Plasmonic anapole states of active metamolecules

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Anapole states, accompanied by strong suppression of light scattering, have attracted extensive attention in recent years due to their supreme performance in enhancing both linear and nonlinear optical effects. Although both low- and high-order anapole states are observed in the dielectric particles with high refractive index, so far few studies have touched on the topic of plasmonic anapole states. Here we demonstrate theoretically and numerically that the ideal plasmonic anapole states (strong suppression of electric dipole scattering) can be achieved in metallic metamolecules via increasing the coupling strength between Cartesian electric dipole and toroidal dipole moments of the system. The increasing coupling is based on compensation of ohmic losses in a plasmon system by introducing of a gain material, the influence of which is well described by the extended coupled oscillator model. Due to suppression of dipole radiation losses, the excitation of anapole states in plasmonic systems can result in enhancement of the near fields in subwavelength spatial regions outside of nanoparticles. That is especially important for developments of nonlinear nanophotonic and plasmonic devices and active functional metamaterials, which provide facilities for strong light energy concentration at the nanoscale. Development of the considered anapole effect with increase of metamolecule components is discussed.

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1. INTRODUCTION

The control of light propagation and concentration due to surface-plasmon resonances has great implication for the fundamentals and applications of nanophotonics [1,2]. In this context, Mie resonances from the optical spectral range [3–6], supported by high-index dielectric nanoparticles and nanostructures, also attract significant attention from the research community [7]. There are light scattering similarities between plasmonic (metallic) particles and their dielectric counterparts. In the both cases the light scattering can be considered as a light reradiation by multipole sources excited in a scatterer by external incident waves. In this case the total scattered fields are imagined to be a superposition of the fields generated by every multipole moment of the scatterer. As a result, the total and differential scattering cross sections can be decomposed on separate contributions related to certain multipole moments. Such multipole approaches can significantly simplify the analysis of the light scattering process and provide important information about material, shape, and size parameters of scattering nanoparticles and nanostructures. However, there are several principal differences between optical response of metal and dielectric nanoparticles. If for metal nanoparticles the optical response is determined by excitation of the free electron conductive current absorbing light energy due to ohmic losses, the optical reaction of all-dielectric nanoparticles is associated with excitation of the displacement currents without the losses of absorption. In the last case, the linear light–matter interaction is solely determined by scattering. Importantly, the strong electromagnetic fields are concentrated in the near-field zone around metal nanoparticles at the resonant conditions [8], whereas the enhancement of the fields for all-dielectric nanoparticles at the resonant conditions is realized in their volumes [5]. The different optical reactions of the metal and dielectric nanoparticles result in differences of their functional properties used in practical applications [9,10].

Recently, the anapole states are observed in many high-refractive-index dielectric particles that support both electric and magnetic resonances and have zero damping loss [11–13]. An ideal anapole state is known as a state with complete scattering cancellation in the far-field and nonzero near-field excitation. From the theoretical point of view, the typical properties of anapole states exhibit zero polarized multipole moments and high near-field enhancement inside the region occupied by the
scatterers [11, 14–16]. The zero polarized moments of anapole states on any dimensionality could constitute the basis of dark matter in the universe [17]. Based on the nonradiative traits, anapole states are increasingly applied to the fields of nonlinear nanophotonics, dielectric metamaterials, light harvesting, and sensing [15, 18–23]. In addition, the total suppressed electric dipole (ED) moment, due to the dipole anapole effect, contributes to the achievement of pure magnetic dipole (MD) scattering [14]. Due to the differences between the optical responses of metallic and dielectric nanoparticles and nanostructures, anapole states have so far been carefully studied, mainly in dielectric nanoparticles with a high refractive index and their structures. In particular, strong ohmic losses of metal nanostructures are the main reason for hindering implementation of the anapole analog in metal nanostructures.

Generally, anapole states, including electric and magnetic anapole states, consist of low- and high-order multipoles [15, 24–26]. For example, the excitation of electric and magnetic anapole states in a hybrid metal-dielectric structure is theoretically investigated in our previous work [15]. The key to exciting anapole states relies on the effective generation of toroidal dipole (TD) moments, which can be imagined as high-order dipole terms of the Cartesian multipole decomposition [20, 27–32]. In the case of electric anapole states, they are realized due to the totally destructive interference between Cartesian TD and ED moments, resulting in significant suppression of the total ED moment and corresponding dipole scattering [11, 29, 30]. Importantly, the excitation of the TD moment is accompanied by the circulating magnetic field in scattering systems [33–35]. Therefore, formally the existence of electric anapole states can be expected even in metallic nanoparticles that support optical resonances accompanied by the circulating magnetic field in the metamolecule volume [36, 37]. Nevertheless, the anapole condition cannot be fully satisfied in the lossy system due to the different sensitivities of different-order multipole moments to ohmic losses.

In this paper, we demonstrate both analytically and numerically that the plasmonic anapole states (suppression of the electric dipole scattering) can be excited in metallic metamolecules consisting of several nanoparticles by enhancing the destructive interference between Cartesian ED and TD moments of the total system. It is worth mentioning that the excitation of anapole states in this work is independent of high-refractive-index dielectric nanoparticles. The ideal plasmonic anapole state is achieved when the coupling strength between Cartesian ED and TD moments reaches its maximum value. Due to the different sensitivities of ohmic losses between Cartesian ED (low-order electric mode) and TD (higher-order electric mode) moments [33], the coupling strength can be enhanced by the reasonable compensation of ohmic losses of the nanosystem (0.008 eV² for the passive case and 0.014 eV² for the active case). This process is well characterized by the extended coupled oscillator (ECO) model [38]. Compared to the anapole states of high-refractive-index nanoparticle origin, the plasmonic anapole states induce stronger intensity of near fields outside nanoparticles due to the participation of surface plasmons.

2. SUPPRESSION OF ELECTRIC DIPOLE SCATTERING

Schematic diagrams of the Au and Au–SiO₂ heptamer are shown in the insets of Fig. 1. The structural parameters selected in the calculation are compatible with current nanofabrication technology [39]. The heights of Au and SiO₂ nanodisks are \( h_1 = 50 \text{ nm} \) and \( h_2 = 30 \text{ nm} \), respectively. The diameter of the central disk is \( 2R = 208 \text{ nm} \). It is symmetrically surrounded by six nanodisks with an equal diameter of \( 2r = 195 \text{ nm} \). The gap between the central disk and the surrounding ones is set to \( g = 20 \text{ nm} \). A monochromatic plane wave with time dependence in the view of \( e^{-\text{i} \omega t} \) polarized along the \( y \) axis illuminates the sample from the Au side, where \( \omega \) is the angular frequency. The heptamer is suspended in air with refractive index \( n = 1.33 \). The ideal plasmonic anapole state is achieved when the coupling strength between Cartesian ED and TD moments reaches its maximum value. Due to the different sensitivities of ohmic losses between Cartesian ED (low-order electric mode) and TD (higher-order electric mode) moments [33], the coupling strength can be enhanced by the reasonable compensation of ohmic losses of the nanosystem (0.008 eV² for the passive case and 0.014 eV² for the active case). This process is well characterized by the extended coupled oscillator (ECO) model [38]. Compared to the anapole states of high-refractive-index nanoparticle origin, the plasmonic anapole states induce stronger intensity of near fields outside nanoparticles due to the participation of surface plasmons.

Fig. 1. Scattering cross sections and their spherical multipole decomposition calculated for (a) Au heptamer, (b) passive Au–SiO₂ heptamer with gain coefficient \( \kappa = 0 \), and (c) active Au–SiO₂ heptamer with gain coefficient \( \kappa = 0.28 \). The structure schematics are shown in the insets.
where $\psi$ denotes the order of the multipole components, such as dipole ($l = 1$) and quadrupole ($l = 2$). $\mathbf{J}_{\text{sc}}(\mathbf{r}) = \mathbf{J}_{\text{Ed}}(\mathbf{r}) + \mathbf{J}_{\text{Td}}(\mathbf{r})$ is the electric field amplitude of the incident light. $\phi$ and $\theta$ are the azimuthal and polar angles, respectively. The electric and magnetic multipole scattering cross sections are expressed as $C_E = \int d^3r|\mathbf{E}_{\text{sc}}(\mathbf{r})|^2$ and $C_M = \int d^3r|\mathbf{H}_{\text{sc}}(\mathbf{r})|^2$, respectively. We consider that all of the electric and magnetic multipoles are located at the center of mass of the Au–SiO$_2$ nanomolecules.

The scattering cross sections and corresponding spherical multipole expansions are shown in Fig. 1. The total scattering cross section of the Au heptamer exhibits a dip at the wavelength about 850 nm with nonzero spherical ED scattering [see Fig. 1(a)]. To dope the gain material, the low-refractive-index SiO$_2$ layer is introduced to the Au heptamer [see Figs. 1(b) and 1(c)]. Since SiO$_2$ has weak optical response at the near-infrared wavelengths, the SiO$_2$ disks in the nanostructure basically play the role of host medium of the gain materials. The introduction of the SiO$_2$ layer leads to the redshift of the dip. In the passive nanostructure, the total scattering cross section exhibits a dip at the wavelength about 900 nm with nonzero spherical ED scattering [see Fig. 1(b)]. In contrast, when a gain material with the coefficient of 0.28 is introduced to the nanosystem, the ED scattering is completely suppressed at the wavelength of total scattering dip as shown in Fig. 1(c).

### 3. Excitation of Plasmonic Anapole

For explanation of the ED scattering suppression we use the multipole decomposition in the framework of the Cartesian multipole determined in the long wavelength approximation (LWA) [29]. In this case the full Cartesian ED moment can be presented as a sum of infinite series of dipole terms [44,45]. For a relatively small scatterer, the series can be restricted to only the first several terms. If we take only the first two terms, the full electric dipole moment can be approximated by the following expression: $\mathbf{D} = \mathbf{p} + ik\mathbf{T}$ [29]. Then the electric dipole part of the scattering cross section is given by
the high-refractive-index (silicon) nanoparticles. In addition, the excitation of anapole states in plasmonic systems can result in enhancement of the near fields in subwavelength spatial regions outside of nanoparticles. That is especially important for developments of nonlinear nanophotonic, plasmonic devices and functional metamaterials, which provide facilities for strong light energy concentration at the nanoscale. The tremendous near-field enhancements are advantageous for boosting Raman scattering [46], fluorescence [47], and nonlinear effects [15]. Moreover, compared to the high-refractive-index nanoparticles, the anapole mode can be excited in the metallic metamolecules with smaller volume.

4. COUPLED OSCILLATORS MODEL

In order to better understand the achievement of anapole condition via loss-compensation mechanism, we adopt an ECO model to analyze the process of the compensation of ohmic losses. In reality, the anapole mode belongs to the category of Fano resonance, which can be well characterized by the ECO model. Herein, the two-coupled oscillators model can be expressed as [38]

\[
\ddot{x}_1 + \gamma_1 \dot{x}_1 + \omega_1^2 x_1 - \nu_{12} x_2 = 0.5P_{\text{tot}} + \alpha_1 E_{\text{ext}},
\]

\[
\ddot{x}_2 + \gamma_2 \dot{x}_2 + \omega_2^2 x_2 - \nu_{21} x_1 = 0.5P_{\text{tot}} + \alpha_2 E_{\text{ext}},
\]

where \(x_1\) and \(x_2\) are the displacements from the equilibrium position of the first and second oscillators with the harmonic solution form of \(x_1(t) = c_1 e^{i\omega_1 t}\) and \(x_2(t) = c_2 e^{i\omega_2 t}\) [48]. \(\gamma_1\) and \(\gamma_2\) are the damping coefficients of the first and second oscillators accounting for intrinsic losses, respectively. \(\omega_1\) and \(\omega_2\) are the natural frequencies (eigenmodes) of the two oscillators. \(\nu_{12}\) and \(\nu_{21}\) describe the coupling strength between the two oscillators \((\nu_{12} = \nu_{21})\). \(P_{\text{tot}} = \alpha_1 x(t) + \alpha_2 x(t)\) is the total dipole moment of the system describing the radiative damping, where \(\alpha_1\) and \(\alpha_2\) are the polarizabilities corresponding to the plasmon amplitude to their induced dipole moments. The oscillators are driven by the harmonic forces \(\alpha_1 E_{\text{ext}}\) and \(\alpha_2 E_{\text{ext}}\), where \(E_{\text{ext}} = E_0 e^{i\omega t}\). Note that the first and second oscillators indicate the Cartesian ED and TD moments, respectively.
The total scattering cross section is $|c_1 + c_2|^2$ [38], where $c_1$ and $c_2$ are obtained by solving Eqs. (4) and (5).

The total scattering spectra of the Au–SiO$_2$ heptamer are calculated via the FEM and ECO models, respectively. According to Fig. 3(a), the $\kappa$-dependent total scattering cross sections are well fitted by the ECO model (the fitting parameters from the ECO model are shown in Table 1). The first oscillator models the radiative mode, which has a high damping coefficient $\gamma_1$. In contrast, the second oscillator models the

### Table 1. Parameters from ECO Model for Different Gain Coefficients [Fig. 3(a)]

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$E_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>11.300</td>
<td>0.173</td>
<td>0.008</td>
<td>1.230</td>
<td>0.163</td>
<td>1.675</td>
<td>1.387</td>
<td>0.008</td>
</tr>
<tr>
<td>0.09</td>
<td>10.800</td>
<td>0.170</td>
<td>0.008</td>
<td>1.100</td>
<td>0.162</td>
<td>1.655</td>
<td>1.382</td>
<td>0.010</td>
</tr>
<tr>
<td>0.18</td>
<td>10.100</td>
<td>0.170</td>
<td>0.008</td>
<td>0.970</td>
<td>0.158</td>
<td>1.630</td>
<td>1.380</td>
<td>0.012</td>
</tr>
<tr>
<td>0.28</td>
<td>10.100</td>
<td>0.168</td>
<td>0.009</td>
<td>0.920</td>
<td>0.154</td>
<td>1.610</td>
<td>1.380</td>
<td>0.014</td>
</tr>
<tr>
<td>0.37</td>
<td>10.300</td>
<td>0.169</td>
<td>0.009</td>
<td>1.030</td>
<td>0.152</td>
<td>1.630</td>
<td>1.385</td>
<td>0.009</td>
</tr>
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</table>

The total scattering cross sections and their spherical multipole decomposition calculated for Au-SiO$_2$ thirteen polymer with the near-field distribution $E/E_0$ on top of Au disks (at the plane of $z = 50$ nm) for an active nanosystem (a) $\kappa = 0$ and (b) $\kappa = 0.32$. 

![Fig. 3](image3.png)

![Fig. 4](image4.png)
The coupling strength between Cartesian ED and TD moments. According to Fig. 3(b), the damping coefficients ($\gamma_1$ and $\gamma_2$) of the two oscillators decrease with the increase of gain coefficient. In other words, the compensation of ohmic losses leads to the decrease of total damping losses of the oscillators. Meanwhile, the decrease of the damping loss leads to the increase of the coupling strength between the two oscillators as shown in Fig. 3(c). However, when gain coefficient is higher than 0.28 ($\kappa > 0.28$), the damping loss of the first oscillator starts to increase. The damping loss of the second oscillator remains in decay. According to Fig. 3, the increase of damping loss of the first oscillator and the decrease of damping loss of the second one not only result in the stronger scattering peak of the low energy but also lead to the reduction of the coupling strength $n_{12}$. Thus, when the gain coefficient reaches 0.28, the coupling strength between the two oscillators comes to its maximum value where the anapole condition is achieved, leading to the completely destructive interference.

5. HIGH-ORDER PLASMONIC ANAPOLE STATE

In addition, we also demonstrate that a high-order anapole state is also achieved in metallic metamolecules by increasing the coupling strength between Cartesian ED and TD moments. In Au–SiO$_2$ thirteen polymer, all the diameters of the center five disks are 208 nm, and the surrounding disks with the diameters of 195 nm are symmetrically distributed around the center disk with the separation $g = 20$ nm. Figures 4(a) and 4(b) indicate that if the gain coefficient reaches 0.32, the anapole state at about 931 nm is achieved. Besides, according to the near-field distributions of $F/E_0$ plotted in Figs. 4(a) and 4(b), the anapole state at 931 nm is the high-order anapole state. The enhancement of the induced near fields of high-order anapole mode origin is much stronger than that of low-order anapole mode origin, which suggests the superiority of the high-order mode in concentrating light in subwavelength regions. The excitation of the higher-order anapole state suggests that the anapole states supported by high-refractive-index particles [49] can be also achieved in the subtly designed metallic structures. Furthermore, the generation of mechanisms of many nonlinear effects from different modes are vastly different, which makes great implications for the excitations of different anapole states. The excitations of low- and high-order anapole states greatly contribute to designing different types of functional metamaterials.

6. CONCLUSION

In summary, it has been theoretically shown that low- and high-order anapole states can be excited in metallic metamolecules. As both the electric and magnetic modes are supported by the metallic metamolecules, it offers the necessary conditions of the origin of the anapole states. The excitation of the ideal plasmonic anapole state depends on the effective excitation of the TD moment and fine-tuning it to ensure the completely destructive interference with the Cartesian ED moment. The totally destructive interference is achieved by enhancing the coupling strength. We have overcome a major limitation of plasmonic (metal) systems in realization of anapole states, related with strong ohmic absorption of light, by doping of the system by a gain material. Note that the spectral position and width of the gain effects depend on the materials used, and the environment and can vary widely. In this case, from the practical point of view, the implementation of the anapole state can be achieved in a required spectral gain range by adjusting the dimensions and material parameters of the total structure. The developed coupled oscillator model with active terms has been used for clarification of the physical process in the anapole state formation. Compared to the high-refractive-index nanoparticles, the plasmonic anapole modes can be excited in the metallic metamolecules with greater enhancement of near fields, which indicate the remarkable energy concentration performance and have more important implications for enhancing Raman scattering, fluorescence, and nonlinear effects. The theoretical results in this paper can be realized with the current nanofabrication technology, which not only open a route to study the anapole modes but also provide a new way of thinking to achieve the total suppression of noise modes and increase the signal-to-noise ratio of the target modes. Application of the considered structures, as buildings blocks for material developments, can extend varied physical approaches [50–52] to the creation of new materials and metamaterials with special functional properties.

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