PHOTONICS Research

Rogue wave light bullets of the three-dimensional inhomogeneous nonlinear Schrödinger equation

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We discover single and homocentric optical spheres of the three-dimensional inhomogeneous nonlinear Schrödinger equation (NLSE) with spherical symmetry, which is a novel model of light bullets that can present a three-dimensional rogue wave. The isosurface of this light bullet oscillates along the radius direction and does not travel with the evolution of time. The localized nature of rogue wave light bullets both in space and in time, which is in complete contrast to the traveling character of the usual light bullets, is due to the localization of the rogue wave in the one-dimensional NLSE. We present also an investigation of the stability of the optical sphere solutions. The lower modes of perturbation are found to display transverse instabilities that break the spherical symmetry of the system. For the higher modes, the optical sphere solutions can be classified as stable solutions.

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1. INTRODUCTION

In recent years, considerable effort has been devoted to the investigation of optical solitary waves [1] due to their fundamental impact on nonlinear wave propagation, spawning exciting applications along the way such as supercontinuum light sources [2], soliton lasers [3], and an improved understanding of the development and control of rogue waves (RWs) [4]. A special type of solitary waves, the RW phenomenon, which was first observed in the ocean, is a rare, short-lived, and high-energy event with amplitude much higher than the average wave crests around it [5]. The typical feature of RWs is that they suddenly appear and increase up to a very high and abnormal amplitude to finally disappear without a trace [6,7]. The experimental observation and theoretical analysis of RWs have ranged from Bose–Einstein condensates (BECs) [8,9] to optical systems [10–12], oceans [13], superfluids [14], and plasmas [15,16]; see more details in two recent review articles [17,18]. One possible mathematical model to describe such RWs is the rational solution of one-dimensional nonlinear Schrödinger (NLS) type equations. Moreover, research has diversified, also addressing optical solitary waves in higher-dimensional media, which display a more complex phenomenology due to the increased degrees of freedom [19].

In this context, the formation of self-trapped wave packets or light bullets, is one of the most exciting yet experimentally unsolved problems in optics [20,21]. Light bullets are spatiotemporal solitons that form when a suitable nonlinearity arrests both spatial diffraction and temporal group-velocity dispersion. Despite considerable theoretical work, experimental research on light bullets is rare, and one of the main reasons is that in nonlinear propagation, three-dimensional light bullets tend to disintegrate due to inherent instabilities. However, different situations are found in BECs and nonlinear optics with temporally or spatially modulated parameters. In particular, it was shown in Ref. [22] that complete stabilization of a cylindrical (2+1)-dimensional [(2+1)D] spatial soliton can be secured in a layered medium with nonlinearity management. A scheme for stabilizing spatiotemporal solitons in media with cubic self-focusing nonlinearity and dispersion management was proposed in Ref. [23]. The formation of tandem structures, which are composed of periodically alternating linear dispersive and nonlinear layers, was studied in Refs. [24,25]. Moreover, alteration of atomic scattering length achieved by Feshbach resonance has been used to dynamically stabilize higher-dimensional bright solitons [26]. Thus, the study of the (2+1)D and (3+1)D variable coefficients NLS equations (NLSEs) has recently been one of the central issues in the field of nonlinear optics. One of the interesting challenges concerns how to characterize the nonlinear light bullets on analytical level [27–29]. In general, the analytical study of the multidimensional light bullets is impeded by the lack of the corresponding integrable systems. Therefore, several approaches have been recently developed to overcome this limitation. The traveling wave and light bullet soliton solutions to the generalized NLSE in (3+1)D for a cubic nonlinearity were first developed in Ref. [30] for anomalous dispersion and were generalized in Ref. [31] for normal dispersion by using the F-expansion technique. Exact solutions for varying potential and nonlinearity were found in Ref. [32] by similarity transformations. Nonautonomous rogue wave solutions have also been found for the generalized NLSEs with variable coefficients in three-dimensional spaces [33] based on the similarity analysis idea.

Very recently, the spatiotemporal dynamics of RW solutions in a composite (2+1)D were investigated in Ref. [34]. A novel type of light bullets, which take the shape of RWs and travel on a finite (2+1)D space-time background, has been obtained. It was shown that both the fundamental and second-order RWs have a directional preference or a bullet nature that can propagate in a certain direction with transverse double localization. Such special (2+1)D RW behavior has been called rogue wave bullets. We shall in this paper proceed along this direction to get rogue wave bullets of a new inhomogeneous (3+1)D integrable system where coefficients depend on time and transverse radial coordinates. The main result of the present work is the possibility to obtain a single optical sphere and homocentric optical spheres for an inhomogeneous (3+1)D NLSE with spherical symmetry.

2. THE THREE-DIMENSIONAL ROGUE WAVE LIGHT BULLETS

The three-dimensional inhomogeneous NLSE with variable coefficients can be written in a dimensionless form:

$$i\left(\frac{\partial}{\partial t}\psi\right)+\beta(r,t)\nabla^2\psi-v(r,t)\psi-g(r,t)|\psi|^2\psi=0, \quad (1)$$

where $\psi(r,t)$ is the complex envelope of the optical field, $r=\sqrt{x^2+y^2+z^2}$ is the distance from a point (x,y,z) to the origin of the coordinates, and $\nabla^2\psi=r^{-2}\partial/\partial r(r^2\partial\psi/\partial r)$ is the 3D Laplacian describing beam diffraction or group velocity dispersion in a 1D time-domain configuration. The external potential $v(r,t)$ and nonlinear coefficient $g(r,t)$ are real-valued functions of time and spatial coordinates. This equation arises in many fields such as nonlinear optics [1,21] and BECs [32,35–37]. The 1D version of Eq. (1) was considered in Refs. [38–40], where the soliton, together with first- and second-order RW solutions, was obtained. We present here 3D RW solutions to the NLSE in (3+1)D. In order to investigate the dynamic properties of the optical rogue wave solution for Eq. (1), we perform a specific reduction, namely,

$$\psi(r,t)=\rho(r)\exp[i\phi(r,t)]\Phi[X(t),T(r,t)], \quad (2)$$

where the functions $\rho(r)$ and $\phi(r,t)$ represent the amplitude and the phase, respectively. The complex function Φ satisfies the following NLSE with constant coefficients:

$$i\left[\frac{\partial}{\partial X}\Phi(X,T)\right]+\frac{\partial^2}{\partial T^2}\Phi(X,T)+2\epsilon|\Phi(X,T)|^2\Phi=0, \quad (3)$$

which is obtained by substituting Eq. (2) into Eq. (1) with the following specific transformation:

$$T=\alpha r+t,\quad X=t,\quad \rho=\frac{1}{r\sqrt{\alpha}},\quad \phi=-\frac{\alpha r}{2}-\frac{t}{2}, \quad (4)$$

$$\beta=\frac{1}{\alpha^2},\quad g=-2\epsilon\alpha r^2,\quad v=\frac{1}{4}. \quad (5)$$

Here $\epsilon=\pm1$ and α is a positive constant.

According to the above transformation defined by Eqs. (2), (4), and (5), we set $\alpha=1,\epsilon=1$, and then Eq. (1) leads to a solvable three-dimensional inhomogeneous NLSE with spatial nonlinearities:

$$i\frac{\partial}{\partial t}\psi+\left(\frac{\partial^2}{\partial r^2}\psi+2r\frac{\partial}{\partial r}\psi\right)-14\psi+2r^2|\psi|^2\psi=0. \quad (6)$$

This equation is a solvable model due to the established transformation and the solvability of the NLSE, which is the main result of this paper. We shall focus on rational-like solutions of Eq. (6), which provide novel localized optical spheres and thus generate new kinds of light bullets. These optical spheres oscillate along the radius direction and do not travel like the usual light bullets.

In optics, spatially inhomogeneous nonlinearities can be realized in various ways [41]. In a BEC, Eq. (6) describes the evolution of matter waves, where the spatially modulated nonlinearity landscape can be generated by the Feshbach resonance in nonuniform external fields [42,43]. Nonlinearity can also be modulated in optical structures, e.g., in photonic crystal fibers with the holes infiltrated with a highly nonlinear material, for example, index-matching nonlinear liquids [44,45].

Here we use the lowest-order rational solution of Eq. (3) [46–49], which serves as a prototype of rogue waves, to construct the optical spheres of Eq. (6). Setting Φ to be the first-order rogue wave [46–49] of the NLSE, $\alpha=1,\epsilon=1$, and, according to the transformations defined by Eqs. (2), (4), and (5), then the first-order rational-like solution of Eq. (6) can be rewritten as

$$\psi=\psi_{\mathrm{rw}}=\frac{-4r^2-8rt-20t^2+3+16it}{(4r^2+8rt+20t^2+1)r}\exp\left[-\frac{1}{2}i(r-3t)\right]. \quad (7)$$

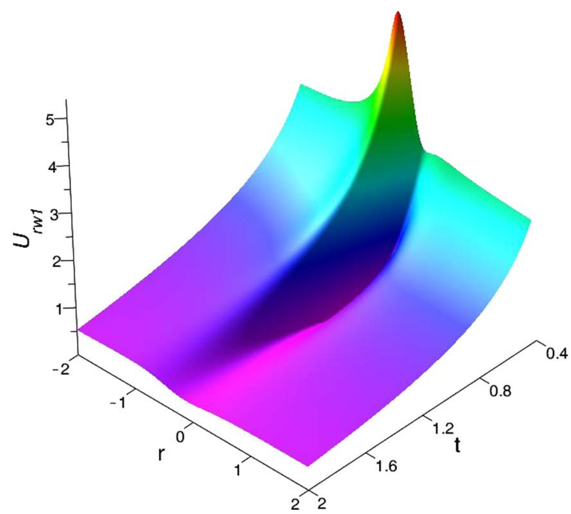

Fig. 1. Evolution of U_{rw1} on the (r,t) plane. It is obvious to find U_{rw1} exhibiting oscillations along r when t is very small.

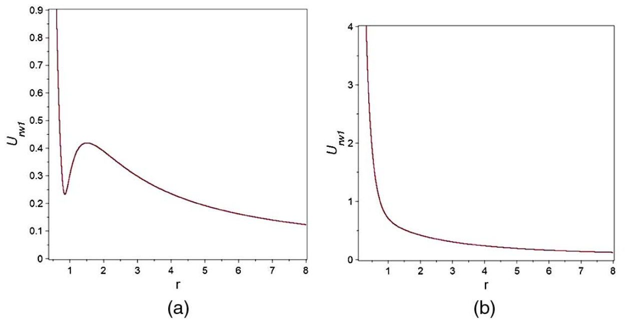

Fig. 2. Profiles of $U_{\rm rw1}$ along r for different values of t. (a) Two extreme points are (0.851,0.233) and (1.507,0.419) for $t = 0.05$; (b) there is no extreme point for $t = 0.25$.

There is only one singularity of ψ at $r = 0$, which is demonstrated by Figs. 1 and 2 and which originates from the $\rho = \frac{1}{r\sqrt{\alpha}}$ in the transformation from Eq. (2).

We next investigate the features of the amplitude $U_{\rm rw1} = |\psi_{\rm rw}|$ of the 3D RW solution from Eq. (7). Indeed, there exists a critical point t_0 such that $U_{\rm rw1}$ oscillates with respect to r when $t < t_0$, but it is a monotonic function of r when $t > t_0$.

It is easy to show that $U_{\rm rw1}$ presents oscillations for very small t and approaches $\frac{1}{r}$ when t goes to ∞, so the continuity of $U_{\rm rw1}$ guarantees the existence of a critical point t_0 of the turning of the monotonicity with respect to t. We found numerically that there exists a critical point $t_0 \in [0.118,0.119]$, such that, if $t > t_0$, $U_{\rm rw1}$ monotonically decreases with respect to r, and if $t < t_0$, it oscillates with respect to r. This feature is complementary to the pioneering RW structure, which appears from nowhere and disappears without a trace [6,7]. This can be confirmed by profiles of $U_{\rm rw1}$ on the (r, t) plane in Fig. 1. According to our analysis, if $t > 0.119$, the isosurface of $U_{\rm rw1}$ is just a single sphere. But for $0 \le t < 0.118$, the isosurfaces can form three homocentric spheres with suitable values of $U_{\rm rw1}$. For example, when $t = 0.25$, $U_{\rm rw1}$ decreases with respect to r [see Fig. 2(b)]. However, when $t = 0.05$, $U_{\rm rw1}$ exhibits oscillations with respect to r and thus presents two extreme points at 0.851 and 1.507; their values are 0.233 and 0.419, respectively [see Fig. 2(a)]. So, by setting $U_{\rm rw1} \in (0.233,0.419)$, the isosurface has three homocentric optical spheres (see Fig. 3). Of course, by setting $U_{\rm rw1} \notin (0.233,0.419)$, the isosurface of $U_{\rm rw1}$ is a single sphere.

The radius of these spheres increases to a certain value and then reaches an upper limit, which corresponds to localized feature of the rogue wave. In this process, the radius r of the sphere may have oscillation. Equivalently, r is not a monotonic function of time t, although it is bounded. In other words, the isosurface of $U_{\rm rw1}$ forms size-bounded sphere, which is a strong reflection of the localized nature of rogue waves in three dimensions. Moreover, the polynomial form of the rogue wave in the one-dimensional NLSE is reflected by the oscillation of the radius r of the isosurface. Therefore, the behavior of the sphere of the isosurface represents the nature of the first-order rogue wave of the NLSE: polynomial and having a doubly localized property. The asymptotical radius of the isosurface valued at $U_{\rm rw1} = k$ is given by $r_{\rm as} = \frac{1}{k}$.

For example, for the isosurface of $U_{\rm rw1}$ valued at 0.5, there are only two extreme points at ($t = 0.03404$, $r = 0.68574$) and ($t = 3.24987$, $r = 2.00604$) in the profile of r varying with t (see Fig. 4). Note that the asymptotic value of r reaches

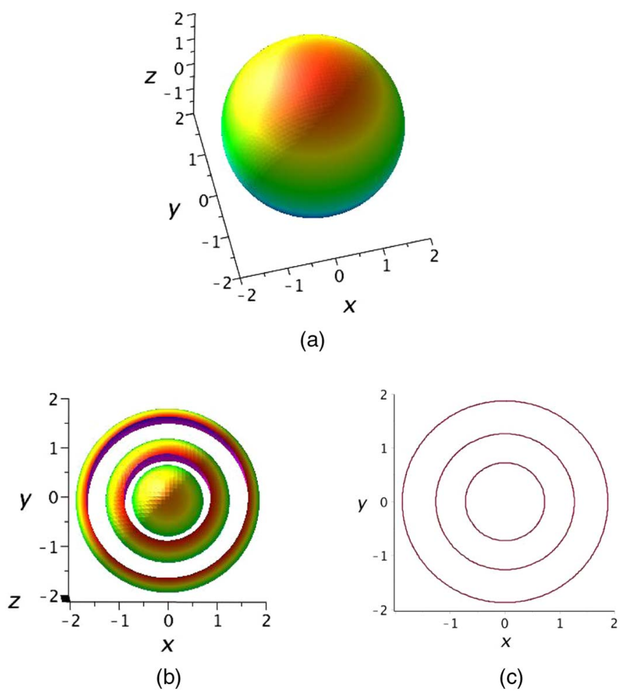

Fig. 3. Profiles of $U_{\rm rw1}$. (a) The isosurface of $U_{\rm rw1}$ at $t = 0.05$ when $U_{\rm rw1} = 0.4$; (b) the inside of (a) plotted from a bird's-eye view and $z \in [-0.9,0.9]$; (c) the contour line of $U_{\rm rw1}$ at $z = 0$. The latter two panels verify that there are three homocentric spheres.

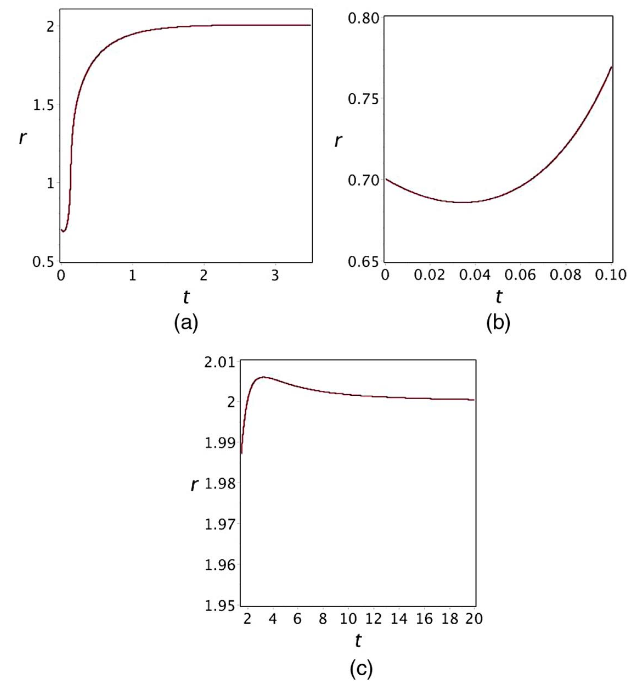

Fig. 4. Radius of the sphere for the isosurface given by $U_{\rm rw1} = 0.5$. Panel (b) is plotted for very small t of panel (a), and there is a minimum $r_{\rm min} = 0.68574$. Panel (c) is plotted for large t of panel (a), and there is a maximum $r_{\rm max} = 2.00604$. The asymptotical value of r is $r_{\rm as} = 2$.

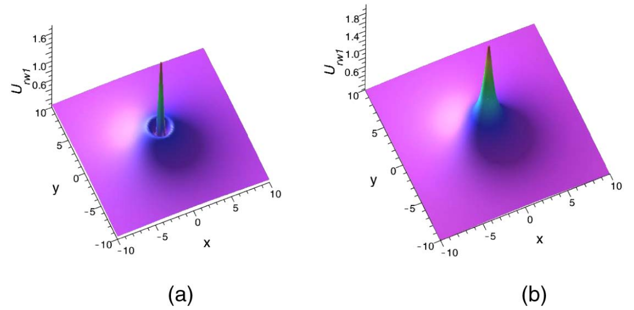

Fig. 5. Localized profiles of the U_{rw1} with $z = 0.5$ in the (x, y) plane: (a) $t = 0$; (b) $t = 0.5$.

2 for large t, but the maximum value of r is larger than 2 (see Fig. 4). The animation (Visualization 1) of an isosurface valued at $U_{\mathrm{rw1}} = 0.5$ confirms remarkably the enlargement of the radius r and asymptotical value r_{as} of the sphere. Due to very tiny variation of r around the two extremes, the animation cannot show the oscillation of r significantly, although it happened during the increase of r. However, this can be confirmed by a two-dimensional animation (Visualization 2) of contours of the sphere $U_{\mathrm{rw1}} = 0.5|_{z=0}$ with respect to t.

The U_{rw1} is also very well localized in the (x, y) plane, although the peak changes with respect to t, which can be confirmed in Fig. 5 and the animation (Visualization 3). We can see from the animation that the peak of U_{rw1} is oscillating with time t. We have found here that if $r < 0.35$, the peak decreases with time; if $r \in [0.35, 0.85]$, the peak oscillates with time; if $r > 0.85$, the peak increases with time. So there exists a critical value of time at which the profile and the amplitude of the solution remain unchanged; see Fig. 6. Note that 0.35 and 0.85 are just approximate values. As we know, $\lim_{t=\infty} U_{\mathrm{rw1}} = \frac{1}{r}$, so $U_{\mathrm{rw1}}|_{z=0.5}$ has an asymptotic maximum value of 2 as confirmed by the animation.

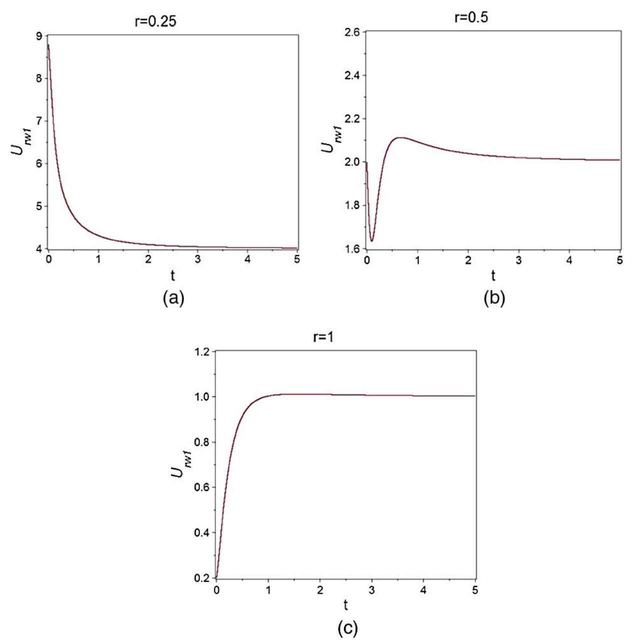

Fig. 6. Profiles of U_{rw1} with respect to t for different values of r: (a) $r = 0.25$; (b) there are two extreme points (0.007,1.635) and (0.663,2.113) for $r = 0.5$; (c) $r = 1$.

3. THE STABILITY OF THE OPTICAL SPHERE SOLUTIONS

In order to determine the stability of the optical sphere solutions with respect to perturbations that break the initial spherical symmetry, we consider the Laplacian of Eq. (1) in spherical coordinates and choose an appropriate perturbation in the form [50]

$$\psi(r, \theta, \phi, t) = \psi_{\mathrm{rw}}(r, t) + \mu g(r, t) \Upsilon_l^m(\theta, \phi). \quad \mathbf{(8)}$$

Here μ is a small expansion parameter and $g(r, t)$ is a radial perturbation function. The spherical harmonic function is defined as $\Upsilon_l^m(\theta, \phi) = P_l^m(\theta) \cos(m\phi)$, where P_l^m is the associated Legendre function with $l \geq m \geq 0$. Note that the angular perturbation function must be real to obtain the linearized equation that follows. Inserting Eq. (8) into Eq. (1) and linearizing in the small parameter μ, we obtain the following linear partial differential equation for the evolution of the radial perturbation function:

$$i\frac{\partial g}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial g}{\partial r}\right) - \frac{l(l+1)}{r^2} g - \frac{1}{4} g + 2r^2 (|\psi_{\mathrm{rw}}|^2 g + \psi_{\mathrm{rw}}^2 g^*) = 0. \quad \mathbf{(9)}$$

The asterisk denotes the complex conjugation. Note that the angular dependence appears solely by virtue of parameter l, and the azimuthal index m does not appear in the perturbation equation. Since this is a linear equation in g, we expect the solution to be of the form [51] $g(r, t) = g(r)e^{\lambda t}$ as $t \to \infty$, where λ is the maximum positive (real) eigenvalue and $g(r)$ is the corresponding eigenfunction. Starting with a small random initial condition for $g(r, 0)$, Eq. (9) is integrated by using a Crank–Nicolson algorithm [52]. Figure 7(a) shows the numerically calculated growth rates λ as a function of the spherical harmonic modes l. As can be seen on this figure, for the modes $l < 70$, the positive growth rate decreases with parameter l and the optical sphere solution is unstable. However, for the modes $l > 70$, the growth constant can become zero, which means that the optical sphere solution can be classified as stable for these modes. Figure 7(b) shows the radial perturbation function corresponding to the most unstable mode $l = 4$. It is shown that an exponentially growing radial profile for $|g|$ emerges and affects only the edges of the optical spheres,

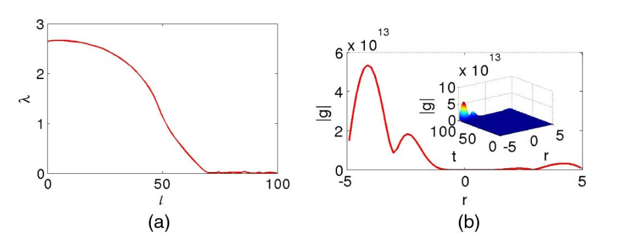

Fig. 7. (a) Growth rates λ as a function of the spherical harmonic modes l. (b) Dominant unstable $l = 4$ radial perturbation eigenmode emerging from a small random initial condition.

leaving the central core relatively untouched. Note that stable vortex solitons have also been obtained in 3D NLSE with spatially inhomogeneous nonlinearities [53].

4. CONCLUSION

In conclusion, we have shown that in the (3+1)-dimensional NLSE with varying coefficients, localized solutions in the form of rational formulas can exist owing to a specific transformation that allows us to reduce the dimensionality of the equation from (3+1) dimensions to (1+1) dimensions. These solutions are localized both in space and in time, and thus their corresponding isosurfaces are single spheres or homocentric spheres, which oscillate along the radius direction and are completely different from the well-known standard traveling light bullets. They can be interpreted as prototypes of RW light bullet solutions in the (3+1)-dimensional time-space. The other properties of the new nonautonomous RW light bullets have been studied analytically. Our analytical findings are confirmed by numerical plots of these solutions. A linear stability analysis in terms of spherical harmonic modes has been investigated. We have found that the RW light bullet solutions are stable for higher modes and transversely unstable for lower modes of the perturbation. Experimental advances have recently provided a strong incentive in the area of RWs in complex media [54]. Note that a demonstration of the direct observation of RWs in self-excited 3D longitudinal plasma density waves was reported in Ref. [55] by using self-excited dust acoustic waves as a platform. We believe that the results obtained here can stimulate further research on the experiments and help to understand the behavior of 3D RWs in a wide range of nonlinear physical areas.

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