

# PHOTONICS Research

## Dispersive non-Hermitian optical heterostructures

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The effect of material dispersion on the optical properties of one-dimensional non-Hermitian scattering systems is investigated in detail. In particular, multilayer heterostructures with gain and loss (parity-time symmetric or not) are examined by taking into account the dispersion of each layer. The exceptional points and phase transitions are characterized based on the spectrum of the corresponding scattering matrix. We demonstrate that an on-average lossy heterostructure can amplify an incident plane wave in the frequency range associated with the emission frequency of the layer with gain. © 2018 Chinese Laser Press

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### 1. INTRODUCTION

The discovery of parity-time symmetric (PT-symmetric) optics has prompted a surge of research activities for engineering synthetic materials with new properties and functionalities. PT-symmetric systems are non-Hermitian, but can exhibit entirely real spectra as long as they respect the conditions of PT symmetry [1]. PT-symmetric photonic systems [2–5] that rely on engineering the imaginary part of the refractive index to create balanced gain and loss regions, have many exotic features, including power oscillations, unidirectional invisibility, coherent perfect absorption, nonreciprocal light propagation, double refraction, and various intriguing nonlinear effects [6–20]. The research into PT-symmetric structures was primarily focused on the nondispersive materials, whereas the systems with dispersion have only lately attracted growing interest [21,22]. The non-Hermitian optical structures with dispersion were considered in Ref. [22]. It was shown that such structures could be PT-symmetric only for a discrete set of real frequencies. For gain and loss layers with identical parameters, the PT-symmetry condition can be fulfilled at the emission frequency.

It is necessary to note that in practice it is difficult to design structures with perfectly balanced gain and loss. The aim of this paper is to explore the fundamental properties of one-dimensional (1D) layered non-Hermitian systems (with or without PT-symmetry) by taking into account the dispersion for gain and loss media. We derive the characteristic frequencies for practical realization of PT-symmetry, and we investigate the role of material dispersion in non-Hermitian scattering systems,

where the spatial distributions of gain and loss are not subject to any spatial symmetry requirements. We point out the existence of exceptional points (EPs) associated with degenerate scattering eigenstates.

### 2. THEORETICAL ANALYSIS

Let us consider the bilayer composed of two slabs of thicknesses,  $d_g$  and  $d_l$ , with frequency-dependent dielectric permittivities corresponding to gain and loss materials. The full stack of thickness  $L$  is surrounded by a homogeneous medium with dielectric permittivity  $\epsilon_a$  at  $z \leq -d_g$  and  $z \geq d_l$  (Fig. 1). The system is assumed to be of infinite extent in the  $x$ - $y$  plane.

The gain and loss media are described by the Lorentzian permittivity. The dielectric permittivity components for the layer of type  $j$  (for gain medium  $j = 1$ ; for loss medium  $j = 2$ ) have the form [22],

$$\epsilon_j(\omega) = \epsilon_{0j} - \frac{\alpha_j \omega_{0j} \gamma_j}{\omega^2 - \omega_{0j}^2 + i\omega\gamma_j}, \quad (1)$$

where  $\epsilon_{0j}$  is the background permittivity of the medium,  $\omega_{0j}$  is the emission frequency,  $\gamma_j$  is the gain/absorption linewidth, and  $\alpha_j$  is the gain/absorption coefficient.

First, let us consider a PT-symmetric optical system. In this case, we assume that two slabs have an identical thickness,  $d = d_g = d_l$ , and the distributed loss and gain are in perfect balance. The PT-symmetry condition for such a structure is defined by the condition for the dielectric permittivities of the layers,

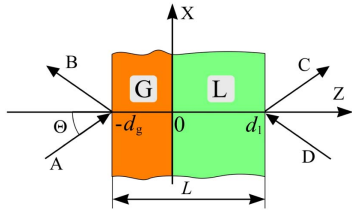


Fig. 1. Geometry of the problem.

$$\text{Re } \varepsilon_1(\omega) = \text{Re } \varepsilon_2(\omega), \text{Im } \varepsilon_1(\omega) = -\text{Im } \varepsilon_2(\omega). \quad (2)$$

We use positive parameters  $\alpha_2$  and  $\gamma_2$  for the absorbing medium with  $\text{Im } \varepsilon_2(\omega) > 0$ . To satisfy Kramers–Kronig relations, in the case of amplifying media with  $\text{Im } \varepsilon_1(\omega) < 0$ , we take  $\alpha_1 < 0$  and  $\gamma_1 > 0$ , which ensures the causality of dielectric permittivity [22]. The last equation in Eq. (2) leads to the following relation between the gain and absorption coefficients of two layers,

$$|\alpha_1| = \alpha_2 \frac{\omega_{02}\gamma_2^2[(\omega^2 - \omega_{01}^2)^2 + \gamma_1^2\omega^2]}{\omega_{01}\gamma_1^2[(\omega^2 - \omega_{02}^2)^2 + \gamma_2^2\omega^2]}. \quad (3)$$

The dependence of the absorption coefficient on the frequency is presented in Fig. 2(a). The parameters of the constituent gain layer correspond to the parameters of the medium with semiconductor quantum dots [23,24]:  $\omega_{01} = 1$  PHz and  $\gamma_1 = 0.067$  PHz. For the loss region, we consider the Lorentz dielectric with  $\omega_{02} = 1.2$  PHz and  $\gamma_2 = 0.14$  PHz. The numerical simulations were done for the wavelength range between 942 nm and 18.85  $\mu\text{m}$ . Figure 2(a) shows that the absorption coefficient reaches the minimum at the emission frequency  $\omega_{01}$  of the gain region. The peak magnitude of  $|\alpha_1|$  corresponds to the emission frequency  $\omega_{02}$ .

Substitution of Eq. (3) into Eq. (2), which is a condition imposed on the real part of permittivity of the system, leads to the difference between the background permittivities of the gain and loss media,

$$\Delta\varepsilon = \varepsilon_{02} - \varepsilon_{01} = \alpha_2 \frac{\omega_{02}\gamma_2 \left[ \omega^2 - \omega_{02}^2 + \frac{\gamma_2}{\gamma_1}(\omega^2 - \omega_{01}^2) \right]}{(\omega^2 - \omega_{02}^2)^2 + \gamma_2^2\omega^2}. \quad (4)$$

From Eq. (4), it immediately follows that for  $\Delta\varepsilon = 0$ , the PT-symmetry condition for an optical system can be fulfilled only for the frequency  $\omega_s = \sqrt{(\omega_{02}^2\gamma_1 + \omega_{01}^2\gamma_2)/(\gamma_1 + \gamma_2)}$ . The dependence of  $\Delta\varepsilon = \varepsilon_{02} - \varepsilon_{01}$  on the frequency for three different values of the absorption coefficient is displayed in

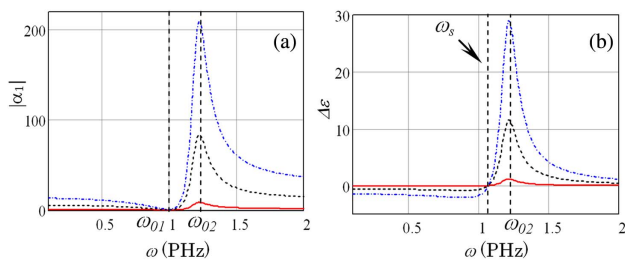


Fig. 2. Absorption coefficient  $|\alpha_1|$  and parameter  $\Delta\varepsilon$  as the functions of frequency at  $\alpha_2 = 0.2$  (solid red line),  $\alpha_2 = 2$  (dashed black line), and  $\alpha_2 = 5$  (dash-dot blue line).

Fig. 2(b). It is evident that for  $\Delta\varepsilon \neq 0$ , we can get a maximum of two frequencies for which the PT-symmetry condition is fulfilled. The maximal difference between the background permittivities corresponds to the frequency  $\omega = \omega_{02}$ .

Since the layers are assumed to be homogeneous in the  $x$ - $y$  plane, the fields can be divided to TE and TM polarized waves with their field profiles  $y$ -independent ( $\partial/\partial y = 0$ ); therefore, they can be analyzed separately. Here, we investigate the electromagnetic scattering properties of the medium under a TM plane-wave illumination incident at angle  $\Theta$  (Fig. 1). It is assumed that the stack is surrounded by air with the relative permittivity  $\varepsilon_a = 1$ . The field distributions are obtained as the solutions of the Helmholtz equation. To get the scattering characteristics, we use the Fresnel approach, taking into account the restrictions for the thickness of the amplifying layer [25]. The reflection and transmission coefficients are readily obtained by satisfying the continuity conditions for the tangential field components at the stack interfaces. We note that when the gain layer thickness exceeds some critical value, lasing begins to develop, and the characteristics of the system must be described in the nonlinear approximation using a field-dependent dielectric constant [25]. In this case, the solution of the problem cannot be obtained by the aforementioned Fresnel method. It can be shown that for frequency values near the emission frequency  $\omega_{01}$ , the critical thickness of the gain layer is much less than the wavelength of electromagnetic wave.

Wave scattering in the proposed system is modeled using the corresponding optical scattering matrix ( $S$ -matrix), which sequentially relates the fields at the system interfaces,

$$\begin{pmatrix} B \\ C \end{pmatrix} = \hat{S} \begin{pmatrix} A \\ D \end{pmatrix} = \begin{pmatrix} R^{(L)}(\omega) & T(\omega) \\ T(\omega) & R^{(R)}(\omega) \end{pmatrix} \begin{pmatrix} A \\ D \end{pmatrix}, \quad (5)$$

where  $A$  and  $C$  are amplitudes of the forward,  $B$  and  $D$  are amplitudes of the backward propagating waves outside the stack,  $R^{(L,R)}(\omega)$  are stack reflection coefficients for wave incident from the left (L) and right (R), and  $T(\omega)$  is the transmission coefficient. In the most general case, the eigenvalues of the  $S$ -matrix are given by

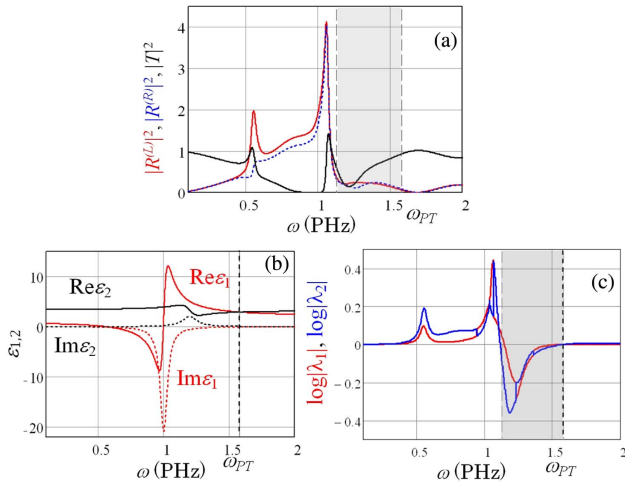
$$\lambda_{1,2} = \frac{R^{(L)}(\omega) + R^{(R)}(\omega) \pm \sqrt{[R^{(L)}(\omega) - R^{(R)}(\omega)]^2 + 4T(\omega)^2}}{2}. \quad (6)$$

The eigenvalues of the  $S$ -matrix of PT-symmetric structure must have reciprocal modules, namely the condition  $|\lambda_1\lambda_2| = 1$  should be satisfied. In the PT-symmetric phase, they are unimodular for symmetric phases  $|\lambda_1| = |\lambda_2| = 1$ . As it was demonstrated before [6], 1D PT-symmetric structures can undergo spontaneous symmetry-breaking transitions in terms of the eigenvalues and eigenvectors of the corresponding  $S$ -matrix. In the PT-symmetry breaking points, which are usually referred to as exceptional points, both eigenvalues coalesce and bifurcate, and the broken phases correspond to  $|\lambda_1| = 1/|\lambda_2| > 1$ . Apparently, two eigenvalues become degenerate and lead to EP in the unit circle of the complex plane when  $[R^{(L)}(\omega) - R^{(R)}(\omega)]^2 + 4T(\omega)^2 = 0$ .

### 3. NUMERICAL RESULTS

The reflectance for both left and right incidence and the transmittance of TM waves incident at angle  $\theta = 15^\circ$  are displayed in Fig. 3(a) for a non-Hermitian system with slabs of identical thickness that are about one order of magnitude less than the wavelength of an electromagnetic wave. The constituent layer parameters for this stack are the same as in Fig. 2. The spectral properties of dielectric permittivities  $\epsilon_j$  are illustrated in Fig. 3(b). The dependences of  $S$ -matrix eigenvalues on the frequency are displayed in Fig. 3(c). The frequency  $\omega_{PT} = 1.58$  PHz corresponds to the PT-symmetric phase. It is evident that in the proximity to the emission frequencies of the layers, the reflectivity exhibits noticeable asymmetry. We can observe that the gain-dominated systems [non-shaded frequency ranges in Fig. 3 for which  $\text{Im}(\epsilon_1 d_g + \epsilon_2 d_l) < 0$ ] can behave as a laser oscillator. At the frequencies  $\omega = 0.56$  PHz and  $\omega = 1.06$  PHz, we have peaks of the eigenvalues of scattering matrix and high amplification of reflected and transmitted waves.

Inspired by the so-called, “loss-induced transparency” phenomena [4], we examine now the increase of the transmittance in a loss-dominated system [a system for which  $\text{Im}(\epsilon_1 d_g + \epsilon_2 d_l) > 0$ ]. In this case and for specific frequencies that are close to the emission frequency of the gain layer, we can understand from Fig. 4(a) that it is possible to have amplification even though we increase the absorption coefficient. It can be seen that the existence of the gain layer leads to the wave-amplifying behavior of the loss-dominated system. The analysis of the energy flux density ( $z$ -component of Poynting vector) distribution through the loss-dominated system with  $\alpha_2$

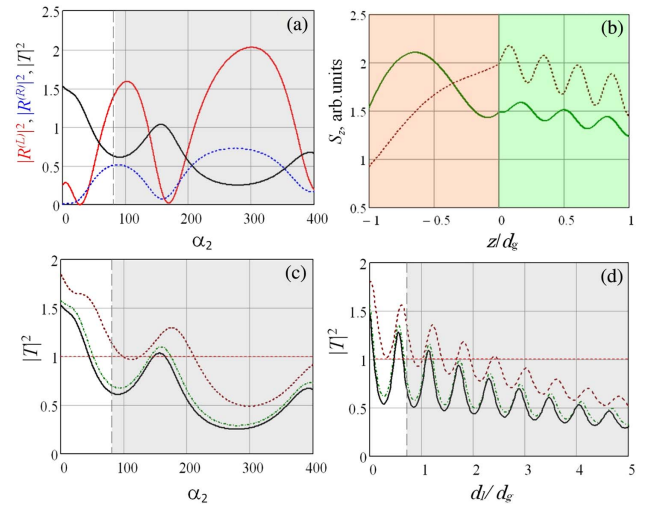


**Fig. 3.** (a) Reflectance [ $R^{(L)}(\omega)$ - solid red line and  $R^{(R)}(\omega)$ - dashed blue line] and transmittance (solid black line) of TM wave incident at  $\theta = 15^\circ$  on 1D system with  $\omega_{01} = 1$  PHz,  $\omega_{02} = 1.2$  PHz,  $\gamma_1 = 0.067$  PHz,  $\gamma_2 = 0.14$  PHz,  $|\alpha_1| = 20.86$ ,  $\alpha_2 = 2$ ,  $\epsilon_{01} = 2$ ,  $\epsilon_{02} = 3.22$ , and  $d_g = d_l = 500$  nm. (b) Real (solid lines) and imaginary (dashed lines) parts of dielectric permittivities of the layers near emission frequencies of the gain (red lines) and loss (black lines) regions. (c) Modulus of eigenvalues of  $S$ -matrix for 1D system as a function of frequency; red and blue lines correspond to the different eigenvalues of the same scattering matrix. The range of frequencies in (a) and (c) corresponding to the loss dominated system is gray shaded.

corresponding to the minimum and maximum of transmittance shows that the amplification phenomena in the loss-dominated system are connected with substantial wave amplification by the gain region [Fig. 4(b)]. The transmittance of TM wave incident at three different values of the incidence angles on non-Hermitian system with identical layer thicknesses is displayed in Fig. 4(c). We observe that the transmittance of the system increases with the angle of incidence. As a result, the maximum amplification of the transmittance can be obtained for a wider range of absorption coefficient values in loss-dominated systems. Similar effects can be observed for a fixed parameter  $\alpha_2 = 115$  and an increasing thickness of the absorbing layer [Fig. 4(d)].

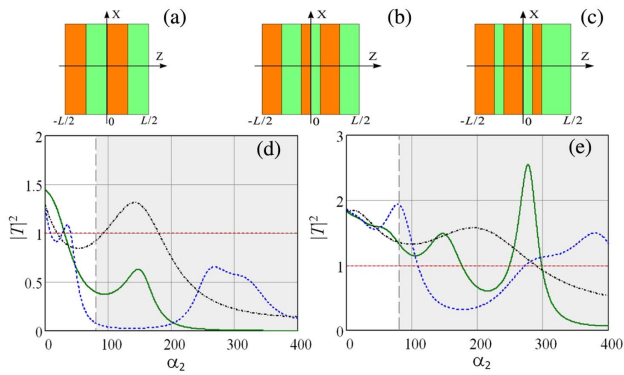
To further investigate this effect of amplification in a loss-dominated multilayer heterostructure, the transmittance coefficient  $|T|^2$  of TM waves incident at angles  $\theta = 0^\circ$  and  $\theta = 60^\circ$  is displayed in Fig. 5 for periodic [Fig. 5(a)] and two random [Figs. 5(b) and 5(c)] stacks of alternating layers with gain and loss. The total thickness of all three stacks coincides with the thickness of the bilayer in Fig. 4(c). Considerable differences in  $|T|^2$  of the layered system are evident in Figs. 5(d) and 5(e). Amplification of the transmitted wave can be observed in the gain-loss multilayers, depending on the angle of incidence [Fig. 5(e)].

As mentioned before, in PT-symmetric systems the eigenvalues of the scattering matrix identify the exceptional points at which the symmetry-breaking transitions occur. To illustrate the effect of the gain/absorption coefficient, the dependence



**Fig. 4.** (a) Reflectance [ $R^{(L)}(\omega)$ - solid red line and  $R^{(R)}(\omega)$ - dashed blue line] and transmittance (solid black line) of TM wave incident at  $\theta = 0^\circ$  and  $\omega = 0.92$  PHz on non-Hermitian system with  $\omega_{01} = 1$  PHz,  $\omega_{02} = 2$  PHz,  $\gamma_1 = 0.067$  PHz,  $\gamma_2 = 0.14$  PHz,  $\alpha_1 = 2$ ,  $\epsilon_{01} = 2$ ,  $\epsilon_{02} = 2.2$ , and  $d_g = d_l = 500$  nm as a function of absorption coefficient. (b) Distribution of the energy flux density of an electromagnetic field in the system with the default parameters and  $\alpha_2 = 92.3$  (solid green line) and  $\alpha_2 = 156.5$  (dashed brown line); the area corresponding to gain material is red shaded, the area corresponding to loss material is green shaded. (c), (d) Transmittance of TM wave incident at  $\theta = 0^\circ$  (solid black line),  $\theta = 30^\circ$  (dash-dot green line),  $\theta = 60^\circ$  (dashed brown line), and  $d_g = 500$  nm. The range of frequencies in (a), (c), and (d) corresponding to the loss dominated system is gray shaded.

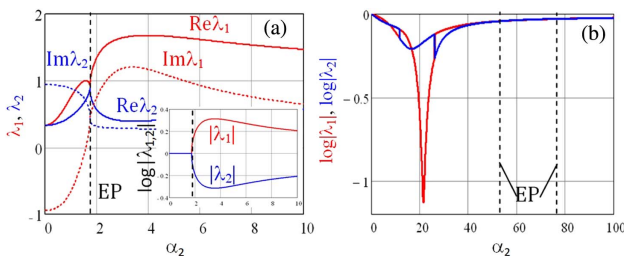




**Fig. 5.** (a) Geometry of non-Hermitian periodic stack. (b), (c) Geometries of non-Hermitian random stacks. (d), (e) Transmittance of TM wave through the stacks incident at  $\omega = 0.92$  PHz and (d)  $\theta = 0^\circ$ , (e)  $\theta = 60^\circ$ ; dash-dot black curve, four layers with  $d_g = d_l = 250$  nm, geometry of the system is presented in (a); dashed blue curves, six layers with thicknesses  $d_{g1} = d_{g3} = d_{l1} = d_{l3} = 200$  nm and  $d_{g2} = d_{l2} = 100$  nm, geometry of the stack is presented in (b); solid green curves, six layers with thicknesses  $d_{g1} = d_{g2} = 200$  nm,  $d_{l1} = d_{l2} = d_{g3} = 100$  nm, and  $d_{l3} = 300$  nm, geometry of the stack is presented in (c). The relevant layer parameters for the stacks are the same as in Fig. 4. The range of frequencies in (d) and (e) corresponding to loss dominated system is gray shaded.

of  $S$ -matrix eigenvalues for PT-symmetric bilayer with identical parameters of gain and loss regions ( $\omega_0 = \omega_{01} = \omega_{02}$ ,  $\gamma = \gamma_1 = \gamma_2$ ,  $\epsilon_0 = \epsilon_{01} = \epsilon_{02}$ ,  $d_g = d_l$ ,  $\alpha_2 = |\alpha_1|$ ) at frequency  $\omega = \omega_0$  is presented in Fig. 6(a). The point of degeneracy of two eigenvalues is the EP. The numerical examination of the modulus of eigenvalues [see inset in Fig. 6(a)] shows that with an increase in the gain/absorption coefficient, the symmetry breaking occurs.

In the general case of a non-Hermitian system, the analysis of the eigenvalue spectrum dependence on the parameter  $\alpha_2$  demonstrates that the EPs can be obtained for specific parameters of the structure and incident wave [see Fig. 6(b)]. For the non-Hermitian bilayer, starting from some values of  $\alpha_2$ , we can get a set of EPs. Moreover, at high values of the absorption coefficient, where the loss layer has a high reflectivity, the eigenvalues



**Fig. 6.** Eigenvalues of the  $S$ -matrix as a function of absorption coefficient for (a) PT-symmetric bilayer with  $\omega_{01} = \omega_{02} = \omega = 1$  PHz,  $\gamma_1 = \gamma_2 = 0.067$  PHz,  $\epsilon_{01} = \epsilon_{02} = 2$ , and  $d_g = d_l = 500$  nm at  $\theta = 0^\circ$  (solid and dashed lines of the same color correspond to the real and imaginary parts of eigenvalues) and (b) non-Hermitian bilayer with  $\omega_{01} = 1$  PHz,  $\omega_{02} = 2$  PHz,  $\gamma_1 = 0.067$  PHz,  $\gamma_2 = 0.14$  PHz,  $\alpha_1 = 2.4$ ,  $\epsilon_{01} = 2$ ,  $\epsilon_{02} = 2.2$ , and  $d_g = d_l = 500$  nm at  $\omega = 2.65$  PHz and  $\theta = 0^\circ$ . Red and blue lines correspond to the eigenvalues  $\lambda_1$  and  $\lambda_2$ . Vertical dashed lines indicate the position of EPs.

of the  $S$ -matrix are not sensitive to variations of  $\alpha_2$ . It is observed that, for larger angles of wave incidence, the EPs correspond to the higher frequencies and absorption coefficients. The composition of the stack has a significant impact on the number of EPs. In the case of a random stack of alternating layers with gain and loss, we get just one EP at a particular frequency.

## 4. CONCLUSIONS

In summary, the basic scattering properties of dispersive non-Hermitian systems, where the spatial distributions of gain and loss are not subject to any spatial symmetry requirements, are systematically examined. We demonstrate that the proper combination of the parameters of constitutive materials of the system helps to implement the PT symmetry at desirable frequencies of incident waves. It is shown that the dispersive system with nonidentical parameters of materials with gain and loss could be PT-symmetric maximum for two real frequencies. One of our main results is that, for a frequency range close to the emission frequency of the gain layer by changing the parameters of the layers and incident waves and composition of the stack, we can have amplification of a transmitted wave in an on-average lossy non-Hermitian structure. The analysis of the eigenvalue spectrum of non-PT symmetric systems demonstrates the existence of EPs associated with degenerate scattering eigenstates.

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