Experimental observation of an anomalous weak value without post-selection

Mu Yang,^{1,2} Qiang Li,^{1,2} Zheng-Hao Liu,^{1,2} Ze-Yan Hao,^{1,2} Chang-Liang Ren,^{3,4} Jin-Shi Xu,^{1,2,5} Chuan-Feng Li,^{1,2,6} and Guang-Can Guo^{1,2}

¹CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China ²CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China ³Center for Nanofabrication and System Integration, Chongqing Institute of Green and Intelligent Technology, Chinese Academy of Sciences, Chongqing 400714, China ⁴e-mail: renchangliang@cigit.ac.cn ⁵e-mail: jsxu@ustc.edu.cn ⁶e-mail: cfli@ustc.edu.cn

Received 24 March 2020; revised 26 June 2020; accepted 3 July 2020; posted 7 July 2020 (Doc. ID 393480); published 20 August 2020

Weak measurement has been shown to play important roles in the investigation of both fundamental and practical problems. Anomalous weak values are generally believed to be observed only when post-selection is performed, i.e., only a particular subset of the data is considered. Here, we experimentally demonstrate that an anomalous weak value can be obtained without discarding any data by performing a sequential weak measurement on a single-qubit system. By controlling the blazing density of the hologram on a spatial light modulator, the measurement strength can be conveniently controlled. Such an anomalous phenomenon disappears when the measurement strength of the first observable becomes strong. Moreover, we find that the anomalous weak value cannot be observed without post-selection when the sequential measurement is performed on each of the components of a two-qubit system, which confirms that the observed anomalous weak value is based on sequential weak measurement of two noncommutative operators. © 2020 Chinese Laser Press

https://doi.org/10.1364/PRJ.393480

1. INTRODUCTION

The concept of weak measurement [1-4] was introduced by Aharonov, Albert, and Vaidman in 1988. Their theory is based on the von Neumann measurement with a very weak coupling between two quantum systems. Compared to the strong measurement, weak measurement theory is a very important nonperturbative theory of quantum measurements. Over several decades of development, there have been many notable investigations from both fundamental and practical perspectives. On the one hand, weak measurement provides novel insights into a number of fundamental quantum effects, including the role of the uncertainty principle in the Hardy's paradox [5-7], the double-slit experiment [8,9], the three-box paradox [10], and Leggett-Garg inequalities [11-14]. On the other hand, it is considered to be useful for the signal amplification while decreasing or retaining the technical noise [15,16] in parameter estimations, such as amplification measurements of small transverse [17,18] and longitudinal shifts [19-21], optical nonlinearities [22], and the Poynting vector field [23].

In all these applications, the strange characteristic that the weak value obtained in the weak measurement can even exceed the eigenvalue range of a typical strong or projective

2327-9125/20/091468-07 Journal © 2020 Chinese Laser Press

measurement and is generally complex (also known as "anomalous weak value"), is usually considered to play a vital role. The standard weak value is defined with post-selection, and the anomalous weak value is usually observed by post-selecting a small subset of data. Therefore, the anomalous weak value is a result of post-selection, which is widely accepted in the community. However there are still great controversies on this point, especially for the validity of weak value technology in quantum metrology [24-26]. With the in-depth research, it is shown that anomalous amplification [27] and unusual smoothed estimation [28] can be realized without the requirement of post-selection of weak measurement techniques in high precise metrology protocols. This result gives us some enlightenments, but it still does not answer the question: can anomalous weak values be attained without post-selection? In fact, it is correct that post-selection is necessary to observe the anomalous weak values in a single weak measurement case. But, it is not the case in general. Recently, a theoretical investigation pointed out that an anomalous weak value can be obtained without post-selection by sequential weak measurements [29,30], which provided a more general insight and was deemed as "yet another surprise" [31].

In this work, we experimentally obtain anomalous weak values in a sequential weak measurement without post-selection, i.e., without discarding any data, in an all-optical system. The photonic polarizations and transverse momenta are chosen to be the system states and pointers, respectively. A sequential weak measurement on the product of two single-qubit observables is realized for arbitrary measurement strength controlled by the phase patterns on a liquid crystal spatial light modulator (SLM). The counter-intuitive average value of joint pointer's deflection is observed when the measurement strength of the first observable is weak, while the anomalous value disappears when the measurement strength of the first observable increases. Moreover, we further perform a sequential weak measurement on two observables each of which belongs to the components of a two-qubit system and find that an anomalous weak value can never be observed based on commutative sequential weak measurements without post-selection.

2. THEORETICAL FRAMEWORK AND PROTOCOL

The standard form of a weak value is given by

$$\langle A_{\psi}^{f} \rangle_{W} := \frac{\langle f | \hat{A} | \psi \rangle}{\langle f | \psi \rangle}, \tag{1}$$

which is the mean value of observable \hat{A} when weakly measured between a pre-selected state $|\psi\rangle$ and a post-selected state $|f\rangle$ [1]. As introduced in Refs. [30,32], in particular, a trivial, deterministic measurement of the identity operator I amounts to performing no post-selection. A weak value with no postselection, can be defined as

$$\langle A^I_{\psi} \rangle_{W} := \langle \psi | \hat{A} | \psi \rangle.$$
 (2)

Obviously, the weak value with no post-selection is equal to the expectation (average) value of \hat{A} . The result of these cases is restricted to lie in $[\eta_{\min}(\hat{A}), \eta_{\max}(\hat{A})]$ ($\eta_{\min(\max)}$ represents the minimum (maximum) eigenvalue), which means anomalous weak values cannot be observed. It is therefore generally considered that anomalous weak values can only be observed by post-selecting a small subset of data.

However, such an interpretation is invalid for sequential weak measurements [30]. The sequential weak value $\langle (A_1A_2)_{\psi}^I \rangle_W$ with no post-selection is defined $(\hat{A}_1 \text{ and } \hat{A}_2 \text{ are independent observables})$ as follows:

$$\langle (A_1 A_2)^I_{\psi} \rangle_W := \langle \psi | \hat{A}_1 \hat{A}_2 | \psi \rangle.$$
(3)

We should note that when \hat{A}_1 and \hat{A}_2 are not commute, and for good choices of A_1 , A_2 , and $|\psi\rangle$, $\langle (A_1A_2)_{\psi}^I \rangle_W$ will not be contained within the interval $[\eta_{\min}(\hat{A}_1, \hat{A}_2), \eta_{\max}(\hat{A}_1, \hat{A}_2)]$, where $\eta_{\min(\max)}(\hat{A}_1, \hat{A}_2) = \min(\max)_{k,l}\eta_k(\hat{A}_1)\eta_l(\hat{A}_2)$ (k, l denote the indexes of eigenvalues). Due to the sequential nature of the weak measurements, an anomalous weak value for sequential weak measurements can occur without post-selection.

We consider two pointers interacting with a quantum system one after another as illustrated in Fig. 1. Suppose a qubit system initially prepared in the general state $|\psi\rangle$, while twopointer states are prepared in $|\phi_1\rangle$ and $|\phi_2\rangle$, respectively. The evolution operator between the system and pointer is



Fig. 1. Theoretical protocol. The system initially in the state $|\psi(0)\rangle$ is sequentially weak coupled with two pointers in the initial states of $|\phi_1(0)\rangle$ and $|\phi_2(0)\rangle$, respectively. The time sequence is denoted as t_1 , t_2 , and t_3 . The pointers are measured individually or jointly.

denoted as $\hat{U}_i = \exp(i\gamma_i \hat{A}_i \otimes \hat{F}_i)$, where $i \in \{1, 2\}$ denotes the *i*th weak measurement, γ_i is the interaction coupling, and \hat{F}_i is an operator of the pointer. The result of the sequential weak measurements can be linked to the anomalous weak value without post-selection $\langle (A_1A_2)_{\psi}^I \rangle_W$.

Specifically, let us consider a qubit system initially prepared in the state $|\psi\rangle = |0\rangle$, assuming \hat{A}_i to be the projection observable on the system with $\hat{A}_i = |a_i\rangle\langle a_i|$, where $|a_i\rangle = \frac{1}{2}|0\rangle - (-1)^i \frac{\sqrt{3}}{2}|1\rangle$, and to each measurement is associated a momentum operator $\hat{F}_i = \hat{p}_i$ of the pointers. Considering a Gaussian pointer [33] with transverse wavefunction $\phi_i = \exp(-x^2/\sigma_i^2)$ (σ_i is a constant), the joint pointer average position is given by [30]

$$\langle \hat{x}_1 \otimes \hat{x}_2 \rangle = \frac{1}{16} \left(1 - 3e^{-\frac{\gamma_1^2}{8\sigma_1^2}} \right) \gamma_1 \gamma_2,$$
 (4)

where \hat{x}_i represents the *i*th position operator. Here we define the inferred values of $M = \langle \hat{x}_1 \otimes \hat{x}_2 \rangle / \gamma_1 \gamma_2$. Since each output pointer observable has an average shift in the range of [0,1], the average values are naturally expected within the range [0,1]. However, when the first measurement is sufficiently weak, $\gamma_1 \to 0$, $\langle \hat{x}_1 \otimes \hat{x}_2 \rangle \approx -\frac{1}{8} \gamma_1 \gamma_2$ lied out of the interval, which shows the average product of the pointer displacements is proportionally linked to the anomalous weak value, where $M \to \text{Re}[\langle (A_1 A_2)_{\psi}^I \rangle_W] = -\frac{1}{8}$. Interestingly, in the real weak limit, as $\gamma_1 \to 0$, then $\langle \hat{x}_1 \otimes \hat{x}_2 \rangle \to 0$ also, unless γ_2 is increased to compensate it. In contrast, when the first measurement is strong $(\gamma_1 \to \infty)$, $\langle \hat{x}_1 \otimes \hat{x}_2 \rangle \approx \frac{1}{16} \gamma_1 \gamma_2$ is consistent with the product of the two observables' expectation values, where $M \to \langle \psi | A_1 | \psi \rangle | \langle a_1 | a_2 \rangle |^2 = \frac{1}{16}$.

3. EXPERIMENTAL SETUP AND RESULTS

We experimentally demonstrate sequential weak measurements [34] by using the SLM, in which a phase φ ($\varphi = \alpha \vec{r} \mod 2\pi$, $\alpha \in N$) that changes linearly along the \vec{r} direction is implemented to the input photons. The evolution can be described as

$$\hat{U}_r = \exp(-i\gamma_{\exp}|H\rangle\langle H|\otimes \hat{p}_{\vec{r}}), \tag{5}$$

where $\hat{p}_{\hat{r}} = -i \frac{\partial}{\partial \hat{r}}$ (setting $\hbar = 1$) represents the momentum operator, $|H\rangle$ represents the horizontal polarization of the photon, and $\gamma_{\exp} = k\alpha$ ($k \in \mathbb{R}$) represents the coupling strength which can be conveniently tuned by changing the blazing density of the hologram on the SLM (see Appendices A and B for more details). We set $\gamma_1 = \gamma_2 = \gamma_{\exp}$ in the experiment. In the experiment, the photon's polarizations and transverse

In the experiment, the photon's polarizations and transverse field momenta $(\hat{p}_{\bar{x}}, \hat{p}_{\bar{y}})$ are chosen to be the system \hat{A}_i and pointers \hat{F}_i , respectively. The experimental setup is illustrated in Fig. 2(a). Single photons (SPs) from an intrinsic defect in a GaN crystal are filtered by a single mode fiber [35] (see Appendices A and B for more details). The zero-delay time of the second-order correlation function $g^2(0)$ is measured to be 0.262 and fitted to be 0.025, which clearly demonstrates the single photon property.

A half-wave plate (HWP₁) with the optical axis set to be 30° is used to prepare the polarization state of the photon to be $|a_1\rangle = \frac{1}{2}|H\rangle + \frac{\sqrt{3}}{2}|V\rangle$, where $|V\rangle$ represents the vertical polarization. Then the photons are focused on the right screen of SLM for the first weak coupling by a convex lens with a focal length of 150 mm. The hologram loaded on the right area of

SLM is a vertical blazed grating, and satisfies $\varphi_{\text{right}}(x, y) = \alpha \vec{x} \mod 2\pi$ ($\alpha \in \mathbb{N}$). In the experiment, $\gamma_{\exp} = 0.0237\alpha$ (see Appendices A and B for details). This progress can be defined as $\hat{U}_1 = \exp(i\gamma_{\exp}\hat{A}_1 \otimes \hat{p}_{\vec{x}})$.

Similarly, through a lens with a focal length of 75 mm, photons are re-focused on the left screen of SLM for the second weak coupling, and the evolution becomes $\hat{U}_2 = \exp(i\gamma_{\exp}\hat{A}_2 \otimes \hat{p}_{\tilde{y}})$. The left hologram is a horizontal grating, which satisfies $\varphi_{\text{left}}(x, y) = \alpha \vec{y} \mod 2\pi$. Another HWP₂ with the optical axis set to be -30° is placed before the screen to rotate $|H\rangle(|V\rangle)$ to the state of $|a_2\rangle = \frac{1}{2}|H\rangle - \frac{\sqrt{3}}{2}|V\rangle$ $(|a_2^{\perp}\rangle = \frac{\sqrt{3}}{2}|H\rangle - \frac{1}{2}|V\rangle)$. The third Fourier lens is used to translate the photons back to the coordinate space, which are directly detected by an intensified charge coupled device (ICCD) without post-selection of polarizations.

Figure 2(b) shows several spatial distributions of the photons with different coupling strengths. The scale of images is 1024×1024 and the pixel length is $13.5 \ \mu\text{m}$. The coupling strength γ_{exp} varies from 0 to 0.37 mm. When γ_{exp} is less than 0.1 mm, the coupling strength is so small that the transverse coordinate deflection is much less than the Rayleigh distance.





With the increase of coupling strength, the deflections of the transverse coordinates are clearly observed, including downward deflection $\phi(x + \gamma_{exp}, y)$, rightward deflection $\phi(x, y + \gamma_{exp})$ and down-rightward deflection $\phi(x + \gamma_{exp}, y + \gamma_{exp})$ after sequential weak measurements \hat{U}_1 and \hat{U}_2 , respectively. The insets in Fig. 2(b) show the theoretical output patterns, of which the similarity is quantitatively determined by the correlation value (Corr) [36] compared with the experimental images. The Corr is larger than 0.96 when $\gamma_{exp} > 0.2366$; the errors are mainly due to the aberrations in optical systems and the inaccuracy of wave plate rotation.

To quantitatively describe the anomalous weak values, we define the transverse coordinate pointer's position $\langle \hat{K} \rangle$ as the mean of the transverse field coordinate as follows:

$$\langle \hat{K} \rangle = \sum_{x,y} K \cdot |\phi'(x,y)|^2 / \sum_{x,y} |\phi'(x,y)|^2,$$
 (6)

where $\hat{K} \in {\hat{x}, \hat{y}, \hat{x} \otimes \hat{y}}$. $\phi'(x, y)$ is the final photonic transverse wave function and $\hat{x} \otimes \hat{y}$ represents the joint pointer position operator [37]. The result of the sequential measurement is proportional to the product of the pointer positions.

Figure 3 shows the deflection of the pointer as a function of the coupling strength. The coupling strength γ_{exp} varies from 0 to 0.711 mm. The pointer positions of horizontal and vertical transverse coordinates $\langle \hat{x} \rangle$ and $\langle \hat{y} \rangle$) are shown in Fig. 3(a) with the brown and blue dots representing the experimental results and the brown and blue lines representing the corresponding theoretical predictions, respectively. No matter what the coupling strength is, we can find that $\langle \hat{x} \rangle$ and $\langle \hat{y} \rangle$ is larger than 0. The joint pointer positions $\langle \hat{x} \otimes \hat{y} \rangle$ are shown in Fig. 3(b). The green dots represent the experimental results with the green line representing the corresponding theoretical prediction. Anomalous phenomenon $\langle \hat{x} \otimes \hat{y} \rangle < 0$ can be observed when the coupling strength γ_{exp} is less than 0.331 mm, which is shown as the red dots. When $\gamma_{exp} = 0.189$ mm, the reversed deflection of the joint pointer reaches the maximum. The values are smaller than prediction, which may be due to the slight tilt of the ICCD camera and the optical aberrations. The error bars represent the corresponding standard deviation $[\Delta(\langle \hat{x} \otimes \hat{y} \rangle)]$. Figure 3(c) shows the inferred values of $M = \langle \hat{x} \otimes \hat{y} \rangle / \gamma_{exp}^2$. When $\gamma_{exp} \rightarrow 0$, *M* equals the real part of the weak value $\operatorname{Re}[\langle (A_1A_2)_{\psi}^I \rangle_W] = -\frac{1}{8}$. However, the experimental errors $\Delta(\langle \hat{x} \otimes \hat{y} \rangle)/\gamma_{\exp}^2$ also get amplified with $\gamma_{\exp} \to 0$, which makes the inferred weak values get unreliable. In spite of this, when 0.13 mm $< \gamma_{exp} < 0.3$ mm, *M* still approximates to the anomalous weak value $-\frac{1}{8}$, and the experimental tendency here is consistent with the theoretical prediction.

Moreover, as a comparison, considering the situation of a sequential weak measurement of two observables, which are measured on each of two qubits respectively, there is no such anomalous deflection at all. Supposing a sequential measurement of the commuting observables $\hat{A}_1 \otimes I$ and $I \otimes \hat{A}_2$ is performed on the bipartite system, the average weak value is equal to the expectation value of $\hat{A}_1 \otimes \hat{A}_2$ in the absence of any postselection, with $\langle (\hat{A}_1 \otimes \hat{A}_2)_{\psi}^{I} \rangle_W = \langle \psi | \hat{A}_1 \otimes \hat{A}_2 | \psi \rangle$, which can never exceed the range of eigenvalues of the observable.

We experimentally investigate the case of weak measurement on two individual photons, which corresponds to the sequential weak measurement on different parts of a bipartite



Fig. 3. Deflections of the pointer's position and the normalized result of sequential weak measurements in the one-qubit system. (a) The brown and blue dots represent the experimental results of the pointer positions $\langle \hat{x} \rangle$ and $\langle \hat{y} \rangle$, respectively. The brown and blue lines represent the corresponding theoretical predictions. (b) The green dots represent the experimental results of the joint pointer position $\langle \hat{x} \otimes \hat{y} \rangle$ with the green solid line represent the anomalous joint pointer position. (c) The black dots represent the inferred values of $M = \langle \hat{x} \otimes \hat{y} \rangle / \gamma_{exp}^2$, while the oretical prediction is shown as a black line.

system. We first apply the \hat{U}_1 to a photon and obtain the deflection of the pointer's positions in the x direction $(\langle \hat{x} \rangle)$. We then implement the \hat{U}_2 to the other photon and obtain the deflection of the pointer's positions in the y direction $(\langle \hat{y} \rangle)$. The experimental results of $\langle \hat{x} \rangle$ (brown dots) and $\langle \hat{y} \rangle$ (blue dots) with the corresponding theoretical predictions represented as brown and blue lines, respectively, are shown in Fig. 4(a). The experimental results of the joint pointer deflection $\langle \hat{x} \otimes \hat{y} \rangle$ (green dots) and the corresponding theoretical prediction (green line) are shown in Fig. 4(b), in which the values of $\langle \hat{x} \otimes \hat{y} \rangle$ are all larger than zero. Error bars represent the corresponding standard deviations. All position deflections are larger than zero and satisfy $\langle \hat{x} \rangle = \langle \hat{y} \rangle = \sqrt{\langle \hat{x} \otimes \hat{y} \rangle}$. There is not any anomalous phenomenon for sequential weak measurement with any coupling strength for two qubit systems, since the two observables are commutative in the sequential case.



Fig. 4. Deflections of pointer positions via sequential weak measurements in the two-qubit system. (a) The brown and blue dots represent the experimental results with the brown and blue lines representing the corresponding theoretical predictions, respectively. (b) The green dots represent the joint average pointer positions $\langle \hat{x} \otimes \hat{y} \rangle$ with the green line representing the corresponding theoretical prediction.

4. CONCLUSION AND DISCUSSION

We have experimentally carried out a sequential measurement of two observables in a single-qubit system and a two-qubit system for arbitrary measurement strength with the SLM. The anomalous weak values are obtained without post-selection, in which the paradox of average pointer deflection is successfully observed in a single-qubit system. Such an anomalous phenomenon disappears when the measurement strength of the first observable becomes strong. On the other hand, when a sequential weak measurement is commutative, the anomalous weak value can never be observed without post-selection.

The experimental method of sequential weak measurement is implemented by taking the photon polarizations and transverse field momenta as the system and pointers, respectively. Compared with other methods [38,39], the advantage of this approach is that the coupling strength can be conveniently adjusted, so one can obtain weak values at different coupling strengths.

The tendencies of experimental data coincide with the theoretical predictions. Since the region to detect deflections of photon distributions associated with the anomalous weak value is less than 10^{-3} mm², the output results are easily disturbed. Moreover, the error of weak values will be rapidly increased as the coupling strength decreases. This effect can be mitigated by increasing γ_2 . This would give an idea of the tradeoff between the fact that, theoretically, one obtains a more anomalous weak value the further one goes to the weak regime, and the fact that the observable effect also becomes smaller in the weak limitation.

By controlling their measurement strengths from weak to strong, different results of a sequential measurement of two observables in weak and strong limits have been clearly shown. The anomalous weak values emerge in the sequential weak measurements. Recently, weak values and sequential weak measurements have played crucial roles in understanding fundamental problems such as the information paradox in black holes [40–43], time [44], and geometric phase [45]. The sequential measurement technology demonstrated in this work may be useful for exploring a series of fundamental physical concepts.

APPENDIX A: WEAK MEASUREMENT BASED ON THE LIQUID CRYSTAL SPATIAL LIGHT MODULATOR

Figure 5(a) shows the process of weak measurement. The polarization of photons is treated as the system state and



Fig. 5. Weak measurement based on the liquid crystal spatial light modulator (SLM). (a) The input photons are transformed from the coordinate space to the momentum space by a Fourier lens and focused on the screen of SLM. A phase that changes linearly along the *x* direction is applied on photons by the SLM, and then photons are re-transformed from the momentum space to the coordinate space by another Fourier lens. The photon wave packet will be transversely shifted slightly, which is known as weak measurement. (b) The relation between grating densities α on SLM and coupling strength γ_{exp} .

momenta (\hat{p}_x, \hat{p}_y) of the transverse field as the pointers. The initial state is prepared as

$$|\Psi\rangle = |\phi(x, y)\rangle(a|H\rangle + b|V\rangle),$$
 (A1)

where $|\phi(x, y)\rangle$ represents the photon's spatial wave function, $|H(V)\rangle$ means the photonic horizontal (vertical) polarization, and (a, b) are dimensionless complex coefficients. Through the first lens, the input photons are transformed from the coordinate space into the momentum space, which is denoted as

.

$$\mathcal{F}[|\phi(x,y)\rangle](a|H\rangle + b|V\rangle) = |U(\eta,\xi)\rangle(a|H\rangle + b|V\rangle),$$
(A2)

where $|U(\eta, \xi)\rangle$ means the wave function in momentum representation. By using the liquid crystal spatial light modulator (SLM) to add phases $\varphi_{\text{SLM}} = \alpha \text{xmod} 2\pi$ ($\alpha \in \mathbb{N}$) on the horizontal polarized photons, where grating density is determined by α , the state becomes

$$a|U(\eta,\xi)\rangle e^{i\gamma_{\exp}\eta}|H\rangle + b|U(\eta,\xi)\rangle|V\rangle),$$
 (A3)

where $\gamma_{exp} = k\alpha$ ($k \in \mathbb{R}$) represents coupling strength. Figure 5(b) shows the relationship between the grating parameter α and the coupling strength γ_{exp} . The coefficient k can be deduced from the fitting line of (α , γ_{exp}), which reads as k = 0.0237. Through another Fourier lens, photons are re-transformed to the coordinate space, which can be expressed as

$$a\mathcal{F}^{-1}[|U(\eta,\xi)\rangle e^{i\gamma_{\exp}\eta}]|H\rangle + b|\mathcal{F}^{-1}[U(\eta,\xi)\rangle]|V\rangle$$

= $a|\phi(x-\gamma_{\exp},y)\rangle|H\rangle + b|\phi(x,y)\rangle|V\rangle.$ (A4)

In summary, this process simulates a weak interaction evolution \hat{U} , which satisfies

$$\hat{U}|\phi(x,y)\rangle(a|H\rangle + b|V\rangle) = a|\phi(x-\gamma_{exp},y)\rangle|H\rangle + b|\phi(x,y)\rangle|V\rangle,$$
 (A5)

where $\hat{U}_x = \exp(-i\gamma_{\exp}|H\rangle\langle H \otimes |\hat{p}_{\vec{x}})$, and $\hat{p}_{\vec{x}} = -i\frac{\partial}{\partial \vec{x}}$ (assuming $\hbar = 1$) represents the horizontal momentum operator.

APPENDIX B: PREPARATION OF THE SINGLE PHOTON EMITTER

The experimental setup of the single photon emitter (SPE) is shown in Fig. 6. The intrinsic defect in the GaN sample is excited by a 532 nm continuous wave laser. The laser is focused by an objective with a high-NA of 0.9 after reflected by a dichroic mirror (DM). The fluorescence is collected by the same objective and is filtered by a bandpass filter with the central



Fig. 6. Experimental setup of the single photon emitter (SPE).

wavelength 808 nm. The single photons are then coupled into a single mode fiber (SMF), which are guided to the sequential weak measurements.

Funding. National Key Research and Development Program of China (2017YFA0305200, 2016YFA0302700, 2017YFA0304100); National Natural Science Foundation of China (11774335,61327901, 61490711, 61725504, 11821404, U19A2075, 11605205); Youth Innovation Promotion Association of the Chinese Academy of Sciences (2015317); Key Research Program of Frontier Sciences, Chinese Academy of Sciences (CAS) (QYZDY-SSW-SLH003); Science Foundation of the CAS (ZDRW-XH-2019-1); Anhui Initiative in Quantum Information Technologies (AHY060300, AHY020100); Fundamental Research Funds for the Central Universities (WK2030380017, WK2470000026); Natural Science Foundation of Chongqing (cstc2015jcyjA00021, cstc2018jcyjAX0656); Entrepreneurship and Innovation Support Program for Chongqing Overseas Returnees (cx2017134, cx2018040); Fund of CAS Key Laboratory of Microscale Magnetic Resonance; Fund of CAS Key Laboratory of Quantum Information.

Acknowledgment. This work was partially carried out at the USTC Center for Micro and Nanoscale Research and Fabrication.

Disclosures. The authors declare no conflicts of interest.

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36. $\operatorname{Corr}(A, B) = \frac{\sum_{i,j} (A_{i,j} - \tilde{A})^{(B_{i,j} - \tilde{B})}}{\sqrt{\sum_{i,j} (A_{i,j} - \tilde{A})^2 \sum_{i,j} (B_{i,j} - \tilde{B})^2}}$, where $A_{i,j}(B_{i,j})$ represents the gray level of image A(B) at pixel (i, j), and $\tilde{A}(\tilde{B})$ means the average

value.

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