{cb} = 0$, the second term

becomes purely imaginary, i.e., $-i\Omega_{ca}\Omega_{cb}/\gamma_c$, and the effective Hamiltonian is the same as Eq. (2). Under the resonant conditions, the perfect nonreciprocal transition, i.e., $J_{ab} = 0$ and $J_{ba} \neq 0$ (or $J_{ba} = 0$ and $J_{ab} \neq 0$), is obtained when $\Phi = \pi/2$ (or $\Phi = -\pi/2$) with $\Omega_{ab} = \Omega_{ca}\Omega_{cb}/\gamma_c$. More generally, we have $|J_{ab}| < |J_{ba}|$ for $0 < \Phi < \pi$ and $|J_{ab}| > |J_{ba}|$ for $-\pi < \Phi < 0$.

To understand the nonreciprocal transition further, we take a view on the dynamical behavior of the transition probabilities between levels $|a\rangle$ and $|b\rangle$. The time-evolution operator for the Hamiltonian H is given by $U(t) = \exp(-iHt)$, and the probabilities for transitions $|a\rangle \to |b\rangle$ and $|b\rangle \to |a\rangle$ can be defined by $T_{ba}(t) \equiv |\langle b|U(t)|a\rangle|^2$ and $T_{ab}(t) \equiv |\langle a|U(t)|b\rangle|^2$, respectively. They are plotted as functions of time $\Omega_{ab}t$ in Figs. 2(a)–2(c). We can see that the transition probabilities are time dependent and the nonreciprocal behaviors emerge after a short time ($\sim 1/\gamma_c$). It is clear that $T_{ab}(t) \ll T_{ba}(t)$ for $\phi = \pi/2$, $T_{ab}(t) \gg T_{ba}(t)$ for $\phi = -\pi/2$, and $T_{ab}(t) = T_{ba}(t)$ for $\phi = 0$. The isolation for the nonreciprocal transition defined by $I(t) \equiv T_{ab}(t)/T_{ba}(t)$ is plotted as a function of time $\Omega_{ab}t$ in Fig. 2(d). One can achieve $I(t) > 10^6$ for $\Phi = -\pi/2$ and $I(t) < 10^{-6}$ for $\Phi = \pi/2$ at time $\Omega_{ab}t = 1$.

Furthermore, the dependence of the transition probabilities $T_{ab}(t)$ and $T_{ba}(t)$ on the synthetic magnetic flux Φ is shown in Fig. 3(a). At time $\Omega_{ab}t = 1$, we have $T_{ab}(t) > T_{ba}(t)$ for synthetic magnetic flux $0 < \Phi < \pi$; in the contrast, we have $T_{ab}(t) < T_{ba}(t)$ for synthetic magnetic flux $-\pi < \Phi < 0$. As shown in Fig. 3(b), under the resonant condition $\Delta_{cb} = \Delta_{ca} = \Delta_{ab} = 0$, the optimal isolation $I(t)$ is obtained with synthetic magnetic flux $\Phi = \pm\pi/2$, which is consistent with the analytical predictions.

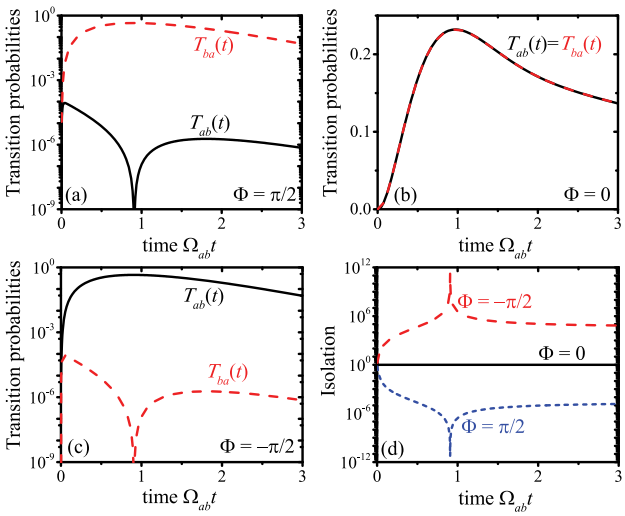

Fig. 2. The transition probabilities $T_{ab}(t)$ and $T_{ba}(t)$ are plotted as functions of the time $\Omega_{ab}t$ for: (a) $\Phi = \pi/2$, (b) $\Phi = 0$, and (c) $\Phi = -\pi/2$. (d) The isolation $I(t)$ is plotted as a function of time $\Omega_{ab}t$ for $\Phi = \pi/2, 0, -\pi/2$. The other parameters are $\gamma_a = \gamma_b = \Omega_{ab}/10$, $\gamma_c = 100\Omega_{ab}$, $\Omega_{ca} = \Omega_{bc} = 10\Omega_{ab}$, and $\Delta_{cb} = \Delta_{ca} = \Delta_{ab} = 0$.

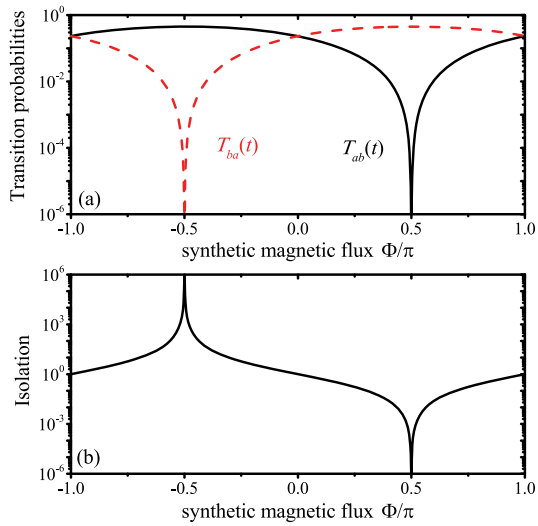

Fig. 3. (a) The transition probabilities $T_{ab}(t)$ and $T_{ba}(t)$ and (b) the isolation $I(t)$ are plotted as functions of the synthetic magnetic flux Φ at time $\Omega_{ab}t = 1$. The other parameters are $\gamma_a = \gamma_b = \Omega_{ab}/10$, $\gamma_c = 100\Omega_{ab}$, $\Omega_{ca} = \Omega_{bc} = 10\Omega_{ab}$, and $\Delta_{cb} = \Delta_{ca} = \Delta_{ab} = 0$.

4. SINGLE-PHOTON NONRECIPROCAL TRANSPORT

As an important application, we will discuss how to realize a single-photon nonreciprocal transport between two 1D semi-infinite CRWs by the nonreciprocal transition effect. We assume that two 1D semi-infinite CRWs, with creation operators $a_j^\dagger$ and $b_j^\dagger$ and frequencies $\omega_{w,a}$ and $\omega_{w,b}$ for the jth cavity modes, are coupled by a ∇-type three-level atom ($|a\rangle$, $|b\rangle$, and $|g\rangle$) with nonreciprocal transition $|a\rangle \leftrightarrow |b\rangle$ as shown in Fig. 4. Here, g_a (g_b) is the coupling strength between CRW-a (CRW-b) and the transition $|a\rangle \leftrightarrow |g\rangle$ ($|b\rangle \leftrightarrow |g\rangle$) with frequency ω_{ag} (ω_{bg}). The system can be described by the total Hamiltonian under the rotating wave approximation $H_{\text{tot}} = \sum_{l=a,b} H_l + \tilde{H}_{\text{eff}} + H_{\text{int}}$. Here, in the rotating reference frame with respect to $H_{\text{rot}} = \omega_{ag}(\sum_j a_j^\dagger a_j + |a\rangle\langle a|) + \omega_{bg}(\sum_j b_j^\dagger b_j + |b\rangle\langle b|)$, the Hamiltonian H_l for the CRW-l is given by

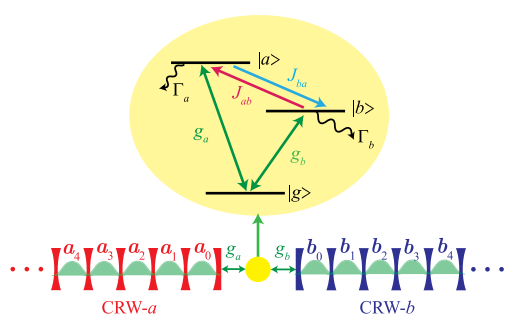

Fig. 4. Schematic of two 1D semi-infinite CRWs connected by a three-level atom characterized by $|a\rangle$, $|b\rangle$, and $|g\rangle$. CRW-a (CRW-b) couples to the three-level atom through the transition $|a\rangle \leftrightarrow |g\rangle$ ($|b\rangle \leftrightarrow |g\rangle$) with strength g_a (g_b).

$$H_l = \Delta_l \sum_{j=0}^{+\infty} l_j^\dagger l_j - \xi_l \sum_{j=0}^{+\infty} (l_j^\dagger l_{j+1} + \text{H.c.}), \tag{6}$$

with homogeneous intercavity coupling constants ξ_l and cavity-atom detunings $\Delta_l = \omega_{w,l} - \omega_{lg}$ $(l = a, b)$; the effective Hamiltonian $\tilde{H}_{\text{eff}}$ for the ∇-type three-level atom with nonreciprocal transition $|a\rangle \leftrightarrow |b\rangle$ is obtained from Eq. (3) with $\Delta_a = \Delta_b = 0$ as

$$\tilde{H}_{\text{eff}} = J_{ab}|a\rangle\langle b| + J_{ba}|b\rangle\langle a| - i\Gamma_a|a\rangle\langle a| - i\Gamma_b|b\rangle\langle b|, \tag{7}$$

and the interaction Hamiltonian H_{int} between the zeroth cavity modes and the three-level atom is described by

$$H_{\text{int}} = g_a a_0|a\rangle\langle g| + g_b b_0|b\rangle\langle g| + g_a a_0^\dagger|g\rangle\langle a| + g_b b_0^\dagger|g\rangle\langle b|. \tag{8}$$

The efficiency for nonreciprocity transport can be described by the scattering flow [40–43] $I_{l'l}$ for a single photon from CRW-l to CRW-l' $(l = a, b)$. The detailed calculations of the scattering flow $I_{l'l}$ can be found in Appendix C. Nonreciprocal single-photon transport appears when $I_{ba} \neq I_{ab}$, which implies that the scattering flow from CRW-a to CRW-b is not equal to that along the opposite direction.

First of all, let us find the optimal conditions for perfect single-photon nonreciprocity, i.e., $I_{ab} = 0$ and $I_{ba} = 1$, analytically. For simplicity, we assume that the two semi-infinite CRWs have the same parameters, i.e., $\xi \equiv \xi_a = \xi_b$, $k \equiv k_a = k_b$, $g \equiv g_a = g_b$, and they are coupled to the atom resonantly with $\Delta_a = \Delta_b = 0$ and $\Gamma \equiv \Gamma_a = \Gamma_b$. Then, $I_{ab} = 0$ can be obtained by setting $J_{ab} = 0$. Through a detailed derivation (see Appendix D), the condition for $I_{ba} = 1$ is $|\sin k| = 1$, i.e., $k = \pi/2$ $(0 < k < \pi)$, in the case that $|J_{ba}| = 2\Gamma$ and $g^2 = \Gamma\xi$. This fits the numerical simulation very well as shown in Figs. 5(a) and 5(b). Luckily, the parameters J_{ab} and J_{ba} as shown in Eqs. (4) and (5) depend on the parameters of the external driving fields, and the optimal conditions for perfect single-photon nonreciprocity can be achieved simultaneously by tuning the strengths and frequencies of the external driving fields.

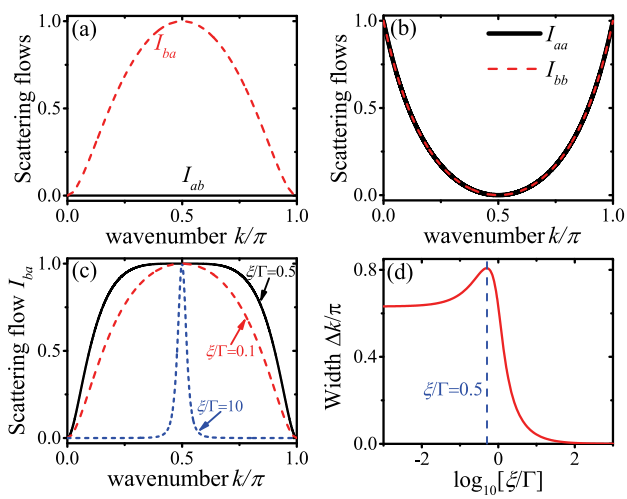

Fig. 5. (a) Scattering flows I_{ab} (black solid curve) and I_{ba} (red dashed curve), (b) I_{aa} (black solid curve) and I_{bb} (red dashed curve), are plotted as functions of the wavenumber k/π for $\xi/\Gamma = 0.1$. (c) Scattering flow I_{ab} is plotted as a function of the wavenumber k/π for different ξ/Γ. (d) The width of the wavenumber Δk for single-photon nonreciprocity is plotted as a function of $\log_{10}(\xi/\Gamma)$ given in Eq. (9). The other parameters are $J_{ba} = 2\Gamma$, $J_{ab} = 0$, $\xi = \Gamma$, $\Delta_a = \Delta_b = 0$, $g^2 = \Gamma\xi$, $\phi = \pi/2$.

Now let us discuss the width of the wavenumber for single-photon nonreciprocity; see Appendix E. We define the width of the wavenumber Δk for single-photon nonreciprocity as the full width at half-maximum (FWHM) by setting $I_{ba} = 1/2$ for $k = k_{\text{half}} \in [0, \pi/2)$:

$$\Delta k \equiv \pi - 2k_{\text{half}}. \tag{9}$$

Under the conditions $|J_{ba}| = 2\Gamma$ and $g^2 = \Gamma\xi$, there is a maximum FWHM for single-photon nonreciprocity at $\xi = \Gamma/2$, and the maximum FWHM $\Delta k_{\max} \approx 0.81\pi$ is obtained with $k_{\text{half}} = \arcsin(2\sqrt{2} - 1 - 2\sqrt{2 - \sqrt{2}})$ in excellent agreement with Figs. 5(c) and 5(d).

5. CONCLUSIONS AND DISCUSSION

In summary, we have shown theoretically that nonreciprocal transition can be observed between two nondegenerate energy levels. A general method has been presented to realize nonreciprocal transition between two nondegenerate energy levels based on a combination of synthetic magnetism and reservoir engineering. As a simple example, we explicitly show an implementation involving an auxiliary energy level, i.e., a cyclic three-level atom system. The generic method for realizing a nonreciprocal transition can be applied to design nonreciprocal phonon devices. A single-photon nonreciprocal transporter has been proposed by the nonreciprocal transition effect. The atom-mediated nonreciprocal devices based on the nonreciprocal transition are suitable for applications in building hybrid quantum networks.

To realize a nonreciprocal transition with a cyclic three-level atom, one ingredient is breaking the symmetry of the potential of the atom. The cyclic three-level transition has been proposed and observed in chiral molecules [44–51]. In addition, the potential of the atom can also be broken by applying an external magnetic field. We can consider a qubit circuit composed of a superconducting loop with three Josephson junctions [52,53] that encloses an applied magnetic flux $\Phi_e = f\Phi_0$ ($\Phi_0 \equiv h/2e$ is the superconducting flux quantum, where h is Planck's constant and $f \equiv \Phi_e/\Phi_0$ is the reduced magnetic flux; e is the charge quantity of one electron). When the reduced magnetic flux f is a half-integer, the potential of the artificial atom is symmetric, and the interaction Hamiltonian has odd parity. However, when f is not a half-integer, the symmetry of the potential is broken, and the interaction Hamiltonian does not have well-defined parity. In this case, transitions can occur between any two levels.

Alternatively, cyclic transitions in a three-level atom can be realized by a single nitrogen-vacancy (NV) center embedded in a mechanical resonator [54]. Three eigenstates ($|0\rangle$ and $|\pm 1\rangle$) of the spin operator along the NV's symmetry axis z (i.e., $S_z|m\rangle = m|m\rangle$) are selected as a three-level atom [55,56]. The two degenerate levels $|\pm 1\rangle$ can be split by applying an external magnetic field along z. We can use

microwave magnetic fields to drive the transitions between $|0\rangle$ and $|\pm 1\rangle$; the magnetic dipole-forbidden transition $|+1\rangle \leftrightarrow |-1\rangle$ can be driven by a time-varying strain field through the mechanical resonator [57,58].

Besides the implementations in a cyclic three-level atom, the nonreciprocal transition can also be implemented in the other physical systems, such as a four-level atom system [59], two qubits in a one-dimensional waveguide [60], and even qubit arrays [61]. The nonreciprocal transition can be extended to explore lasing without inversion [62–65], quantum nonreciprocal physics [66–68], and topological phases [69] in a single multilevel atom or qubit array.

APPENDIX A: HAMILTONIAN FOR CYCLIC THREE-LEVEL ATOM

We consider a cyclic three-level atom ($|a\rangle$, $|b\rangle$, and $|c\rangle$) driven by three classical coherent fields (at rates Ω_{ij}, phases ϕ_{ij}, frequencies ν_{ij}, with $i,j = a,b,c$ and $\nu_{cb} = \nu_{ab} + \nu_{ca}$) that is described by a Hamiltonian given by

$$\begin{aligned}\tilde{H} &= (\omega_{ab} - i\gamma_a)|a\rangle\langle a| - i\gamma_b|b\rangle\langle b| + (\omega_{cb} - i\gamma_c)|c\rangle\langle c| \\ &\quad + (\Omega_{ab}e^{i\phi_{ab}}e^{-i\nu_{ab}t}|a\rangle\langle b| + \Omega_{cb}e^{i\phi_{cb}}e^{-i\nu_{cb}t}|c\rangle\langle b| \\ &\quad + \Omega_{ca}e^{i\phi_{ca}}e^{-i\nu_{ca}t}|c\rangle\langle a| + \text{H.c.}),\end{aligned} \tag{A1}$$

where ω_{ij} is the frequency difference between levels $|i\rangle$ and $|j\rangle$, and the three levels can decay to the other levels with the decay rates γ_i $(i = a,b,c)$.

In the rotating frame respect to the operator $W = e^{-i(\nu_{ab}|a\rangle\langle a| + \nu_{cb}|c\rangle\langle c|)t}$, we have

$$\begin{aligned}H &= W^\dagger \tilde{H} W + i\frac{\mathrm{d}W^\dagger}{\mathrm{d}t}W \\ &= (\Delta_{ab} - i\gamma_a)|a\rangle\langle a| - i\gamma_b|b\rangle\langle b| + (\Delta_{cb} - i\gamma_c)|c\rangle\langle c| \\ &\quad + \Omega_{ab}e^{i\phi_{ab}}|a\rangle\langle b| + \Omega_{cb}e^{i\phi_{cb}}|c\rangle\langle b| + \Omega_{ca}e^{i\phi_{ca}}|c\rangle\langle a| + \text{H.c.},\end{aligned} \tag{A2}$$

with the detuning $\Delta_{ij} \equiv \omega_{ij} - \nu_{ij}$ $(i,j = a,b,c)$. By local redefinition of the eigenstates, i.e., $e^{i\phi_{cb}}\langle b| \to \langle b|$ and $e^{-i\phi_{ca}}|a\rangle \to |a\rangle$, the Hamiltonian can be rewritten as Eq. (2) in the main text with the synthetic magnetic flux $\Phi \equiv \phi_{ab} - \phi_{cb} + \phi_{ca}$.

APPENDIX B: ADIABATIC ELIMINATION

We will derive the effective Hamiltonian Eq. (3) by eliminating the level $|c\rangle$ (the engineered reservoir) adiabatically. The state vector for these three levels at time t can be written as

$$|\psi\rangle = A(t)|a\rangle + B(t)|b\rangle + C(t)|c\rangle. \tag{B1}$$

The coefficients $|A(t)|^2$, $|B(t)|^2$, and $|C(t)|^2$ denote occupying probabilities in states $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively. Then the dynamical behaviors for the coefficients can be obtained by the Schrödinger equation, i.e., $i|\dot{\psi}\rangle = H|\psi\rangle$, given by

$$\dot{A}(t) = (-i\Delta_{ab} - \gamma_a)A(t) - i\Omega_{ab}e^{i\Phi}B(t) - i\Omega_{ca}C(t), \tag{B2}$$

$$\dot{B}(t) = -\gamma_b B(t) - i\Omega_{ab}e^{-i\Phi}A(t) - i\Omega_{cb}C(t), \tag{B3}$$

$$\dot{C}(t) = (-i\Delta_{cb} - \gamma_c)C(t) - i\Omega_{ca}A(t) - i\Omega_{cb}B(t). \tag{B4}$$

Under the assumption that the decay of the state $|c\rangle$ is much faster than decay of the states $|a\rangle$ and $|b\rangle$ with the conditions $\min\{\omega_{ca}, \omega_{cb}\} \gg \gamma_c \gg \max\{\Omega_{ca}, \Omega_{cb}, \gamma_a, \gamma_b\}$, we can adiabatically eliminate level $|c\rangle$ with $\dot{C}(t) = 0$ as

$$C(t) = \frac{-i\Omega_{ca}}{\gamma_c + i\Delta_{cb}}A(t) - \frac{i\Omega_{cb}}{\gamma_c + i\Delta_{cb}}B(t). \tag{B5}$$

By substituting Eq. (B5) into Eqs. (B2) and (B3), then the dynamical equations of $A(t)$ and $B(t)$ become

$$\begin{aligned}\dot{A}(t) &= -\left[i\left(\Delta_{ab} - \frac{\Omega_{ca}^2\Delta_{cb}}{\gamma_c^2 + \Delta_{cb}^2}\right) + \left(\gamma_a + \frac{\Omega_{ca}^2\gamma_c}{\gamma_c^2 + \Delta_{cb}^2}\right)\right]A(t) \\ &\quad - \left[i\Omega_{ab}e^{i\Phi} + \frac{\Omega_{ca}\Omega_{cb}(\gamma_c - i\Delta_{cb})}{\gamma_c^2 + \Delta_{cb}^2}\right]B(t),\end{aligned} \tag{B6}$$

$$\begin{aligned}\dot{B}(t) &= -\left[-i\frac{\Omega_{cb}^2\Delta_{cb}}{\gamma_c^2 + \Delta_{cb}^2} + \left(\gamma_b + \frac{\Omega_{cb}^2\gamma_c}{\gamma_c^2 + \Delta_{cb}^2}\right)\right]B(t) \\ &\quad - \left[i\Omega_{ab}e^{-i\Phi} + \frac{\Omega_{ca}\Omega_{cb}(\gamma_c - i\Delta_{cb})}{\gamma_c^2 + \Delta_{cb}^2}\right]A(t).\end{aligned} \tag{B7}$$

Physically, the dynamic equations in Eqs. (B6) and (B7) correspond to the Schrödinger evolution of the effective Hamiltonian Eq. (3) in the main text.

APPENDIX C: SCATTERING FLOW

To study the nonreciprocal single-photon transport, we discuss the scattering of a single photon in the system with the total Hamiltonian in the rotating reference frame with respect to H_{rot} as

$$H_{\text{tot}} = \sum_{l=a,b} H_l + \tilde{H}_{\text{eff}} + H_{\text{int}}, \tag{C1}$$

where the Hamiltonians H_l, $\tilde{H}_{\text{eff}}$, and H_{int} are given in Eqs. (6)–(8) in the main text. As the total number of photons in the system is a conserved quantity (without dissipation), we consider the stationary eigenstate of a single photon in the system as

$$|E\rangle = \sum_{j=0}^{+\infty}[u_a(j)a_j^\dagger + u_b(j)b_j^\dagger]|g,0\rangle + A|a,0\rangle + B|b,0\rangle, \tag{C2}$$

where $|0\rangle$ indicates the vacuum state of the 1D semi-infinite CRWs, $u_l(j)$ denotes the probability amplitude in the state with a single photon in the jth cavity of the CRW-l, and A (B) denotes the probability amplitude in the atom state $|a\rangle$ ($|b\rangle$). Substituting the stationary eigenstate $|E\rangle$ in Eq. (C2) and the total Hamiltonian H_{tot} into the eigenequation $H_{\text{tot}}|E\rangle = E|E\rangle$, we can obtain the coupled equations for the probability amplitudes as

$$\Delta_a u_a(0) - \xi_a u_a(1) + g_a A = E u_a(0), \tag{C3}$$

$$\Delta_b u_b(0) - \xi_b u_b(1) + g_b B = E u_b(0), \tag{C4}$$

$$-i\Gamma_a A + g_a u_a(0) + J_{ab}B = EA, \tag{C5}$$

$$-i\Gamma_b B + g_b u_b(0) + J_{ba}A = EB, \tag{C6}$$

$$\Delta_l u_l(j) - \xi_l[u_l(j+1) + u_l(j-1)] = E u_l(j), \tag{C7}$$

with $j > 0$ and $l = a, b$.

If a single photon with energy E is incident from the infinity side of CRW-l, the ∇-type three-level atom will result in photon scattering between different CRWs or photon absorption by the dissipative of the atom. The general expressions of the probability amplitudes in the CRWs ($j \geq 0$) are given by

$$u_l(j) = e^{-ik_l j} + s_{ll} e^{ik_l j}, \tag{C8}$$

$$u_{l'}(j) = s_{l'l} e^{ik_{l'} j}, \tag{C9}$$

where $s_{l'l}$ denotes the single-photon scattering amplitude from CRW-l to CRW-l' ($l, l' = a, b$). Substituting Eq. (C8) or Eq. (C9) into Eq. (C7), the eigenvalue of the semi-infinite CRW-l in the rotating reference frame is given by [40]

$$E = \Delta_l - 2\xi_l \cos k_l, \quad 0 < k_l < \pi, \tag{C10}$$

where k_l is the wavenumber of the single photon in the CRW-l. Without loss of generality, we assume that $\xi_l > 0$ and $0 < k_l < \pi$ for semi-infinite CRW-l.

Now let us derive the scattering amplitudes for single-photon scattering by the atom with nonreciprocal transition. By solving Eqs. (C5) and (C6), the coefficients A and B can be expressed by

$$A = \frac{(E + i\Gamma_b) g_a u_a(0) + J_{ab} g_b u_b(0)}{(E + i\Gamma_a)(E + i\Gamma_b) - J_{ba} J_{ab}}, \tag{C11}$$

$$B = \frac{(E + i\Gamma_a) g_b u_b(0) + J_{ba} g_a u_a(0)}{(E + i\Gamma_a)(E + i\Gamma_b) - J_{ba} J_{ab}}. \tag{C12}$$

Substituting A and B into Eqs. (C3) and (C4), we have

$$(\Delta_a - E + \overline{\Delta}_a) u_a(0) + J'_{ab} u_b(0) = \xi_a u_a(1), \tag{C13}$$

$$(\Delta_b - E + \overline{\Delta}_b) u_b(0) + J'_{ba} u_a(0) = \xi_b u_b(1), \tag{C14}$$

with the effective coupling strengths $J'_{ll'}$ and frequency shifts $\overline{\Delta}_l$ induced by the ∇-type three-level atom defined by

$$J'_{ab} = \frac{J_{ab} g_a g_b}{(E + i\Gamma_a)(E + i\Gamma_b) - J_{ba} J_{ab}}, \tag{C15}$$

$$J'_{ba} = \frac{J_{ba} g_a g_b}{(E + i\Gamma_a)(E + i\Gamma_b) - J_{ba} J_{ab}}, \tag{C16}$$

$$\overline{\Delta}_a = \frac{(E + i\Gamma_b) g_a^2}{(E + i\Gamma_a)(E + i\Gamma_b) - J_{ba} J_{ab}}, \tag{C17}$$

$$\overline{\Delta}_b = \frac{(E + i\Gamma_a) g_b^2}{(E + i\Gamma_a)(E + i\Gamma_b) - J_{ba} J_{ab}}. \tag{C18}$$

When a single photon is input from CRW-a, we have $u_a(j) = e^{-ik_a j} + s_{aa} e^{ik_a j}$ and $u_b(j) = s_{ba} e^{ik_b j}$, and the scattering amplitudes s_{aa} and s_{ba} satisfy the following equations:

$$(\xi_a e^{-ik_a} + \overline{\Delta}_a) s_{aa} + J'_{ab} s_{ba} = -\xi_a e^{ik_a} - \overline{\Delta}_a, \tag{C19}$$

$$J'_{ba} s_{aa} + (\xi_b e^{-ik_b} + \overline{\Delta}_b) s_{ba} = -J'_{ba}. \tag{C20}$$

Similarly, when a single photon is input from CRW-b, we have $u_b(j) = e^{-ik_b j} + s_{bb} e^{ik_b j}$ and $u_a(j) = s_{ab} e^{ik_a j}$, and the scattering amplitudes s_{ab} and s_{bb} satisfy the following equations:

$$(\xi_a e^{-ik_a} + \overline{\Delta}_a) s_{ab} + J'_{ab} s_{bb} = -J'_{ab}, \tag{C21}$$

$$J'_{ba} s_{ab} + (\xi_b e^{-ik_b} + \overline{\Delta}_b) s_{bb} = -\xi_b e^{ik_b} - \overline{\Delta}_b. \tag{C22}$$

Equations (C19)–(C22) can be expressed concisely in matrix form as

$$LS = R, \tag{C23}$$

with the scattering matrix

$$S = \begin{pmatrix} s_{aa} & s_{ab} \\ s_{ba} & s_{bb} \end{pmatrix} \tag{C24}$$

and coefficient matrices

$$L = \begin{pmatrix} \xi_a e^{-ik_a} + \overline{\Delta}_a & J'_{ab} \\ J'_{ba} & \xi_b e^{-ik_b} + \overline{\Delta}_b \end{pmatrix}, \tag{C25}$$

$$R = -\begin{pmatrix} \xi_a e^{ik_a} + \overline{\Delta}_a & J'_{ab} \\ J'_{ba} & \xi_b e^{ik_b} + \overline{\Delta}_b \end{pmatrix}. \tag{C26}$$

The solutions of Eq. (C23) are given by

$$s_{aa} = \frac{J'_{ab} J'_{ba} - (\xi_a e^{ik_a} + \overline{\Delta}_a)(\xi_b e^{-ik_b} + \overline{\Delta}_b)}{(\xi_a e^{-ik_a} + \overline{\Delta}_a)(\xi_b e^{-ik_b} + \overline{\Delta}_b) - J'_{ab} J'_{ba}}, \tag{C27}$$

$$s_{ba} = \frac{2i\xi_a J'_{ba} \sin k_a}{(\xi_a e^{-ik_a} + \overline{\Delta}_a)(\xi_b e^{-ik_b} + \overline{\Delta}_b) - J'_{ab} J'_{ba}}, \tag{C28}$$

$$s_{ab} = \frac{2i\xi_b J'_{ab} \sin k_b}{(\xi_a e^{-ik_a} + \overline{\Delta}_a)(\xi_b e^{-ik_b} + \overline{\Delta}_b) - J'_{ab} J'_{ba}}, \tag{C29}$$

$$s_{bb} = \frac{J'_{ab} J'_{ba} - (\xi_a e^{-ik_a} + \overline{\Delta}_a)(\xi_b e^{ik_b} + \overline{\Delta}_b)}{(\xi_a e^{-ik_a} + \overline{\Delta}_a)(\xi_b e^{-ik_b} + \overline{\Delta}_b) - J'_{ab} J'_{ba}}. \tag{C30}$$

To quantify the efficiency for nonreciprocity transport, we define the scattering flow [41–43] of a single photon from CRW-l to CRW-l' as

$$I_{l'l} \equiv |s_{l'l}|^2 \frac{\xi_{l'} \sin k_{l'}}{\xi_l \sin k_l}, \tag{C31}$$

where $\xi_l \sin k_l$ ($\xi_{l'} \sin k_{l'}$) is the group velocity in the CRW-l (CRW-l').

APPENDIX D: PERFECT SINGLE-PHOTON NONRECIPROCITY

In this section, we will derive the conditions for perfect nonreciprocal single-photon transport, i.e., $I_{ab} = 0$ and $I_{ba} = 1$, analytically. For simplicity, we assume that the two semi-infinite CRWs have the same parameters, i.e., $\xi \equiv \xi_a = \xi_b$, $k \equiv k_a = k_b$, $g \equiv g_a = g_b$, and they are coupled to the atom resonantly with $\Delta_a = \Delta_b = 0$ and $\Gamma \equiv \Gamma_a = \Gamma_b$. $I_{ab} = 0$ can be obtained by setting $J_{ab} = 0$ or $J'_{ab} = 0$. In this case, we have

$$I_{ba} = |s_{ba}|^2 = \left| \frac{2J_{ba} g^2 \xi \sin k}{[\xi e^{-ik}(-2\xi \cos k + i\Gamma) + g^2]^2} \right|^2. \tag{D1}$$

So the condition for $I_{ba} = 1$ is

$$|\sin k| = \frac{(|J_{ba}| - \Gamma)g^2 \pm \sqrt{\Theta}}{4(g^2 - \xi^2)\xi}, \tag{D2}$$

with

$$\Theta = (|J_{ba}| - \Gamma)^2 g^4 - 4(g^2 - \xi^2)[4(\xi^2 - g^2)\xi^2 + \Gamma^2\xi^2 + g^4]. \tag{D3}$$

As a simple example, the maximum scattering flow $I_{ba} = 1$ can be obtained at the maximum group velocity $|\sin k| = 1$, with

$$|J_{ba}| = \frac{(g^2 + \Gamma\xi)^2}{2g^2\xi}. \tag{D4}$$

Furthermore, if $g^2 = \Gamma\xi$, then we have

$$|J_{ba}| = 2\Gamma. \tag{D5}$$

APPENDIX E: MAXIMUM FULL WIDTH AT HALF-MAXIMUM

We will derive the maximum full width at half-maximum (FWHM) for perfect nonreciprocal single-photon transport. The half-maximum of the scattering flow I_{ba} is given by

$$I_{ba} = |s_{ba}|^2 = \left| \frac{2J_{ba}g^2\xi \sin k_{\text{half}}}{[\xi e^{-ik}(-2\xi \cos k_{\text{half}} + i\Gamma) + g^2]^2} \right|^2 = \frac{1}{2}. \tag{E1}$$

Under the conditions that $|J_{ba}| = 2\Gamma$ and $g^2 = \Gamma\xi$, we have

$$2(\Gamma - \xi)\xi|\sin k_{\text{half}}|^2 + (1 - 2\sqrt{2})\Gamma^2|\sin k_{\text{half}}| + 2(\xi - \Gamma)\xi + \Gamma^2 = 0. \tag{E2}$$

Defining $\eta \equiv \xi/\Gamma$ and $\zeta \equiv 4(1 - \eta)\eta$, Eq. (E2) can be rewritten as

$$|\sin k_{\text{half}}| = \frac{-(1 - 2\sqrt{2}) \pm \sqrt{(1 - 2\sqrt{2})^2 - \zeta(2 - \zeta)}}{\zeta}. \tag{E3}$$

The condition for maximum width $\Delta k_{\max}$ is

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\Big|\sin k_{\text{half}}| = \frac{\mathrm{d}|\sin k_{\text{half}}|}{\mathrm{d}\zeta}\frac{\mathrm{d}\zeta}{\mathrm{d}\eta} = 0, \tag{E4}$$

which is satisfied with

$$\eta = \frac{1}{2} \Rightarrow \xi = \frac{\Gamma}{2}. \tag{E5}$$

That is to say, the maximum width $\Delta k_{\max}$ is obtained at $\xi = \Gamma/2$ with

$$|\sin k_{\text{half}}| = 2\sqrt{2} - 1 - 2\sqrt{2 - \sqrt{2}}, \tag{E6}$$

and the maximum FWHM $\Delta k_{\max}$ is

$$\Delta k_{\max} \equiv \pi - 2 \arcsin\left(2\sqrt{2} - 1 - 2\sqrt{2 - \sqrt{2}}\right) \approx 0.81\pi. \tag{E7}$$

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