PHOTONICS Research

Enhancement of the angular rotation measurement sensitivity based on SU(2) and SU(1,1) interferometers

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We investigate the sensitivity of the angular rotation measurement with the method of homodyne detection in SU(2) and SU(1,1) interferometers by employing orbital angular momentum (OAM). By combining a coherent beam with a vacuum beam in an SU(2) interferometer, we get the sensitivity of the angular rotation measurement as $\frac{1}{2\sqrt{N}l}$. We can surpass the limit of the angular rotation measurement in an SU(1,1) interferometer by combining a coherent beam with a vacuum beam or a squeezed vacuum beam when the probe beam has OAM. Without injection, the sensitivity can reach $\frac{1}{2Nl}$. In addition, by employing another construction of an SU(1,1) interferometer where the pump beam has OAM, with the same injection of an SU(1,1) interferometer, the sensitivity of the angular rotation measurement can be improved by a factor of 2, reaching $\frac{1}{4Nl}$. The results confirm the potential of this technology for precision measurements in angular rotation measurements.

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1. INTRODUCTION

An optical interferometer [1,2] is an important tool for precision measurements of phase shifts. Traditional interferometers, so called an SU(2) interferometers, e.g., a Mach–Zehnder (MZ) interferometer, is combined by beam splitters (BSs), which are employed to split the beam into two arms and recombine the beams together. For a normal MZ interferometer, it has two input ports, which are injected usually by one coherent beam in one port. The sensitivity of this interferometer is limited by the quantum noise of the other unused port [3–5]. By injection of the squeezed vacuum beam in the unused port, the sensitivity is improved and the method has been widely applied to, for example, the Laser Interferometer Gravitational-Wave Observatory (LIGO) [6–9]. In 1986, another nonlinear interferometer, which was named as the SU(1,1) interferometer, was proposed by Yurke *et al.* [10]. The SU(1,1) interferometer is usually composed of two nonlinear processes, such as four-wave mixing or parametric amplification (PA). Employment of the SU(1,1) interferometer can surpass the phase shot noise limit (SNL), which is defined as $\frac{1}{\sqrt{N}}$ [11–20], where N is the average photon number. By injecting two vacuum beams, the phase sensitivity can be further enhanced and reach the phase Heisenberg limit.

Compared with the phase measurement, recently, the measurement of angular rotation [21] has attracted much attention based on the interferometers. A photon with the Laguerre–Gaussian (LG) mode is a candidate for quantum information processing [22], which possess both spin angular momentum (SAM) and orbital angular momentum (OAM) [23–28]. The OAM relates to the transverse angular phase of the light in the form of $\exp(il\psi)$, while the SAM is related to the light polarization. Utilizing photon OAM can amplify a mechanical rotation of θ into $l\theta$, where l is the OAM quantum number of the photon. Jha *et al.* employed N-unentangled photons and N-entangled photons to inject the MZ SU(2) interferometer and found resolutions of $\frac{1}{2\sqrt{N}l}$ and $\frac{1}{2Nl}$, respectively [29]. Soon after, Zhang *et al.* obtained superresolving and ultrasensitive angular rotation measurements by employing the quantum measurement strategy [30,31]. However, with the injection of a coherent beam and a vacuum beam, the sensitivity of angular rotation even does not surpass the limit of $\frac{1}{2\sqrt{N}l}$. This means that the sensitivity of the angular rotation measurement is still limited by the SNL of the angular rotation measurement in the SU(2) interferometer. As described above, as SU(1,1) interferometers can significantly improve the phase sensitivity, they could also be good candidates for the angular rotation measurement. To implement the measurement, compared with the precious SU(1,1) interferometer [11], we need inject two

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Dove prisms in two arms of the SU(1,1) interferometers to transform the angular rotation into phase variation.

In this paper, we study the angular rotation measurement both in SU(2) and SU(1,1) interferometers with homodyne detection (HD). When a coherent beam carrying OAM and a vacuum beam enters into the SU(2) interferometer, we get the sensitivity $\frac{1}{2\sqrt{N}l}$, which is defined as SNL. In order to achieve the optimal angular rotation sensitivity, we discuss three kinds of input beams in two sorts of SU(1,1) interferometers. In the first case, the sensitivity of the angular rotation measurement is improved by a factor of $\frac{1}{\sqrt{2G}}$ compared with SNL in an SU(1,1) interferometer where OAM is carried by a probe beam. In the following two cases, the inputs are a coherent and a squeezing beam or a vacuum and a vacuum beam, where the sensitivity can reach $\frac{1}{2Nl}$. Moreover, we also consider the SU(1,1) interferometer where a pump beam carries OAM in the PA process, and the angular rotation sensitivity can be enhanced by a factor of 2, reaching our optimal result of $\frac{1}{4Nl}$.

The paper is organized as follows. In Section 2, we present a description of the angular rotation measurement of an SU(2) interferometer using a coherent beam and a vacuum beam. In Section 3.A, we describe an SU(1,1) interferometer where a probe beam has OAM with the injection of a coherent beam and a vacuum beam, a coherent beam and a squeezing vacuum beam, or a vacuum beam and a vacuum beam. In Section 3.B, the sensitivity of another construction of an SU(1,1) interferometer is investigated where OAM is carried by the pump beam and the probe beam no longer carries OAM with the same injection to the previous interferometer. Section 4 presents our conclusions.

2. SCHEME OF THE SU(2) INTERFEROMETER

We consider an MZ interferometer with a coherent beam in one input port and a vacuum beam in the other input port by employing the method of HD. Different from the usual SU(2) interferometer, two Dove prisms are located in each arm respectively, as shown in Fig. 1. Dove prism 2 orients at angle θ_2 and Dove prism 1 is set at $\theta_1 = 0$. The employment of Dove prisms is that it not only rotates the beam with OAM but also can alternate the value from l to $-l$. In addition, for a coherent beam $|\alpha\rangle$ with OAM l at input-port a and a vacuum beam at input-port b, when a beam with OAM passes through a Dove prism, it will have a phase shift of $2l\theta$, and θ is the rotation angular. The input–output relations of the BS are

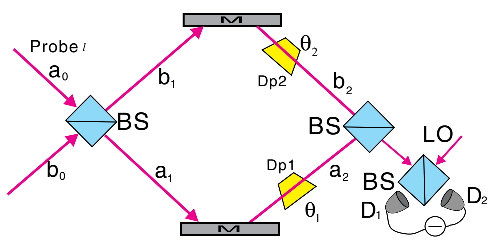

Fig. 1. Scheme for the angular rotation measurement uses a coherent beam carrying OAM and a vacuum beam. The coherent beam comes from input-port a and has an OAM of $l\hbar$ and is detected by the method of HD. M, mirror; DP, Dove prism; LO, local beam.

$$\hat{a}_1 = \sqrt{T}\hat{a}_{0,+l} + i\sqrt{R}\hat{b}_0,\ \hat{b}_1 = i\sqrt{R}\hat{a}_{0,+l} + \sqrt{T}\hat{b}_0. \quad (1)$$

$T = R = \frac{1}{2}$ are the transmissivity and reflectivity of the BS. $a_{0,+l}$ and b_0 are annihilate operators. According to Eq. (1), we obtain the input–output relations of the interferometer, which are described by

$$\hat{a}_{\rm out} = \frac{1}{2}\hat{a}_{0,+l}(e^{2il\theta_2} - 1) + \frac{1}{2}i\hat{b}_0(e^{2il\theta_2} + 1),$$
$$\hat{b}_{\rm out} = i\frac{1}{2}\hat{a}_{0,+l}(e^{2il\theta_2} + 1) + \frac{1}{2}\hat{b}_0(e^{2il\theta_2} - 1). \quad (2)$$

After the first BS, the beam intensity is

$$I_{\rm ps} = \langle \hat{b}_1^+ \hat{b}_1 \rangle = \frac{|\alpha|^2}{2}. \quad (3)$$

We assume that $|\alpha| \gg 1$. The beam intensity that passes through the interferometer is

$$\langle \hat{b}_{\rm out}^+ \hat{b}_{\rm out} \rangle = \frac{\langle \hat{b}_0^+ \hat{b}_0 \rangle [1 + \cos(2l\theta_2)]}{2} = I_{\rm ps}[1 + \cos(2l\theta_2)]. \quad (4)$$

In the next step, according to Ref. [11], the definition of the quadrature amplitude of this interferometer is

$$\hat{X}_{\rm out} = \hat{a}_{\rm out}^+ + \hat{a}_{\rm out}. \quad (5)$$

By assuming that $\alpha = i|\alpha|$, we can easily calculate the quadrature amplitude. Moreover, from Ref. [11], the interferometer works at near $2l\theta_2 = 0$ and we assume that $2l\theta_2 = \delta$, with $\delta \ll 1$. We have

$$\langle \hat{X}_{\rm out}^2 \rangle = |\alpha||\alpha| \sin^2(2l\theta_2) + 1 = \langle \hat{X}_{\rm out} \rangle^2 + 1$$
$$= 2I_{\rm ps}\delta^2 + 1. \quad (6)$$

In Eq. (6), $\langle X_{\rm out} \rangle^2$ corresponds to the signal where the noise is just 1 from vacuum noise, which leads to the signal-to-noise ratio (SNR) as

$$R_{\rm limit} = |\alpha|^2(\delta)^2/1 = 2I_{\rm ps}(\delta)^2 = 2I_{\rm ps}(2l\theta_2)^2. \quad (7)$$

Then the phase sensitivity of an SU(2) interferometer is

$$\delta = \frac{1}{\sqrt{N}} = \delta_{\rm SNL}. \quad (8)$$

Thus, the corresponding sensitivity of the angular rotation measurement is

$$\theta_2 = \frac{1}{2l\sqrt{N}} = \theta_{\rm SNL}. \quad (9)$$

According to Ref. [11], the phase sensitivity in Eq. (8) is defined as the phase SNL. Furthermore, the sensitivity of the angular rotation measurement in Eq. (9) is the same as the result of N-unentangled photons in an SU(2) interferometer. However, when a coherent beam and a vacuum beam enter an SU(2) interferometer by the HD, the phase sensitivity is defined as phase SNL. Thus, this sensitivity of the angular rotation measurement is defined as SNL. In addition, in Ref. [29], with the same construction, it is injected with N-unentangled photons carrying OAM and is directly detected by the detector. The angular rotation measurement of this construction is the relative angular rotation between the two Dove prisms. Here, though the sensitivity of the angular rotation measurement is the same as the previous result, what we measure is the absolute angular rotation of Dove prism 2.

3. SCHEME OF THE SU(1,1) INTERFEROMETER

A. Probe Beam Carries OAM

Now, let us consider an SU(1,1) interferometer. Unlike an SU(2) interferometer in Fig. 1, here BS is replaced by PA as shown in Fig. 2. For the PA process, G is the gain of PA and the relationship between input and output is

$$\hat{a}_1 = \sqrt{G}\hat{a}_{0,+l} + \sqrt{G-1}\hat{b}_0^+,$$
$$\hat{b}_1 = \sqrt{G-1}\hat{a}_{0,+l}^+ + \sqrt{G}\hat{b}_0. \quad (10)$$

The input–output relations of the whole interferometer are

$$\hat{a}_{\text{out}} = [G + (G-1)e^{2il\theta_2}]\hat{a}_{0,+l} + \left[\sqrt{G(G-1)} + \sqrt{G(G-1)}e^{2il\theta_2}\right]\hat{b}_0^+,$$
$$\hat{b}_{\text{out}} = \left[\sqrt{G(G-1)} + \sqrt{G(G-1)}e^{-2il\theta_2}\right]\hat{a}_{0,+l}^+ + [(G-1) + Ge^{-2il\theta_2}]\hat{b}_0. \quad (11)$$

As the above SU(2) interferometer, we can calculate I_{ps} as

$$\hat{I}_{\text{ps}} = \langle \hat{b}_1^+\hat{b}_1\rangle = (G-1)(|\alpha|^2+1) \approx (G-1)|\alpha|^2, |\alpha| \gg 1. \quad (12)$$

The output intensity of the interferometer can be expressed as

$$\langle \hat{b}_{\text{out}}^+\hat{b}_{\text{out}}\rangle = [2G(G-1) + 2G(G-1)\cos(2l\theta_2)](|\alpha|^2+1)$$
$$\approx [2G(G-1) + 2G(G-1)\cos(2l\theta_2)](|\alpha|^2)$$
$$= 2G[1+\cos(2l\theta_2)]I_{\text{ps}}. \quad (13)$$

Next, we examine the quantum noise in the output of the interferometer

$$\langle \hat{X}_{b\text{out}}^2\rangle = 2G(G-1)|\alpha|^2[1-\cos(4l\theta_2)] + 4G^2 - 4G + 1 + 4G(G-1)\cos(2l\theta_2). \quad (14)$$

Here, we assume that $\theta_2 = \theta + \Delta\theta$ and $2l\theta_2 = 2l(\theta+\Delta\theta)$. The interferometer usually works near at the dark fringe with $2l\theta = \pi$. Thus, with a very small angular rotation shift $\Delta\theta \ll 1$, $2l\theta = \pi$, and $2l\Delta\theta = \delta$, we have

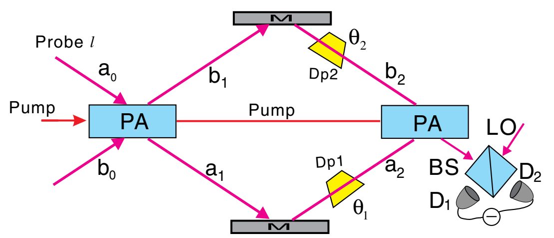

Fig. 2. SU(1,1) interferometer for the angular rotation measurement uses a coherent beam and a vacuum beam. The beam comes from input-port a carries an OAM of $l\hbar$. Note that two PA processes replace two BS compared with Fig. 1.

$$\langle \hat{X}_{b\text{out}}^2\rangle \approx G(G-1)\delta^2(4|\alpha|^2+2)+1. \quad (15)$$

Hence, the SNR for measuring a small phase shift of $\Delta\theta$ is given by

$$R_{\text{limit}} = G(G-1)\delta^2(4|\alpha|^2+2) \approx 4GI_{\text{ps}}\delta^2. \quad (16)$$

Under this condition, we get a phase measurement of angular rotation shift, which is the same as Ref. [11],

$$\delta = \frac{1}{2\sqrt{GI_{ps}}} = \frac{\delta_{\text{SNL}}}{\sqrt{2G}}. \quad (17)$$

Due to the condition that $2l\Delta\theta = \delta$, we get the sensitivity of the angular rotation measurement, which is described by

$$\Delta\theta = \frac{1}{4l\sqrt{GI_{\text{ps}}}} = \frac{\theta_{\text{SNL}}}{\sqrt{2G}}. \quad (18)$$

Obviously, comparing the result with SNL, it is easy to find that with the gain of PA, the angular rotation sensitivity of an SU(1,1) interferometer is enhanced by a factor of $\frac{1}{\sqrt{2G}}$. The beam input is a coherent beam and a vacuum beam. We assume that $|\alpha| = \sqrt{N} \gg 1$, for the technology limitation, where the gain is less than $\sqrt{N}$. While the sensitivity is better than the SNL, it is impossible to reach $\frac{1}{2Nl}$.

Next, we reduce the noise by injecting a squeezed vacuum beam. In the following model, we set the input port a_0 in a coherent state $|\alpha\rangle$, while the input port $\hat{b}_0$ is in a squeezed vacuum beam. In this case,

$$\xi^+\hat{b}_0\xi = \hat{b}_0 \cosh r - \hat{b}_0^+ e^{2i\phi}\sinh r,$$
$$\xi^+\hat{b}_0^+\xi = \hat{b}_0^+ \cosh r - \hat{b}_0 e^{-2i\phi}\sinh r, \quad (19)$$

where ξ is the squeezing operator so that the input beam is $\xi|0\rangle$ and r is the squeezing parameter. For convenience, we assume that $\phi = 0$ and we get the quadrature amplitude

$$\langle \hat{X}_{b\text{out}}^2\rangle \approx 4G(G-1)\delta^2|\alpha|^2 + e^{-r}|\alpha| \gg 1. \quad (20)$$

Hence, we get the signal-to-noise ratio, which is displayed as

$$R_{\text{limit}} = 4G(G-1)\delta^2|\alpha|^2e^r. \quad (21)$$

Then the phase shift is

$$\delta = \frac{1}{2\sqrt{GI_{\text{ps}}e^r}} = \frac{\delta_{\text{SNL}}}{\sqrt{2Ge^r}}. \quad (22)$$

Taking OAM into consideration, we get

$$\Delta\theta = \frac{1}{4l\sqrt{GI_{\text{ps}}e^r}} = \frac{\theta_{\text{SNL}}}{\sqrt{2Ge^r}}. \quad (23)$$

With the squeezed vacuum at the other input port, we get a better sensitivity of the angular rotation measurement with an improvement by a factor of $\sqrt{e^r}$. While we increase the gain of the PA and the squeezing degree of a vacuum squeezing beam, it seems possible to get the result of $\frac{1}{2lN}$. However, in the current experimental condition, to obtain such a result, we need $\sqrt{2Ge^r} = \sqrt{N}$, and it still has challenges to approach it. In our configuration, the phase shift is caused by Dove prisms and OAM. Due to the generation of photons from a squeezing vacuum beam, here, we should assume that the squeezing vacuum beam carries OAM of $-l\hbar$.

Next, we consider an interesting case that there is no injection of a coherent beam at all. Then $|\alpha|^2 = 0$ and

$$I_{\rm ps} = G - 1. \tag{24}$$

The intensity of the outcome is

$$\langle \hat{b}^{+}_{\rm out}\hat{b}_{\rm out}\rangle = [2G(G-1) + 2G(G-1)\cos(2l\theta_2)]. \tag{25}$$

Thus, the quadrature amplitude is

$$\langle \hat{X}^2_{b\rm out}\rangle = 1 + 4G(G-1)\cos(2l\theta_2) \approx 1 + 2G(G-1)\delta^2. \tag{26}$$

Hence, the SNR is

$$R_{\rm limit} = 2G(G-1)\delta^2. \tag{27}$$

In this case, the phase shift is

$$\delta = \frac{1}{\sqrt{2G(G-1)}} = \frac{1}{\sqrt{2I_{\rm ps}(I_{\rm ps}+1)}} \approx \frac{1}{N} = \delta_{\rm HL}. \tag{28}$$

The result is just the Heisenberg limit in phase measurement as Ref. [11]. The angular rotation measurement of the Dove prism is shown as

$$\Delta\theta \approx \frac{1}{2lN}. \tag{29}$$

It is identical to the result of Ref. [29] where N-entangled photons enter into the SU(2) interferometer. Obviously, due to the simple device, it is much easier to realize than that with N-entangled photons. However, the problem in this scheme is that due to no injection of a coherent beam to boost the sensing intensity, N is low. According to Ref. [28], another problem in this interferometer is that $l\hbar$ is bounded by the spontaneous emitted photon from PA. In this case, in fact, the topological charge l is limited in 0, ± 1, and ± 2. In addition, we also need the projection measurement to choose the valid $l\hbar$.

B. Pump Beam Carries OAM

In this section, we consider another construction where the pump beam has the OAM of $l\hbar$, as shown in Fig. 3. Combining the phase-matching conditions with the assumption that the probe beam no longer carries OAM, the beam b has a topological charge of $2l$ [32]. For the PA where the probe beam has no OAM, it follows that

$$\hat{a}_1 = \sqrt{G}\hat{a}_0 + \sqrt{G-1}\hat{b}^{+}_0,$$
$$\hat{b}_1 = \sqrt{G-1}\hat{a}^{+}_0 + \sqrt{G}\hat{b}_0. \tag{30}$$

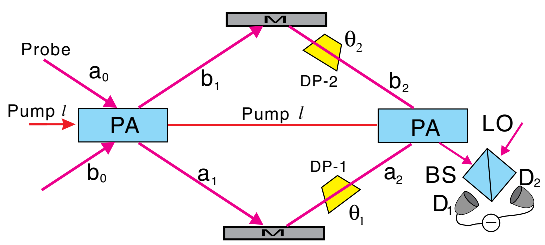

Fig. 3. SU(1,1) interferometer for the angular rotation measurement uses a coherent beam and a vacuum beam. The pump beam has an OAM of $l\hbar$, and the probe beam has no OAM.

Then the relationship of this SU(1,1) interferometer between input and output is given by

$$\hat{a}_{\rm out} = [G + (G-1)e^{-4il\theta_2}]\hat{a}_0 + \left[\sqrt{G(G-1)} + \sqrt{G(G-1)}e^{-4il\theta_2}\right]\hat{b}^{+}_0,$$
$$\hat{b}_{\rm out} = \left[\sqrt{G(G-1)} + \sqrt{G(G-1)}e^{4il\theta_2}\right]\hat{a}^{+}_0 + [(G-1) + Ge^{4il\theta_2}]\hat{b}_0. \tag{31}$$

As we have calculated before, we get the $I_{\rm ps}$ as

$$I_{\rm ps} = \langle \hat{b}^{+}_1\hat{b}_1\rangle = (G-1)(|\alpha|^2+1) \approx (G-1)|\alpha|^2, |\alpha| \gg 1. \tag{32}$$

Then the output intensity is calculated as

$$\begin{aligned}\langle \hat{b}^{+}_{\rm out}\hat{b}_{\rm out}\rangle &= [2G(G-1) + 2G(G-1)\cos(4l\theta_2)](|\alpha|^2+1)\\ &\approx [2G(G-1) + 2G(G-1)\cos(4l\theta_2)]|\alpha|^2\\ &= 2G[1+\cos\,(4l\theta_2)]I_{\rm ps}.\end{aligned} \tag{33}$$

The quadrature amplitude is

$$\langle \hat{X}^2_{b\rm out}\rangle = 2G(G-1)|\alpha|^2[1-\cos(8l\theta_2)] + 4G^2 - 4G + 1 + 4G(G-1)\cos(4l\theta_2). \tag{34}$$

Here, we assume that $\theta_2 = \theta + \Delta\theta$, and the interferometer works near $4l\theta = \pi$. Thus, $4l\theta_2 = 4l(\theta+\Delta\theta) = \pi + \delta$, and then

$$\langle \hat{X}^2_{b\rm out}\rangle \approx G(G-1)\delta^2(4|\alpha|^2+2) + 1. \tag{35}$$

We get the SNR like before as

$$R_{\rm limit} = G(G-1)\delta^2(4|\alpha|^2+2) \approx 4GI_{\rm ps}\delta^2. \tag{36}$$

Hence, the phase shift is derived as

$$\delta = \frac{1}{2\sqrt{GI_{\rm ps}}} = \frac{\delta_{\rm SNL}}{\sqrt{2G}}. \tag{37}$$

We get the angular rotation measurement, which is shown as

$$\Delta\theta = \frac{1}{8l\sqrt{GI_{\rm ps}}} = \frac{\theta_{\rm SNL}}{2\sqrt{2G}}. \tag{38}$$

Obviously, the phase sensitivity in Eq. (37) is same as Eq. (17), and the sensitivity of the angular rotation measurement is enhanced by a factor of 2. Then in the case that the unused mode $\hat{b}_0$ is a vacuum squeezed beam, the quadrature amplitude is

$$\langle \hat{X}^2_{b\rm out}\rangle \approx 4G(G-1)\delta^2|\alpha|^2 + e^{-r}, \tag{39}$$

and the SNR is

$$R_{\rm limit} = 4G(G-1)\delta^2|\alpha|^2e^{r}. \tag{40}$$

Then we get the phase shift as in Ref. [13]:

$$\delta = \frac{1}{2\sqrt{GI_{\rm ps}e^{r}}} = \frac{1}{\sqrt{2GNe^{r}}}. \tag{41}$$

Compared with Eq. (23), the sensitivity of phase is same to each other, while the angular rotation from each other is different, which is shown as

$$\Delta\theta = \frac{1}{8l\sqrt{GI_{\rm ps}e^{r}}} = \frac{1}{4l\sqrt{2GNe^{r}}}. \quad (42)$$

Next, another interesting case is that there is no injection of a coherent beam at all while the pump beam carries OAM $l\hbar$. Then the calculated phase shift is

$$\delta \approx \frac{1}{N} = \delta_{\rm HL}. \quad (43)$$

Therefore, the corresponding sensitivity of the angular rotation measurement is

$$\Delta\theta \approx \frac{1}{4lN}. \quad (44)$$

Compared with the SU(1,1) interferometer where the probe beam carries OAM, the angular rotation measurement of this interferometer is enhanced by a factor of 2 and the phase sensitivity is same when topological charge l is carried by the pump beam. Furthermore, in mode A, the path a carries OAM and we set the angular rotation θ as 0. In the current case, due to the phase-matching condition, the path a no longer carries OAM and the angular rotation of Dove prism 1 can be an arbitrary angle. Since the input is a coherent beam and a squeezing vacuum beam, to meet the phase-matching condition, the squeezing vacuum beam needs to carry OAM of $2l\hbar$. In this case, the input is two vacuum beams, the OAM, which is carried by photons from spontaneous emission, is complex, and we have choose the one that path b has OAM of $2l\hbar$. However, the topological charge l is also limited by PA. In addition, for the N-entangled photons in an SU(2) interferometer, this construction can beat the sensitivity. Although the result $\frac{1}{4Nl}$ has the limit of low photon number due to the loss of a coherent beam, it is still the best result of angular rotation measurements so far. At the same time, it is much easier than the N-entangled photons in an SU(2) interferometer, which is limited by the generation of N-entangled photons.

4. CONCLUSION

In conclusion, we have showed that by the employment of HD, the sensitivity of angular rotation measurements in an SU(2) interferometer with the injection of a coherent beam and a vacuum beam can be same, which is injected by N-unentangled photons with intensity detection, $\frac{1}{2\sqrt{N}l}$. Furthermore, the analysis shows that the SU(1,1) interferometer where OAM is carried by a probe beam can further enhance the performance of the sensitivity with a coherent beam combined with a vacuum beam using the method of HD. The sensitivity can be improved while a vacuum beam is substituted by a vacuum beam or a coherent beam is replaced by a vacuum squeezed beam. In addition, we find that compared with SU(1,1) interferometer where OAM is carried by a probe beam, the sensitivity of the SU(1,1) interferometer where OAM is carried by a pump beam has increased by a factor of 2 and the best result is $\frac{1}{4Nl}$. The work presented here provides such examples to detect small rotation of optical components, including single Dove-prism angular rotation rather than relative angular rotation between two Dove prisms. It will have potential applications both in sensing and performing fundamental studies.

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