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Integral Backstepping 法在四旋翼飞行器抗干扰研究中的应用

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摘要: 针对四旋翼飞行器多输入多输出、强耦合、非线性且欠驱动等特点带来的对外部干扰敏感问题, 提出了基于积分型反步法的控制系统。首先建立了四旋翼飞行器的动力学模型, 然后设计了位置控制回路外环及姿态控制回路内环的双闭环控制结构 PID 控制器, 和由李雅普诺夫稳定性理论证明的 Integral Backstepping 法的控制器, 最后在 Matlab/Simulink 中对这两种控制器进行仿真实验分析。结果表明, 使用 Integral Backstepping 算法的控制器模型在控制精度、调节时间和抗干扰性上都明显优于经典 PID 算法。

关键词: 四旋翼飞行器; 积分型反步法; 李雅普诺夫稳定性理论; PID 控制; 抗干扰

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Application of Integral Backstepping in Anti-disturbance Research of Quad-rotor UAVs

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Abstract: Due to the characteristics of multi-input/multi-output, strong coupling, nonlinearity and underactuation, the quad-rotor UAVs are sensitive to the external disturbances. To solve the problem, a control system based on Integral Backstepping is proposed. Firstly, the dynamic model of quad-rotor aircraft is established. Then, a double closed-loop control structure PID controller is designed with the outer-loop of the position control loop and the inner-loop of the attitude control loop, and a controller based on Integral Backstepping method is also designed, which is proved by Lyapunov stability theory. Finally, a simulation analysis is made to the two kinds of controllers with Matlab/Simulink. The results show that the controller model using Integral Backstepping design is obviously superior to the classical PID algorithm on control precision, regulating time and anti-disturbance performance.

Key words: quad-rotor UAV; Integral Backstepping; Lyapunov stability theory; PID control; anti-disturbance

0 引言

由于无人机可用于救援行动、交通监控和森林管理等, 在过去几年引起了相关研究人员的极大关注, 无论是民用还是军用无人机都成为了航空航天领域发展最快的项目。其中, 垂直起降无人机由于卓越的运动

敏捷性、相对较小的尺寸以及在混乱环境中悬停和操作的能力, 一直是研究的主要对象。飞行控制系统的工作是研究中的基本问题, 对于 4 个电机 6 个自由度欠驱动的四旋翼飞行器来说, 目前国内外使用最广泛的方法有 PID 控制、动态反馈线性化控制、线性二次型最优控制和模糊预测控制^[1]。文献[2]采用双闭环 PID 控制算法, 在高度控制和姿态控制中分别增加加速度环和内环角速度环, 增强了飞控系统的鲁棒性; 文献[3]采用传统的 Backstepping 法, 简化了控制器结构, 且参数更容易调整, 但轨迹跟踪精度下降了 2%。本文在传统 Backstepping 法的基础上, 增加了一个弥补稳态误差的积分项, 利用基于积分型反步法 (Integral

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Backstepping)的控制器来控制飞行器的轨迹以及提高其抗干扰能力。

1 四旋翼飞行器的建模

为了便于对飞行器建立数学模型,可以先做如下理想化的假设:把地球表面视为平面,且飞行器是刚体;飞行器的质量和转动惯量不变;飞行器的中心和重心一致;飞行器只受螺旋桨的升力和重力的影响;忽略风的阻力。

大地坐标系和机体坐标系如图1所示。

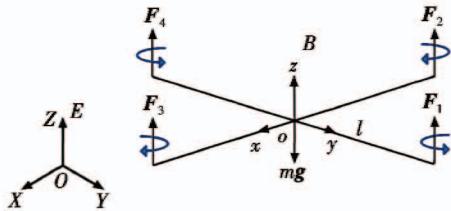


图1 大地坐标系和机体坐标系

Fig. 1 Geodetic coordinate system and body coordinate system

机体坐标系 $oxyz$ 首先要绕 z 轴转过偏航角 ψ , 然后绕 y 轴转过俯仰角 θ , 再绕 x 轴转过滚转角 φ , 最后与惯性坐标系 $OXYZ$ 重合。根据牛顿运动定律, 可以建立四旋翼飞行器沿 $E(X, Y, Z)$ 的三轴力平衡方程和沿机体坐标系三轴的力矩平衡方程式^[4]

$$\begin{cases} \dot{\mathbf{P}} = \mathbf{V} \\ \dot{\mathbf{V}} = \mathbf{g}e_z - \mathbf{TR} \cdot e_z/m \\ \dot{\mathbf{R}} = \mathbf{RS}(\bar{\boldsymbol{\omega}}) \\ \dot{\mathbf{I}_f} \bar{\boldsymbol{\omega}} = -\bar{\boldsymbol{\omega}} \times \mathbf{I}_f \bar{\boldsymbol{\omega}} - \mathbf{G}_a + \tau_a \\ \mathbf{I}_r \dot{\boldsymbol{\omega}}_i = \tau_i - \mathbf{Q}_i \quad i \in \{1, 2, 3, 4\} \end{cases} \quad (1)$$

式中: \mathbf{g} 为重力加速度; $\mathbf{T} = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)$ 为升力; m 为飞行器的质量; \mathbf{R} 为坐标变换矩阵; $\mathbf{P} = (X, Y, Z)^T$ 和 $\mathbf{V}(u, v, w)^T$ 分别表示飞行器在大地坐标系下的坐标及线速度值; $\bar{\boldsymbol{\omega}} = (p, q, r)^T$ 为角速度向量; $\mathbf{S}(\bar{\boldsymbol{\omega}})$ 为一个斜对称矩阵, $\mathbf{S}(\bar{\boldsymbol{\omega}})\mathbf{V} = (\bar{\boldsymbol{\omega}}) \times \mathbf{V}$; \mathbf{G}_a 为陀螺扭矩; $\mathbf{Q}_i = k\omega_i^2$ 为阻力; $\mathbf{I}_f, \mathbf{I}_r$ 分别是机体和旋翼的惯性矩阵。由式(1)得到机体沿 X, Y 和 Z 方向的线运动方程为

$$\begin{cases} \ddot{x} = \frac{T}{m}(\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi) \\ \ddot{y} = \frac{T}{m}(\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi) \\ \ddot{z} = \frac{T}{m}(\cos \varphi \cos \theta) - g \end{cases} \quad (2)$$

通过式(2)计算运动方程, 可得到机体角速度与欧拉角速度的关系方程, 从而得到沿 X, Y 和 Z 方向的

角运动方程为

$$\begin{cases} \ddot{\varphi} = \frac{(I_{fx} - I_{fy})}{I_{fx}} \dot{\theta} \dot{\psi} + \frac{\tau_\varphi}{I_{fx}} + \\ \frac{I_n}{I_{fx}}(-\omega_1 + \omega_2 - \omega_3 + \omega_4) - \frac{K_4}{I_{fx}} \dot{\varphi} \\ \ddot{\theta} = \frac{(I_{fx} - I_{fy})}{I_{fy}} \dot{\varphi} \dot{\psi} + \frac{\tau_\theta}{I_{fy}} - \\ \frac{I_n}{I_{fy}}(-\omega_1 + \omega_2 - \omega_3 + \omega_4) - \frac{K_5}{I_{fy}} \dot{\theta} \\ \ddot{\psi} = \frac{(I_{fx} - I_{fy})}{I_{fx}} \dot{\varphi} \dot{\theta} + \frac{\tau_\psi}{I_{fx}} - \frac{K_6}{I_{fx}} \dot{\psi} \end{cases} \quad (3)$$

定义4个控制量为

$$\begin{cases} U_1 = T_1 + T_2 + T_3 + T_4 \\ U_2 = -T_2 + T_4 \\ U_3 = -T_1 + T_3 \\ U_4 = K_q(T_1 - T_2 + T_3 - T_4) \end{cases} \quad (4)$$

式中: K_q 为旋翼反扭矩系数; U_1 为高度通道控制量; U_2 为 $x\cdot\theta$ 通道控制量; U_3 为 $y\cdot\varphi$ 通道控制量; U_4 为偏航通道控制量。对飞行器的控制可分解为对4个控制通道的控制量的调节^[5]。简化后得到四旋翼飞行器动力学方程为

$$\begin{cases} \ddot{x} = \frac{U_1}{m}(\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi) \\ \ddot{y} = \frac{U_1}{m}(\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi) \\ \ddot{z} = \frac{U_1}{m}(\cos \varphi \cos \theta) - g \\ \ddot{\varphi} = \frac{(I_{fx} - I_{fy})}{I_{fx}} \dot{\theta} \dot{\psi} + \frac{lU_2}{I_{fx}} \\ \ddot{\theta} = \frac{(I_{fx} - I_{fy})}{I_{fy}} \dot{\varphi} \dot{\psi} + \frac{lU_3}{I_{fy}} \\ \ddot{\psi} = \frac{(I_{fx} - I_{fy})}{I_{fx}} \dot{\varphi} \dot{\theta} + \frac{lU_4}{I_{fx}} \end{cases} \quad (5)$$

2 系统总体控制结构设计

将飞行器控制系统分为姿态控制和位置控制两个子系统。姿态控制回路作为内回路, 位置控制回路为外回路^[6]。

在位置控制回路中, U_1 用来控制 x, y 和 z 三轴方向的位移。位置控制回路计算出飞行状态所需的姿态角 $[\varphi_d \theta_d]$, 而 ψ_d 通过单独输入得到, 然后将3个姿态角提供给姿态控制回路, 实现对姿态的跟踪控制, 如图2所示。

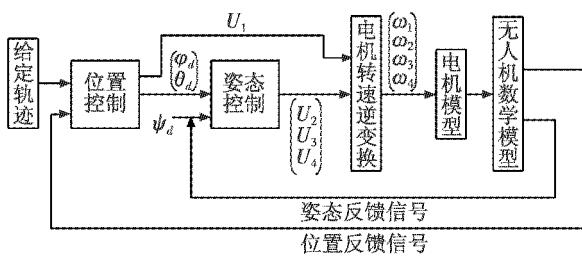


图2 系统总体控制结构图

Fig. 2 Diagram of the overall control structure of the system

2.1 经典 PID 控制设计

PID 控制主要是靠给定值(目标值)和输出(控制结果)的偏差来实现反馈控制^[2]。

PID 控制的表达式为

$$\begin{cases} e(t) = r(t) - y(t) \\ u(t) = k_p \left[e(t) + \frac{1}{T_i} \int e(t) d(t) + T_d \frac{de(t)}{dt} \right] \end{cases} \quad (6)$$

式中: $e(t)$ 为控制偏差; k_p 为比例系数; T_i 为积分时间常数; T_d 为微分时间常数。

首先构造控制量为

$$\begin{cases} U_2 = K_{p\varphi}(\varphi_d - \varphi) - K_{d\varphi}p \\ U_3 = K_{p\theta}(\theta_d - \theta) - K_{d\theta}q \\ U_4 = K_{p\psi}(\psi_d - \psi) - K_{d\psi}r \end{cases} \quad (7)$$

式中: $[\varphi_d \theta_d \psi_d]$ 为期望的姿态角信号; $[\varphi \theta \psi]$ 为传感器测量得到的姿态角信号; $[p \ q \ r]$ 为机体角速度。 $K_{p\varphi}, K_{p\theta}, K_{p\psi}$ 和 $K_{d\varphi}, K_{d\theta}, K_{d\psi}$ 分别为3个通道的比例调节系数和微分调节系数^[7]。

则位置控制回路的输出为

$$\begin{cases} U_x = K_{px}(x_d - x) + K_{dx}(\dot{x}_d - \dot{x}) + \ddot{x} \\ U_y = K_{py}(y_d - y) + K_{dy}(\dot{y}_d - \dot{y}) + \ddot{y} \\ U_z = K_{pz}(z_d - z) + K_{dz}(\dot{z}_d - \dot{z}) + \ddot{z} \end{cases} \quad (8)$$

式中: $[x_d \ y_d \ z_d]$ 为期望位置; $[x \ y \ z]$ 为飞行器位置; $[\dot{x}_d \ \dot{y}_d \ \dot{z}_d]$ 为期望线速度信号; $[\dot{x} \ \dot{y} \ \dot{z}]$ 为飞行器线速度; $[\ddot{x} \ \ddot{y} \ \ddot{z}]$ 是飞行器加速度。 K_{px}, K_{py}, K_{pz} 和 K_{dx}, K_{dy}, K_{dz} 分别为3个通道的比例调节系数和微分调节系数。

定义 T_x, T_y 和 T_z 为 U_1 在惯性坐标系下三轴中的分量,则

$$\begin{cases} T_x = (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi) U_1 \\ T_y = (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi) U_1 \\ T_z = (\cos \varphi \cos \theta) U_1 \end{cases} \quad (9)$$

由式(7)可知,位置回路的控制量可近似等效于飞行器加速度在惯性坐标系下的3个分量,即

$$\begin{cases} U_x = \frac{1}{m} T_x \\ U_y = \frac{1}{m} T_y \\ U_z = \frac{1}{m} T_z - g \end{cases} \quad (10)$$

偏航角 ψ_d 为给定,则可以推出

$$\begin{cases} U_1 = m \sqrt{U_x^2 + U_y^2 + (U_z + g)^2} \\ \varphi_d = \arcsin((U_x \sin \psi_d - U_y \cos \psi_d) \frac{m}{U_1}) \\ \theta_d = \arcsin(U_x m - U_1 \sin \psi_d \sin \varphi_d) \\ \psi_d = U_1 \cos \psi_d \cos \varphi_d \end{cases} \quad (11)$$

由该式计算得到的姿态角指令信号 φ_d, θ_d 可作为姿态控制回路的输入信号。

2.2 Integral Backstepping 理论控制器设计

Backstepping 控制理论是指把复杂非线性系统分解成多个低阶的简单子系统,然后定义若干个中间虚拟量来匹配 Lyapunov 函数对应的子系统,保持系统稳定从最低阶次开始一直后退,最终将所有子系统集合起来完成系统的控制器设计。引入积分作用可以有效消除系统控制过程中的误差^[3]。

首先把四旋翼飞行器旋翼的模态模型写成状态方程 $\dot{\mathbf{X}} = f(\mathbf{X}, u)$ 的形式。为此,引入包含12个状态变量的状态向量 $\mathbf{X} = (x_1, x_2, \dots, x_{12})^T$ 。其中, $x_1 = \varphi, x_2 = \dot{x}_1 = \dot{\varphi}, x_3 = \theta, x_4 = \dot{x}_3 = \dot{\theta}, x_5 = \psi, x_6 = \dot{x}_5 = \dot{\psi}, x_7 = z, x_8 = \dot{x}_7 = \dot{z}, x_9 = x, x_{10} = \dot{x}_9 = \dot{x}, x_{11} = y, x_{12} = \dot{x}_{11} = \dot{y}$ 。

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{\varphi} \\ \frac{(I_{fy} - I_{fx})}{I_{fx}} \dot{\theta} \dot{\psi} + \frac{lU_2}{I_{fx}} \\ \dot{\theta} \\ \frac{(I_{fx} - I_{fy})}{I_{fy}} \dot{\varphi} \dot{\psi} + \frac{lU_3}{I_{fy}} \\ \dot{\psi} \\ \frac{(I_{fx} - I_{fy})}{I_{fx}} \dot{\varphi} \dot{\theta} + \frac{lU_4}{I_{fx}} \\ \dot{z} \\ \frac{U_1}{m} (\cos \varphi \cos \theta) - g \\ \dot{x} \\ \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi) \\ \dot{y} \\ \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi) \end{bmatrix} \quad (12)$$

位置控制回路中, U_1 用来控制无人机的飞行高度,而虚拟控制量 $[U_x, U_y]$ 用来控制无人机在 x, y 轴向上的坐标位置。姿态控制回路中 $[U_2, U_3, U_4]$ 分别控制无人机的滚转、俯仰和偏航飞行运动^[8]。

首先,对无人机的滚转角通道进行控制器设计, $x_1 = \varphi, x_2 = \dot{\varphi}$,微分变换后 $\dot{x}_1 = \dot{\varphi} = x_2, \dot{x}_2 = \ddot{\varphi} = \frac{(I_{fy} - I_{fx})}{I_{fx}} \dot{\theta} \dot{\psi} + \frac{lU_2}{I_{fx}}$, $U_2 = \frac{(I_{fy} - I_{fx})}{I_{fx}} \dot{\theta} \dot{\psi} + \frac{lU_2}{I_{fx}}$, $U_3 = \frac{(I_{fx} - I_{fy})}{I_{fy}} \dot{\varphi} \dot{\psi} + \frac{lU_3}{I_{fy}}$, $U_4 = \frac{(I_{fx} - I_{fy})}{I_{fx}} \dot{\varphi} \dot{\theta} + \frac{lU_4}{I_{fx}}$, $U_1 = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_5 = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_6 = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_7 = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_8 = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_9 = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{10} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{11} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{12} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{13} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{14} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{15} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{16} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{17} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{18} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{19} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{20} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{21} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{22} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{23} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{24} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{25} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{26} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{27} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{28} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{29} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{30} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{31} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{32} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{33} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{34} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{35} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{36} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{37} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{38} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{39} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{40} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{41} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{42} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{43} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{44} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{45} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{46} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{47} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{48} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{49} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{50} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{51} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{52} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{53} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{54} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{55} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{56} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{57} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{58} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{59} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{60} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{61} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{62} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{63} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{64} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{65} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{66} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{67} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{68} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{69} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{70} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{71} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{72} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{73} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{74} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{75} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{76} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{77} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{78} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{79} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{80} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{81} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{82} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{83} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{84} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{85} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{86} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{87} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{88} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{89} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{90} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{91} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{92} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{93} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{94} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{95} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{96} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{97} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{98} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{99} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{100} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{101} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{102} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{103} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{104} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{105} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{106} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{107} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{108} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{109} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{110} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{111} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{112} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{113} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{114} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{115} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{116} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{117} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{118} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{119} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{120} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{121} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{122} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{123} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{124} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{125} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{126} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{127} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{128} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{129} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{130} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{131} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{132} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{133} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{134} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{135} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{136} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{137} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{138} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{139} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{140} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{141} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{142} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{143} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{144} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{145} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{146} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{147} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{148} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{149} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{150} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{151} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{152} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{153} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{154} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{155} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{156} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{157} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{158} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{159} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{160} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{161} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{162} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{163} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{164} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{165} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{166} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{167} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{168} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{169} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{170} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{171} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{172} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{173} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{174} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{175} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{176} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{177} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{178} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{179} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{180} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{181} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{182} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{183} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{184} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{185} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{186} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{187} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{188} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{189} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{190} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{191} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{192} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{193} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{194} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$, $U_{195} = \frac{U_1}{m} (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi)$, $U_{196} = \frac{U_1}{m} (\cos \varphi \cos \theta) - g$, $U_{197} = \frac{U_1}{m} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi)$,<

化为 $\dot{e}_1 = e_2 - \alpha_1 + f_1$ 形式, 其中, $e_2 = \alpha_1 - x_2$ 为角速度误差变量, α_1 为虚拟控制量。选取 Lyapunov 函数为

$$V_1 = \frac{1}{2}\lambda_1\chi_1^2 + \frac{1}{2}e_1^2 \quad (13)$$

$$\dot{V}_1 = \lambda_1\chi_1\dot{\chi}_1 + e_1\dot{e}_1 \quad (14)$$

式中, $\chi_1 = \int_0^t e_1(\tau) d\tau$ 为无人机滚转角误差积分项。则

$$\dot{V}_1 = \lambda_1\chi_1e_1 + e_1\dot{e}_1 \circ \quad (15)$$

虚拟控制量 $\alpha_1 = C_1e_1 + \lambda_1\chi_1 + f_1$, 则 $\dot{e}_1 = e_2 - C_1e_1 - \lambda_1\chi_1$, 得到

$$\dot{V}_1 = e_1e_2 - C_1e_1^2 \quad (16)$$

式中: $C_i (i=1, 2, \dots, 12) > 0$ 为控制增益; 为了保证系统的稳定性, 耦合项 e_1e_2 会在后面消去。整理得

$$\begin{cases} \dot{e}_1 = e_2 - C_1e_1 - \lambda_1\chi_1 \\ f_1 = \dot{x}_{1d} \\ \alpha_1 = C_1e_1 + \lambda_1\chi_1 + \dot{x}_{1d} \\ e_2 = C_1e_1 + \lambda_1\chi_1 + \dot{x}_{1d} - x_2 \end{cases} \circ \quad (17)$$

选取 Lyapunov 函数为

$$V_2 = \frac{1}{2}\lambda_1\chi_1^2 + \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \quad (18)$$

$$\dot{V}_2 = \dot{V}_1 + e_2\dot{e}_2 = e_1e_2 - C_1e_1^2 + e_2\dot{e}_2 \circ \quad (19)$$

为满足系统的稳定性, 必须保证 $\dot{V}_2 < 0$, 可以推出

$\dot{e}_2 = -e_1 - C_2e_2$, 得到

$$\dot{V}_2 = -C_1e_1^2 - C_2e_2^2 \circ \quad (20)$$

由 e_2 和 \dot{e}_2 的表达式推出

$$\begin{cases} C_1\dot{e}_1 + \lambda_1e_1 + \ddot{x}_{1d} - \dot{x}_2 = -e_1 - C_2e_2 \\ U_2 = [I_{fx} \cdot [C_1(\dot{x}_{1d} - x_2) + (\lambda_1 + 1)e_1 + \ddot{x}_{1d} + C_2e_2] - (I_{fy} - I_{fx})\dot{\theta}\dot{\psi}] / l \end{cases} \quad (21)$$

按照上述方法, 可依次求解俯仰角通道、偏航角通道控制量 U_3, U_4 的表达式^[9]分别为

$$\begin{cases} U_3 = [I_{fy} \cdot [C_3(\dot{x}_{3d} - x_4) + (\lambda_2 + 1)e_3 + \ddot{x}_{3d} + C_4e_4] - (I_{fx} - I_{fy})\dot{\phi}\dot{\psi}] / l \\ U_4 = [I_{fy} \cdot [C_5(\dot{x}_{5d} - x_6) + (\lambda_3 + 1)e_5 + \ddot{x}_{5d} + C_6e_6] - (I_{fx} - I_{fy})\dot{\theta}\dot{\phi}] / l \end{cases} \quad (22)$$

式中: $e_3 = x_{3d} - x_3$; $e_4 = C_3e_3 + \lambda_2\chi_2 + \dot{x}_{3d} - x_4$, $\chi_2 = \int_0^t e_3(\tau) d\tau$ 为无人机俯仰角误差积分项; $e_5 = x_{5d} - x_5$; $e_6 = C_5e_5 + \lambda_3\chi_3 + \dot{x}_{5d} - x_6$, $\chi_3 = \int_0^t e_5(\tau) d\tau$ 为无人机偏航角误差积分项。

位置控制通道主要包括 x, y, z 三个轴向上的无人机坐标调整, 最终求得控制量为

$$U_z = \frac{m}{\cos\theta\cos\varphi} \times [g + C_7(\dot{x}_{7d} - x_8) + (\lambda_4 + 1)e_7 + \ddot{x}_{7d} + C_8e_8] \quad (23)$$

$$U_x = \frac{m}{\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi} \times [C_9(\dot{x}_{9d} - x_{10}) + (\lambda_5 + 1)e_9 + \ddot{x}_{9d} + C_{10}e_{10}] \quad (24)$$

$$U_y = \frac{m}{\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi} \times [C_{11}(\dot{x}_{11d} - x_{12}) + (\lambda_6 + 1)e_{11} + \ddot{x}_{11d} + C_{12}e_{12}] \quad (25)$$

式中: $e_7 = x_{7d} - x_7$; $e_8 = C_7e_7 + \lambda_4\chi_4 + \dot{x}_{7d} - x_8$, $\chi_4 = \int_0^t e_7(\tau) d\tau$ 为无人机在 z 轴上的误差积分项; $e_9 = x_{9d} - x_9$; $e_{10} = C_9e_9 + \lambda_5\chi_5 + \dot{x}_{9d} - x_{10}$, $\chi_5 = \int_0^t e_9(\tau) d\tau$ 为无人机在 x 轴上的误差积分项; $e_{11} = x_{11d} - x_{11}$; $e_{12} = C_{11}e_{11} + \lambda_6\chi_6 + \dot{x}_{11d} - x_{12}$, $\chi_6 = \int_0^t e_{11}(\tau) d\tau$ 为无人机在 y 轴上的误差积分项。

3 仿真实验

首先在 Matlab/Simulink 环境下构建两种算法完整的控制系统仿真模型^[10], 然后设定两个飞行器模型的飞行轨迹均为 $X = \cos(t/2)$, $Y = \sin(t/2)$, $Z = 0.2 t$; 初始状态静止于坐标点 $(1, 0, 0)$, 初始姿态角为 0° , 期望偏航角度为 1° , 仿真结果如图 3~图 7 所示。

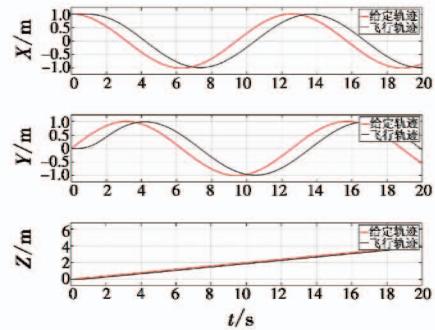


图 3 经典 PID 算法的轨迹跟踪曲线

Fig. 3 Trajectory tracking curves of classical PID algorithm

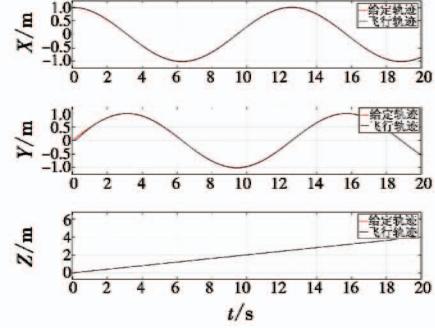


图 4 基于 Integral Backstepping 算法的轨迹跟踪曲线

Fig. 4 Trajectory tracking curves of Integral Backstepping algorithm

当飞行器悬停时, 为测试系统抗外界干扰能力, 在 t 处于 $8 \sim 12$ s 的时间范围内, 对 3 个通道的力矩输入

端分别加入幅值为 $0.2 \text{ N} \cdot \text{m}$ 的干扰力矩。

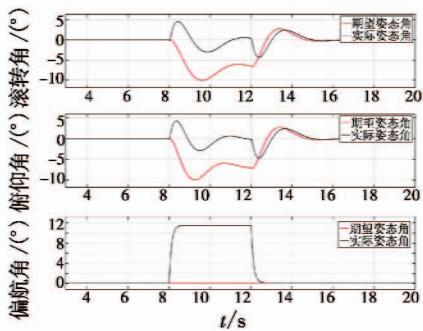


图5 经典 PID 算法控制下系统姿态角抗干扰曲线

Fig. 5 Anti-disturbance curve of system attitude angle under the control of classical PID algorithm

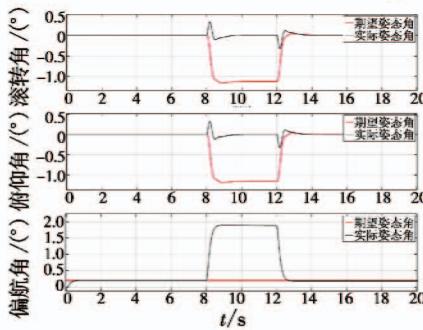


图6 Integral Backstepping 算法控制下系统姿态角抗干扰曲线

Fig. 6 Anti-disturbance curve of system attitude angle under the control of Integral Backstepping algorithm

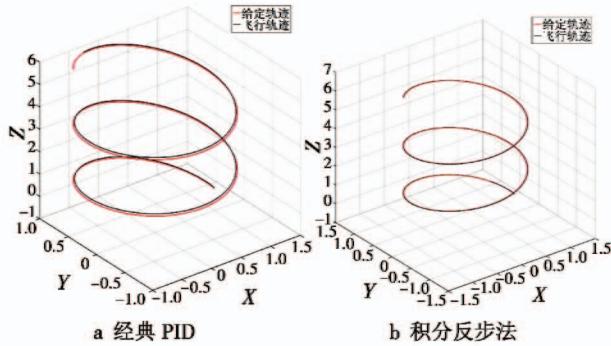


图7 两种算法的三维轨迹跟踪曲线

Fig. 7 The 3D trajectory tracking curves of two algorithms

1) 从图3和图4可以看出,与经典PID控制器设计方案相比,基于Integral Backstepping算法模型的系统稳定性明显增强,在 x , y , z 三个轴向上的位置坐标偏差值大大降低。

2) 从图5和图6可以看出,在第12 s 干扰消除后,PID 控制的飞行器可在3 s 内达到稳定状态,而基于 Integral Backstepping 算法的控制器可在1 s 内达到稳定状态,调节时间大大缩短。

3) 从图7可以看出,通过plot3 函数可以得到飞行器的三维轨迹跟踪曲线,基于 Integral Backstepping

算法的控制器比经典 PID 控制器的跟踪精度更高。

4 结束语

本文首先以合理假设为基础,根据牛顿运动定律建立了较为精确的四旋翼飞行器的动力学模型,然后根据四旋翼飞行器的模型特点,将其分为姿态子系统和位移子系统。由于姿态控制系统在位置控制系统之前,所以先研究了姿态控制系统,后研究了位置控制系统。最后分别使用经典 PID 控制算法、基于 Integral Backstepping 的控制算法在 Matlab/Simulink 环境中进行系统模型的设计,并分别对两种模型进行了仿真和分析。仿真结果验证了两种控制方案的可行性,但从控制性能方面的对比来看,基于 Integral Backstepping 算法的控制模型比经典 PID 控制模型的控制精度更高、调节时间更短、抗干扰能力更强。

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