

# Theoretical investigation of core mode cut-off condition for tapered multicore fiber

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Core mode cutoff is a useful concept not only for a tapered single-core fiber (SCF) but also for a tapered multicore fiber (MCF) to realize cladding mode transmission. In this paper, cut-off conditions of either core mode for tapered SCFs or supermodes for MCFs are theoretically investigated. Rigorous analytical formulas are derived for the modes of SCF by a three-layer waveguide model, and an approximation formula of the cut-off condition is given for the LP<sub>01</sub> mode. The supermodes of MCFs are analyzed by the coupling mode theory, and the cut-off condition is calculated by a numerical method. Simulation results show that the in-phase supermode of MCFs has a similar cut-off condition with that of SCF. Based on this property, a convenient approximate formula is given to estimate the cut-off condition of the in-phase supermode for tapered MCFs. © 2015 Chinese Laser Press

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## 1. INTRODUCTION

Multicore fiber (MCF) has been under intense research recently among the industrial communities due to its higher information capacity in comparison with traditional single-core fibers (SCFs), and it presents great potential to be the next-generation transmission fiber [1–4]. In addition, MCF also seems to be a promising method to realize power scaling with good beam quality, which is meaningful for high-power fiber lasers [5–7]. Unlike traditional SCFs, the cores are arranged separately in MCFs, which results in a totally different fiber mode property [8–10]. The electric field in each core can be coupled to adjacent cores via evanescent wave, so optic signals in the MCFs are actually transmitted by supermodes [11]. There have been many works mainly paying attention to the properties of the supermodes [12–16]. Apart from traditional MCFs, some new structures like trench-assisted MCFs have also been studied to decrease the cross talk between cores [17].

However, all the works mentioned above only care about the electric field in fiber cores, and the effect of the cladding is ignored, so that infinite cladding is taken into consideration. This assumption is reasonable for most cases due to the large diameter of cladding. However, for MCFs with small diameter of cladding, the effect of cladding should be considered, and the whole structure should be seen as a three-layer waveguide (TLW). This phenomenon is obvious, especially for tapered MCFs, which means that core modes may also be cut off in tapered MCFs just like that in SCFs [18–20].

In this present paper, we discuss the cut-off condition of core modes for tapered MCFs. First, rigorous analytical formulas of the core modes and cladding modes are derived for SCFs based on the TLW model. The cutoff of core modes for SCFs is investigated, and an approximate formula is given to estimate the cut-off condition of the LP<sub>01</sub> mode. Second,

supermodes of a three-core fiber (TCF) are analyzed by coupled mode theory, and the cut-off condition is numerically calculated by analysis software (Comsol Multiphysics). Results show that the cut-off condition of the in-phase supermode in MCFs has a similar property with the LP<sub>01</sub> mode in SCFs. Finally, more examples are investigated to verify this property, and a reasonable explanation is given. Based on this property, a convenient formula can be derived to estimate the cut-off condition of the in-phase supermode, which is very useful for tapered MCF applications.

## 2. CASE OF TAPERED SINGLE-CORE FIBER

First, let us consider a tapered single-core step-index fiber. As shown in Fig. 1, the core radius is  $a$  and the cladding radius is  $b$  in the original part of the SCF. While in the tapered part, the core radius is  $a_z$ , and the cladding radius is  $b_z$  at the longitude position  $z$ , where the taper ratio is define as  $TR = d_z/d$ . For a tapered fiber, the refractive index is unchanged (NA is a constant), while the core diameter and cladding diameter are decreased equally. Hence, the normalized frequency  $V$  is also decreased during the process of tapering. It is obvious that a tapered fiber does not have any modes because it is not a uniform waveguide. However, for an adiabatic tapered fiber, we can build a series of equivalent uniform fibers whose cross-sectional geometry and refractive index are the same with the tapered fiber at each longitude position [21,22]. By this method, we can easily analyze fiber mode properties with any TRs.

In the TLW model, the core can be seen as the first layer with index  $n_1$ , and the finite cladding can be seen as the second layer with index  $n_2$ . It should be noted that, in the TLW model, the effect of material outside the cladding should not be ignored, and it should be seen as the third layer with index  $n_3$  ( $n_3 = 1$  for air).

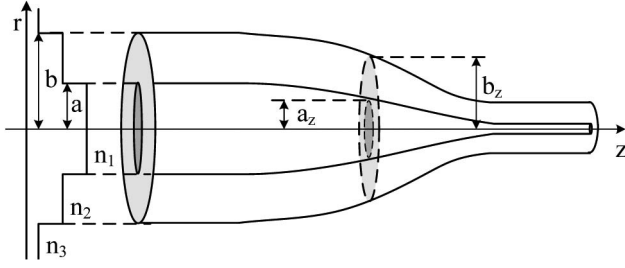


Fig. 1. Refraction index profile of a tapered SCF with TLW model.

The main difference between TLW model and infinite cladding model is whether there are cladding modes. In the infinite cladding model, the electric field in the cladding decreases gradually along the radial direction until 0 at  $R_{\text{clad}} = \infty$ . However, in the TLW model, the finite cladding and outer material can be also seen as a waveguide, so that the electric field in the cladding will no longer satisfy decreasing to 0 at  $R_{\text{clad}} = \infty$ .

It should be mentioned that weak-guidance approximation is not satisfied in the cladding-air interface, where the refractive index difference is about  $\Delta \approx 26\%$  ( $n_2 = 1.45$  for silica glass). However, it is sufficiently accurate here in the analysis of the cut-off condition estimation by scalar wave equation. According to the TLW model, the transverse electric field  $\psi$  is the solution of the scalar wave equation:

$$r^2 \frac{\partial^2}{\partial r^2} \psi + r \frac{\partial}{\partial r} \psi + [(n^2 k_0^2 - \beta^2) r^2 - m^2] \psi = 0, \quad (1)$$

where  $n$  is the refractive index,  $k_0$  is the wavenumber, and  $\beta$  is the propagation constant. The effective index can be obtained with  $n_{\text{eff}} = \beta/k_0$ .

The solutions of Eq. (1) in core or cladding depend on the value of  $n_{\text{eff}}$  relative to  $n$ . Hence, the transverse electric field  $\psi$  is obtained from Eq. (1) as [22]

$$\psi = \begin{cases} AJ_m\left(\frac{U}{a}r\right), & r < a, \\ BI_m\left(\frac{W}{a}r\right) + CK_m\left(\frac{W}{a}r\right), & a \leq r \leq b, \\ DK_m\left(\frac{T}{b}r\right), & r > b, \end{cases} \quad (2)$$

where the fiber normalized modal parameters in different layers are defined as  $U = ak_0 \sqrt{n_1^2 - n_{\text{eff}}^2}$ ,  $W = ak_0 \sqrt{n_{\text{eff}}^2 - n_2^2}$ ,  $Q = bk_0 \sqrt{n_2^2 - n_{\text{eff}}^2}$ , and  $T = bk_0 \sqrt{n_{\text{eff}}^2 - n_3^2}$ .

According to the continuity of  $\psi$  and  $d\psi/dr$  at the core-cladding and cladding-outer interfaces, the eigenvalue equation can be derived as

$$\begin{vmatrix} J_m(U) & -I_m(W) & -K_m(W) & 0 \\ \frac{U}{a}J'_m(U) & -\frac{W}{a}I'_m(W) & -\frac{W}{a}K'_m(W) & 0 \\ 0 & I_m(W\frac{b}{a}) & K_m(W\frac{b}{a}) & -K_m(T) \\ 0 & \frac{W}{a}I'_m(W\frac{b}{a}) & \frac{W}{a}K'_m(W\frac{b}{a}) & -\frac{T}{b}K'_m(T) \end{vmatrix} = 0 \quad (3)$$

for  $n_{\text{eff}} \geq n_2$  and

$$\begin{vmatrix} J_m(U) & -J_m(Q\frac{a}{b}) & -N_m(Q\frac{a}{b}) & 0 \\ \frac{U}{a}J'_m(U) & -\frac{Q}{b}J'_m(Q\frac{a}{b}) & -\frac{Q}{b}N'_m(Q\frac{a}{b}) & 0 \\ 0 & J_m(Q) & N_m(Q) & -K_m(T) \\ 0 & \frac{Q}{b}J'_m(Q) & \frac{Q}{b}N'_m(Q) & -\frac{T}{b}K'_m(T) \end{vmatrix} = 0 \quad (4)$$

for  $n_{\text{eff}} < n_2$ . In these formulas,  $J_m$  and  $N_m$  are the  $m$ -order Bessel function of the first kind and the second kind, respectively.  $I_m$  and  $K_m$  are the  $m$ -order modified Bessel function of the first kind and second kind, respectively.

It can be concluded that there are two eigenvalue equations for every fiber mode, which are core mode ( $n_{\text{eff}} \geq n_2$ ) and cladding mode ( $n_{\text{eff}} < n_2$ ). For a tapered SCF, the fiber core mode will cut off at the point of  $n_{\text{eff}} = n_2$ . As an example, the effective indices of the low-order modes for a few mode SCF ( $a = 10 \mu\text{m}$ ,  $b = 65 \mu\text{m}$ ,  $n_1 = 1.45221$  with  $\text{NA} = 0.08$ ) are calculated based on Eqs. (3) and (4). The results are shown in Fig. 2, where the horizontal coordinate is  $V$  not  $\text{TR}$  for the convenience of comparison. It can be found that the cut-off condition of high-order modes in the TLW model is almost the same as that in the infinite cladding model. (For example,  $V_c \approx 2.4$  for LP11 mode and  $V_c \approx 3.8$  for LP02 and LP21 mode in both cases.) However, the cut-off condition of the LP01 mode is totally different because it will never be cut off in the infinite cladding model.

For a tapered SCF, we always care about the mode evolution property of the LP01 mode. However, the analytic solution given above is not convenient for calculating the exact cut-off point. Another approximate solution is often used for the LP01 mode [19]:

$$V_c \approx \sqrt{\frac{2}{\ln s}} \left(1 + \frac{0.26}{\ln s}\right)^{-1/2}, \quad (5)$$

where  $s = b/a$  is the cladding-core ratio.

### 3. CASE OF TAPERED THREE-CORE FIBER

In this section, the cut-off condition of core modes for tapered TCF is analyzed. Figure 3 gives an example of the index distribution profile for a common isometric distributed TCF. As shown in Fig. 3, the core radius is  $a$ , and the cladding radius is  $b$ . The distance between the adjacent cores is  $\Lambda$ , and an equivalent core with radius  $R$  is defined as the circumcircle of all the cores. Here, the cladding-core ratio is defined as  $S = b/R$ . Similarly, we analyze fiber mode properties in tapered MCF by building a series of equivalent uniform MCFs.

The basic core modes for MCFs are supermodes, which are generated by intercoupling of each core mode. A theoretical calculation for supermodes of MCF can be analyzed based on the coupled mode theory in the case of a weak coupling

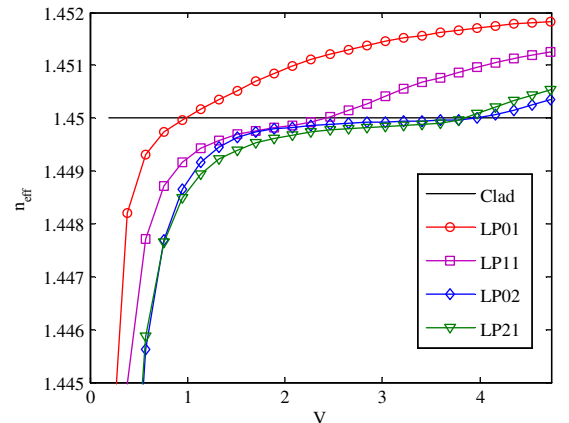


Fig. 2. Effective indices for lower-order modes in SCFs.

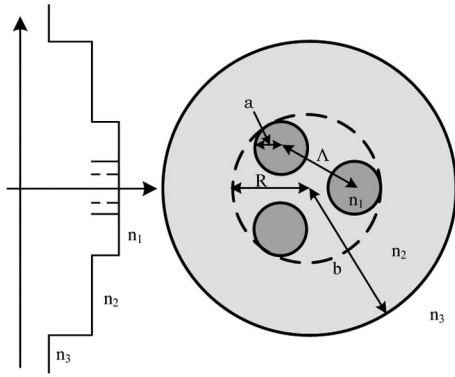


Fig. 3. Cross profile of TCF with TLW model.

condition. Here, the derivation process of supermodes for a TCF is given with scalar approximation.

The electric field of a TCF can be expressed as

$$E^v(x, y, z) = \sum_m A_m(z) E_m(x, y) \exp(i\beta_m z), \quad (6)$$

where  $v$  is the order of the mode and  $E_m(x, y) \exp(i\beta_m z)$  is the mode of the  $m$ th fiber.

A vector can be built based on Eq. (6) as

$$E(z) = \begin{bmatrix} A_1(z) \exp(i\beta_1 z) \\ A_2(z) \exp(i\beta_2 z) \\ A_3(z) \exp(i\beta_3 z) \end{bmatrix} = \begin{bmatrix} E_1(z) \\ E_2(z) \\ E_3(z) \end{bmatrix}. \quad (7)$$

According to coupling mode theory, the electric field can be given as a result of coupling between different core modes:

$$\frac{dE(z)}{dz} = CE(z), \quad C = \begin{bmatrix} i\beta & i\kappa & i\kappa \\ i\kappa & i\beta & i\kappa \\ i\kappa & i\kappa & i\beta \end{bmatrix}. \quad (8)$$

Here, we consider all three fiber cores have the same propagation parameters  $\beta$ , and the mode coupling coefficient of the adjacent fiber cores  $\kappa$  can be derived from [23]

$$\kappa = \frac{NA}{\sqrt{2n_1 a} V^3 K_1^2(W)} \sqrt{\frac{\pi a}{W\Lambda}} \exp\left(-\frac{W}{a}\Lambda\right). \quad (9)$$

Hence, the eigenvectors (supermodes) are derived as

$$\begin{aligned} E^1(x, y, z) &= [E_1(x, y) + E_2(x, y) + E_3(x, y)] \exp[i(\beta + 2\kappa)z], \\ E^2(x, y, z) &= [E_2(x, y) - E_3(x, y)] \exp[i(\beta - \kappa)z], \\ E^3(x, y, z) &= [2E_1(x, y) - E_2(x, y) - E_3(x, y)] \exp[i(\beta - \kappa)z]. \end{aligned} \quad (10)$$

Among all three supermodes, only  $E^1(x, y, z)$  has all positive coefficients, which is called the in-phase supermode. Figure 4 shows the mode field distribution of the supermodes with different TRs for a TCF ( $a = 2.5 \mu\text{m}$ ,  $NA = 0.14$ ,  $\Lambda = 10 \mu\text{m}$ , and  $S = 3$ ). From Figs. 4(e) and 4(f), we can see the similarity between the supermodes in tapered TCF and LP modes in SCFs. For convenience of discussion in the following section, these supermodes are called LP01 and LP11 modes, respectively.

As we can see in Figs. 4(d)–4(f), the relative mode field area is larger, and the distribution is more concentrated than that of Figs. 4(a)–4(c). This is mainly due to the decrease of  $a$  and  $\Lambda$  during the tapering processing. With decreased  $a$ , light

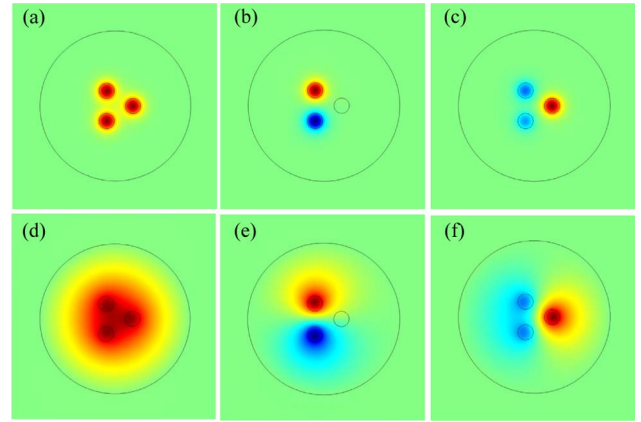


Fig. 4. Mode field distribution of the supermodes for a TCF with different taper ratios. (a)–(c) TR = 100%. (d) TR = 32.5%. (e), (f) TR = 54.3%. (d)–(f) Point of core mode cutoff with  $n_{\text{eff}} = n_2$ .

transmitted in the core area will gradually diffuse to the cladding. With decreased  $\Lambda$ , the distances between adjacent cores are smaller. Both of these decreases will lead to the increase of mode coupling coefficient  $\kappa$ .

Though analytical formulas have been deduced based on the coupled mode theory, it can only be used as a phenomenological approximation solution of supermodes. That is because a tapered MCF may not satisfy the weak-coupling condition. For the case of a strong-coupling waveguide, especially when the effect of cladding cannot be ignored, it is difficult to obtain a rigorous analytical solution. On the other hand, the mode coupling coefficient  $\kappa$  is also complicated to calculate by analytical method for different taper conditions. Hence, the method of numerical analysis is often used with more precise results. Here, the effective indices of the low-order modes of the tapered TCF with the same parameters are calculated by a fully vectorial finite element mode solver (Comsol Multiphysics), with the results shown in Fig. 5, where  $V$  presents the normalized frequency of each fiber core in TCF.

Unlike the curves in Fig. 2, LP01 and LP11 modes for TCF have the same effective indices at the beginning of the taper process, where the mode coupling coefficient is too small for relatively far distance between fiber cores, and every fiber core can be seen as an independent waveguide. Apart from this difference, the evolution process of effective indices for

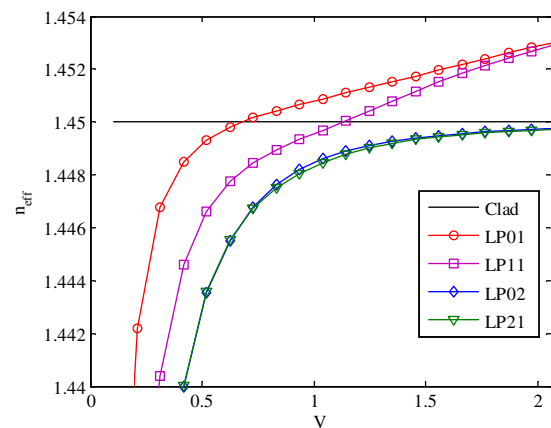


Fig. 5. Effective index of low-order core modes and cladding modes in TCF.

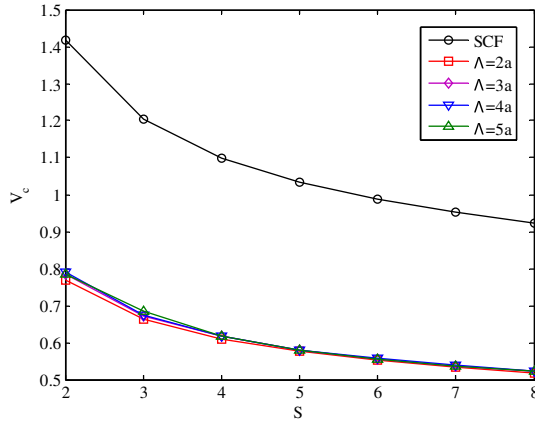


Fig. 6. Cut-off normalized frequency for LP01 mode in TCF.

TCF is similar to that of SCF. Hence, the core mode’s cut-off condition of a tapered MCF also can be similarly defined as the point of  $n_{\text{eff}} = n_2$ . Based on this definition, the core mode’s cut-off condition for tapered TCF is first investigated. Figure 6 gives the cut-off condition of LP01 mode by the parameter of single-core normalized frequency  $V$  for different  $S$  and  $\Lambda$ . (In all the simulations, parameters of the SCF are unchanged with  $a = 2.5 \mu\text{m}$ ,  $\text{NA} = 0.14$ .) It can be concluded that the cut-off condition mainly depends on the cladding-core ratio  $S$ , while the impact of  $\Lambda$  can be ignored.

It is an interesting phenomenon that there seems to be a relationship between the cut-off normalized frequency of SCF and that of TCF. We further calculate the ratio of their cut-off normalized frequency and find it drops very close to a constant with  $R_V \approx \sqrt{3}$ . This approximation is more precise for larger  $S$ .

The cut-off condition of the LP11 mode is shown in Fig. 7. Unlike the LP01 mode, the cut-off condition of the LP11 mode has a relationship with both factors, and the impact of  $\Lambda$  is larger than that of  $S$ . As mentioned above,  $V_c \approx 2.4$  is close to a constant for large  $S$  in SCFs. It is also close to a constant for large  $S$  in TCFs.

#### 4. MORE EXAMPLES AND DISCUSSION

In Section 3, the cut-off conditions of tapered TCFs are investigated in detail. In this section, cases of four-core fibers (FCFs), six-core fibers (ICFs), and seven-core fibers (ECFs) are also investigated by the same method. The fiber cross

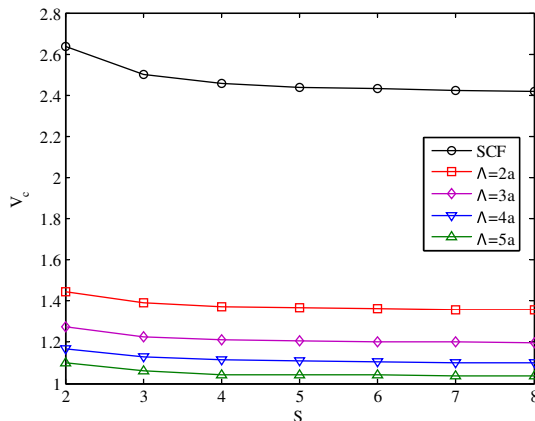


Fig. 7. Cut-off normalized frequency for LP11 mode in TCF.

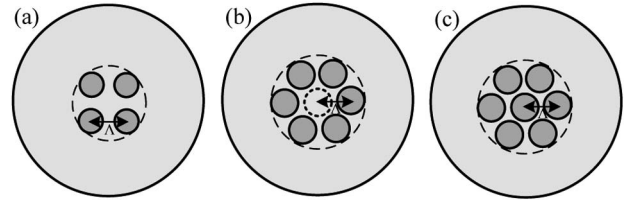


Fig. 8. Cross profiles of (a) FCF, (b) ICF, and (c) ECF.

profiles are shown in Fig. 8, and the equivalent core is also defined as the circumcircle of all the cores, while  $\Lambda$  is defined as the distance between nearest fiber cores.

The in-phase supermodes of these MCFs with different TRs are calculated by the same method, as shown in Fig. 9.

The cut-off conditions of LP01 mode for these MCFs are also calculated by numerical simulations. Results also show the impact of  $\Lambda$  can be ignored, and cut-off conditions mainly depend on the cladding-core ratio  $S$ , as shown in Fig. 10. We further calculate the cut-off normalized frequency ratio of SCF and MCFs and find it drops very close to a constant with  $R_V \approx \sqrt{m}$  ( $m$  is the number of the fiber cores).

Actually, this conclusion is suitable for all tapered MCFs only if the core distribution is symmetrical and the cladding-core ratio is much larger than 1. It can be seen from Figs. 4

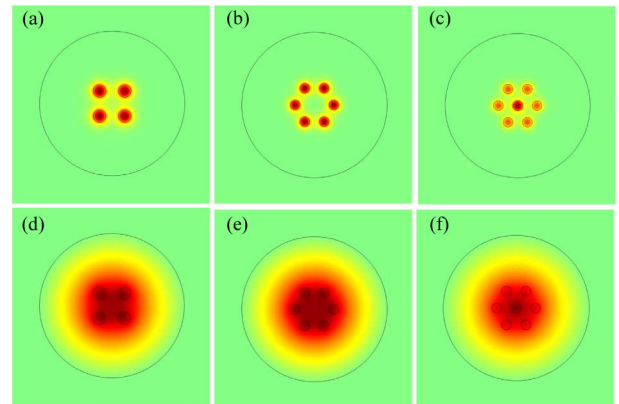


Fig. 9. Mode field distribution of the supermodes for MCFs with different taper ratios. (a)–(c) TR = 100%. (d) TR = 28.9%. (e) TR = 24.4%. (f) TR = 22.2%. (d)–(f) Point of core mode cutoff with  $n_{\text{eff}} = n_2$ .

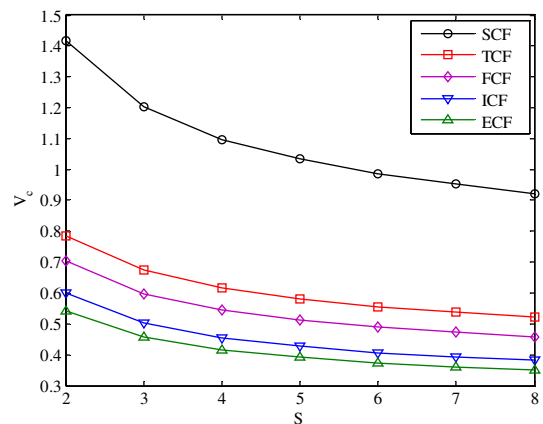


Fig. 10. Cut-off normalized frequency for LP01 mode in SCF and MCFs.

and 9 that the distribution of in-phase supermode is similar to that of the LP01 mode in SCF when it is cut off. As previously mentioned, an equivalent step-index SCF can be built to analyze the in-phase of tapered MCF. The core of this equivalent SCF is the circumcircle of all the multicores with effective radius  $R$ . The refractive index can be expressed as [24]

$$n^2(r) = n_2^2 + (n_1^2 - n_2^2)u(r, t), \quad (11)$$

where  $u(r, t)$  is the dopant concentration function.

For each fiber core in the MCF, its normalized frequency  $V$  can be expressed as

$$V^2 = k_0^2 \int_0^\infty (n^2(r) - n_2^2) \cdot 2r dr. \quad (12)$$

For a MCF with  $m$  fiber cores, the normalized frequency of the equivalent SCF can be expressed as

$$\begin{aligned} V^2 &= k_0^2 \int_0^\infty (n^2(r') - n_2^2) \cdot 2r' dr' \\ &= k_0^2 \cdot m \int_0^\infty (n^2(r) - n_2^2) \cdot 2r dr. \end{aligned} \quad (13)$$

Compared with Eqs. (11) and (12), we can obtain  $V' = \sqrt{m}V$ . This result is consistent with the numerical calculation results. Based on this relationship, the cut-off condition of the in-phase supermode of tapered MCFs can be easily calculated by the parameters of each fiber core. Compared with Eq. (5), it can be estimated by

$$V_c \approx \sqrt{\frac{2}{m \ln S}} \left(1 + \frac{0.26}{\ln S}\right)^{-1/2}, \quad (14)$$

where  $V_c$  is the normalized frequency of a single core in MCF and  $m$  is the number of fiber cores.

However, it is difficult to obtain a similar approximate formula for higher-order modes (such as LP11 mode). This is mainly due to the failure when we try to build an equivalent step-index SCF to replace a MCF with the same mode field distribution of high-order supermodes. This phenomenon can be illustrated by comparing the difference between mode field in Figs. 4(e) and 4(f) and that in a SCF. This difference is larger for larger  $\Lambda$ . This can be the reason why results in Fig. 7 show an important relationship between  $V_c$  and  $\Lambda$ .

## 5. CONCLUSIONS

In this paper, we have theoretically investigated the cut-off condition of core modes for tapered SCFs and MCFs by analytical and numerical methods. Analytical formulas for fiber modes in SCFs are derived based on scalar wave equation. Numerical calculations for MCFs show that the cut-off condition of in-phase supermode has a relationship with that of the LP01 mode in each fiber core. This relationship is explained by building an equivalent SCF. Finally, a convenient formula has been derived to estimate the cut-off condition of the in-phase supermode, which is very useful for tapered MCF applications.

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