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纯反馈非线性系统的鲁棒自适应跟踪控制

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摘 要: 针对一类非仿射函数不连续的纯反馈非线性系统,提出了一种鲁棒自适应控制方法。放宽了对非仿射函数的连续性条件和边界条件,非仿射函数的边界均为未知连续函数,利用动态面控制技术避免了对虚拟控制律反复求导而导致的计算复杂性问题。从理论上证明了所设计的方法能够保证闭环系统所有信号半全局一致终结有界,且通过选择合适的设计参数使系统输出能渐近收敛到原点的任意小邻域内。仿真结果表明了所提出方法的有效性。

关键词: 非仿射纯反馈; 自适应控制; 动态面控制
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Robust Adaptive Tracking Control for Pure-Feedback Nonlinear Systems

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Abstract: A robust adaptive control scheme is designed for a class of pure-feedback nonlinear systems with non-affine functions possibly being discontinuous. The continuous and bounded conditions on non-affine functions are relaxed, and the bounds of non-affine function are unknown continuous functions. Moreover, the problem of computational complexity caused by the repeated differentiations of virtual control laws is eliminated by utilizing dynamic surface control technique. It is proved theoretically that all the signals in the closed-loop control system are semi-globally uniformly ultimately bounded, and the system output can asymptotically converge to an arbitrarily small neighborhood of the equilibrium point by choosing the appropriate design parameters. Finally, simulation examples are provided to demonstrate the effectiveness of the designed approach.

Key words: non-affine pure-feedback; adaptive control; dynamic surface control

0 引言

近年来,非线性系统的控制设计问题引起了各国学者的广泛关注,在严格反馈系统控制领域取得了很多成果[1-6]。但实际中存在很多非仿射纯反馈结构的非线性系统,如生化处理系统、飞行控制系统、Duffing振荡器以及机械系统等,因此研究非仿射纯反馈系统的控制理论具有一定的实际意义。

由于非仿射纯反馈系统的控制问题更复杂且更具挑战性,取得的成果还比较少见[7-19]。已有研究成果中对非仿射纯反馈非线性系统控制设计方法主要依靠均值定理与隐函数定理[10-15]。文献[10-14]针对具有下三角结构的纯反馈非线性系统讨论了自适应神经网络 backstepping 控制方案;文献[15]针对具有非仿射函数和未知死区的纯反馈系统,结合均值定理和 backstepping 技术,提出了一种直接自适应控制方法。上述文献都假定非仿射函数对于输入变量是可导的,甚至导数必须是严格为正或负。针对非仿射函数不可导的纯反馈非线性系统的控制问题,文献[16]提出了能保证非仿射纯反馈非线性系统可控性的连续性函数条件,取消了非仿射非线性函数必须可导的假设;文献[17]在文献[16]的基础上进一步放宽了非仿射函数连续性条件。然而,上述方案未考虑非仿射函数不连续时的控制问题。

基于以上分析,本文就一类非仿射函数不连续的纯反馈非线性系统的控制问题,提出了一种鲁棒自适应神经网络控制方法。本文所提方案具有以下优点:1)取消了非仿射函数必须满足连续性条件;2)考虑了

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控制输入存在死区非线性的情况;3)引入了自适应补偿项消除建模误差、神经网络逼近误差和外界干扰造成的影响;4)从理论上证明了闭环系统的所有信号半全局一致终结有界。

1 问题描述和准备

考虑如下一类纯反馈非线性系统

$$\begin{cases}\dot{x}_i=f_i(\bar{x}_i,x_{i+1})+d_i(t) \quad i=1,2,\cdots,n-1\\ \dot{x}_n=f_n(\boldsymbol{x},v(u))+d_n(t)\\ y=x_1\end{cases} \tag{1}$$

式中:$\bar{\boldsymbol{x}}_i=[x_1,x_2,\cdots,x_i]^{\mathrm{T}}\in\mathbf{R}^i$ 和 $\boldsymbol{x}=[x_1,x_2,\cdots,x_n]^{\mathrm{T}}\in\mathbf{R}^n$ 为系统状态;$v(u)$是输入为 u 的死区模型输出;$y\in\mathbf{R}$ 为系统输出;$f_i(\cdot)$为未知非仿射系统函数;$d_i(t)$为外界干扰。

与以往文献相比,本文建立了一种更广泛的输入非线性模型:定义 $F_i(\bar{x}_i,x_{i+1})=f_i(\bar{x}_i,x_{i+1})-f_i(\bar{x}_i,0)$,其中,$f_i(\bar{x}_i,x_{i+1})$是关于 x_{i+1}的不连续函数,$f_i(\bar{x}_i,0)$是连续函数。为了简洁起见,定义 $x_{n+1}=v(u)$,$\bar{\boldsymbol{x}}_{n+1}=[\boldsymbol{x}^{\mathrm{T}},v(u)]^{\mathrm{T}}$。

假设1 存在正定连续函数 $\underline{F}_i(\bar{x}_i)$,$\bar{F}_i(\bar{x}_i)$,$\underline{F}_i'(\bar{x}_i)$,$\bar{F}_i'(\bar{x}_i)$,任意连续函数 $C_{1i}(\bar{x}_i)$,$C_{2i}(\bar{x}_i)$,$C_{3i}(\bar{x}_i)$,$C_{4i}(\bar{x}_i)$使得

$$\begin{cases}\underline{F}_i(\bar{x}_i)x_{i+1}+C_{1i}(\bar{x}_i)\leqslant F_i(\bar{x}_i,x_{i+1})\leqslant\\ \qquad\bar{F}_i(\bar{x}_i)x_{i+1}+C_{2i}(\bar{x}_i) \quad x_{i+1}\geqslant0\\ \underline{F}_i'(\bar{x}_i)x_{i+1}+C_{3i}(\bar{x}_i)\leqslant F_i(\bar{x}_i,x_{i+1})\leqslant\\ \qquad\bar{F}_i'(\bar{x}_i)x_{i+1}+C_{4i}(\bar{x}_i) \quad x_{i+1}<0\end{cases}。 \tag{2}$$

由式(2)可知,存在定义在[0,1]上未知光滑的正定函数 $\theta_{i1}(\bar{x}_{i+1})$和 $\theta_{i2}(\bar{x}_{i+1})$满足

$$\begin{cases}F_i(\bar{x}_i,x_{i+1})=(1-\theta_{i1}(\bar{x}_{i+1}))C_{1i}(\bar{x}_i)+\theta_{i1}(\bar{x}_{i+1})C_{2i}(\bar{x}_i)+\\ \quad[(1-\theta_{i1}(\bar{x}_{i+1}))\underline{F}_i(\bar{x}_i)+\theta_{i1}(\bar{x}_{i+1})\bar{F}_i(\bar{x}_i)]x_{i+1} \quad x_{i+1}\geqslant0\\ F_i(\bar{x}_i,x_{i+1})=(1-\theta_{i2}(\bar{x}_{i+1}))C_{3i}(\bar{x}_i)+\theta_{i2}(\bar{x}_{i+1})C_{4i}(\bar{x}_i)+\\ \quad[(1-\theta_{i2}(\bar{x}_{i+1}))\underline{F}_i'(\bar{x}_i)+\theta_{i2}(\bar{x}_{i+1})\bar{F}'(\bar{x}_i)]x_{i+1} \quad x_{i+1}<0\end{cases}。 \tag{3}$$

定义两个分段函数 $G_i(\bar{x}_{i+1})$和 $\Delta_i(\bar{x}_{i+1})$为

$$\begin{cases}G_i(\bar{x}_{i+1})=\begin{cases}(1-\theta_{i1}(\bar{x}_{i+1}))\underline{F}_i(\bar{x}_i)+\theta_{i1}(\bar{x}_{i+1})\bar{F}_i(\bar{x}_i) & x_{i+1}\geqslant0\\ (1-\theta_{i2}(\bar{x}_{i+1}))\underline{F}_i'(\bar{x}_i)+\theta_{i2}(\bar{x}_{i+1})\bar{F}_i'(\bar{x}_i) & x_{i+1}<0\end{cases}\\ \Delta_i(\bar{x}_{i+1})=\begin{cases}(1-\theta_{i1}(\bar{x}_{i+1}))C_{1i}(\bar{x}_i)+\theta_{i1}(\bar{x}_{i+1})C_{2i}(\bar{x}_i) & x_{i+1}\geqslant0\\ (1-\theta_{i2}(\bar{x}_{i+1}))C_{3i}(\bar{x}_i)+\theta_{i2}(\bar{x}_{i+1})C_{4i}(\bar{x}_i) & x_{i+1}<0\end{cases}\end{cases} \tag{4}$$

结合式(4),可将 $F_i(\bar{x}_i,x_{i+1})$改写为

$$F_i(\bar{x}_i,x_{i+1})=G_i(\bar{x}_{i+1})x_{i+1}+\Delta_i(\bar{x}_{i+1})。 \tag{5}$$

假设2 定义期望参考轨迹 y_{d},假设 y_{d},$\dot{y}_{\mathrm{d}}$和 $\ddot{y}_{\mathrm{d}}$ 是连续且有界的,那么存在一个未知正数 B_0,使得期望轨迹信号 y_{d},$\dot{y}_{\mathrm{d}}$和 $\ddot{y}_{\mathrm{d}}$ 始终在紧集 Π_0 中并满足

$$\Pi_0:=\{(y_{\mathrm{d}},\dot{y}_{\mathrm{d}},\ddot{y}_{\mathrm{d}}):(y_{\mathrm{d}})^2+(\dot{y}_{\mathrm{d}})^2+(\ddot{y}_{\mathrm{d}})^2\leqslant B_0\}。 \tag{6}$$

假设3 对于任意 $t>0$,存在未知正数 d_i^*,使得 $|d_i(t)|\leqslant d_i^*$,$1\leqslant i\leqslant n$。

引理1[18] 双曲正切函数是连续可导的,并且对于任意 $q\in R$ 和 $\forall\boldsymbol{v}>0$ 有以下不等式成立

$$0\leqslant|q|-q\tanh\left(\frac{q}{\boldsymbol{v}}\right)\leqslant0.278\,5\boldsymbol{v}。 \tag{7}$$

输入 u,输出 $v(u)$的死区模型描述为

$$v(u)=\begin{cases}k_r(u-b_r) & u\geqslant b_r\\ 0 & b_l<u<b_r\\ k_l(u-b_l) & u\leqslant b_l\end{cases}。 \tag{8}$$

假设死区模型具有如下基本性质:1)死区输出 $v(u)$是不可测量的;2)死区的倾斜度 $k_r=k_l=k$;3)死区参数 b_r,b_l 和 k 是未知的有界常数,但它们的符号已知,$b_r>0$,$b_l<0$,$k>0$。

根据上述死区的基本性质,重新定义死区模型为

$$v(u)=ku+\varepsilon_u \tag{9}$$

式中,

$$\varepsilon_u=\begin{cases}-kb_r & u\geqslant b_r\\ -ku & b_l<u<b_r\\ -kb_l & u\leqslant b_l\end{cases}, \tag{10}$$

$|\varepsilon_u|\leqslant\varepsilon^*$,$\varepsilon^*$是未知正常数。

引理2[20] 已知 $V(\cdot)$和 $\varsigma(\cdot)$是$[0,t_f)$上的光滑函数且 $V(t)\geqslant0$,$t_f\in[0,\infty]$,$N(\varsigma)$是 Nussbaum 函数,如果下列不等式成立

$$V(t)\leqslant c_1+\mathrm{e}^{-c_2t}\int_0^t[gN(\varsigma(\tau))+1]\dot{\varsigma}\mathrm{e}^{c_2\tau}\mathrm{d}\tau \tag{11}$$

式中:c_1 为适当的常数;c_2 为正数;g 为非零常数,那么 $V(\cdot)$,$\varsigma(\cdot)$ 和$\int_0^tN(\varsigma(\tau))\dot{\varsigma}\mathrm{d}\tau$ 在区间$[0,t_f)$ 上有界。

本文将采用 RBF 神经网络进行逼近。RBF 神经网络是一个线性参数化的神经网络,其能以任意精度逼近任何连续非线性函数 $\phi(\boldsymbol{Z})$,即

$$\phi(\boldsymbol{Z})=\boldsymbol{\Theta}^{*\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{Z})+\mu \tag{12}$$

式中:输入向量 $\boldsymbol{Z}\in\Omega_Z\subset\mathbf{R}^n$,$n$ 是神经网络的输入维数;μ 为神经网络的逼近误差且满足$|\mu(\boldsymbol{Z})|\leqslant\mu^*$,$\mu^*$是未知正数;$\boldsymbol{\xi}(\boldsymbol{Z})\in\mathbf{R}^l$ 为径向基函数向量,$l>1$ 为神经网络的节点数;$\boldsymbol{\Theta}^*\in\mathbf{R}^l$ 为最优权值向量,即

$$\boldsymbol{\Theta}^* = \arg\min_{\boldsymbol{\Theta}\in\mathbf{R}^l}\left\{\sup_{Z\in\Omega_Z}|\phi(\mathbf{Z}) - \boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\xi}(\mathbf{Z})|\right\} \quad (13)$$

式中，$\boldsymbol{\Theta}$ 为任意的权重向量。

2　控制器设计及稳定性分析

定义式(1)闭环系统的状态跟踪误差为 $e_1 = x_1 - y_d$，$e_i = x_i - \alpha_{if}(i=2,3,\cdots,n)$。设计过程共包含 n 步。在前 $n-1$ 步中设计期望虚拟控制信号 α_{i-1}，再以 α_{i-1} 为输入通过一阶滤波器得到 α_{if}，在第 n 步设计自适应控制器 u。

第 1 步　沿 $e_1 = x_1 - y_d$ 对 e_1 求导可得

$$\dot{e}_1 = f_1(x_1,x_2) + d_1(t) - \dot{y}_d \quad (14)$$

将式(5)和式(12)代入式(14)可得

$$\dot{e}_1 = F_1(x_1,x_2) + f_1(x_1,0) + d_1(t) - \dot{y}_d = G_1(\bar{x}_2)x_2 + \Delta_1(\bar{x}_2) + \boldsymbol{\Theta}_1^{*\mathrm{T}}\boldsymbol{\xi}(x_1) + \mu_1 + d_1(t) - \dot{y}_d。\quad (15)$$

构造虚拟控制律 α_1 以及自适应律为

$$\alpha_1 = -k_1e_1 - \zeta_1\dot{y}_d\tanh(e_1\dot{y}_d/v_1) - \frac{e_1\hat{\Phi}_1}{2a_1^2}\boldsymbol{\xi}^{\mathrm{T}}(x_1)\boldsymbol{\xi}(x_1) - \hat{\theta}_1\tanh(e_1/v_1) \quad (16)$$

$$\begin{cases}\dot{\hat{\theta}}_1 = \gamma_1e_1\tanh(e_1/v_1) - \sigma_1\gamma_1\hat{\theta}_1\\ \dot{\hat{\Phi}}_1 = \dfrac{\beta_1e_1^2}{2a_1^2}\boldsymbol{\xi}^{\mathrm{T}}(x_1)\boldsymbol{\xi}(x_1) - \sigma_1\beta_1\hat{\Phi}_1\end{cases} \quad (17)$$

式中：$k_1>0$，$\beta_1>0$，$\gamma_1>0$，$\sigma_1>0$，$a_1>0$，$v_1>0$ 和 $\zeta_1 \geq G_{1,m}^{-1}$ 均为设计参数；$\hat{\theta}_1$ 为鲁棒项，用于消除神经网络逼近误差和外界干扰影响。

将信号 α_1 通过一个一阶滤波器，滤波器的输出为 α_{2f}，时间常数为 τ_2，即

$$\tau_2\dot{\alpha}_{2f} + \alpha_{2f} = \alpha_1 \quad \alpha_{2f}(0) = \alpha_1(0)\ 。\quad (18)$$

定义滤波器的输出误差为 $y_2 = \alpha_{2f} - \alpha_1$，于是有

$$\dot{y}_2 = \dot{\alpha}_{2f} - \dot{\alpha}_1 = -\frac{y_2}{\tau_2} + \left(-\frac{\partial\alpha_1}{\partial e_1}\dot{e}_1 - \frac{\partial\alpha_1}{\partial\hat{\Phi}_1}\dot{\hat{\Phi}}_1 - \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{\partial\alpha_1}{\partial\dot{y}_d}\ddot{y}_d\right) = -\frac{y_2}{\tau_2} + B_2(e_1,\hat{\Phi}_1,\hat{\theta}_1,y_d,\dot{y}_d,\ddot{y}_d)。\quad (19)$$

根据 $x_1 = e_1 + y_d$，存在未知连续函数

$$\begin{cases}\underline{F}_1(x_1) = \kappa_{\underline{F}_1}(e_1,y_d)\\ \bar{F}_1(x_1) = \kappa_{\bar{F}_1}(e_1,y_d)\\ \underline{F}_1'(x_1) = \kappa_{\underline{F}_1'}(e_1,y_d)\\ \bar{F}_1'(x_1) = \kappa_{\bar{F}_1'}(e_1,y_d)\end{cases},\begin{cases}C_{1,1}(x_1) = \kappa_{C_{1,1}}(e_1,y_d)\\ C_{2,1}(x_1) = \kappa_{C_{2,1}}(e_1,y_d)\\ C_{3,1}(x_1) = \kappa_{C_{3,1}}(e_1,y_d)\\ C_{4,1}(x_1) = \kappa_{C_{4,1}}(e_1,y_d)\end{cases}。\quad (20)$$

定义紧集 $\Pi_1 := \{e_1^2 \leq 2\omega\}$，由定义可知，函数 $\kappa_{\underline{F}_1}(e_1,y_d)$，$\kappa_{\bar{F}_1}(e_1,y_d)$，$\kappa_{\underline{F}_1'}(e_1,y_d)$，$\kappa_{\bar{F}_1'}(e_1,y_d)$，$\kappa_{C_{1,1}}(e_1,y_d)$，$\kappa_{C_{2,1}}(e_1,y_d)$，$\kappa_{C_{3,1}}(e_1,y_d)$ 和 $\kappa_{C_{4,1}}(e_1,y_d)$ 在紧集 $\Pi_1\times\Pi_0$ 上存在最大值和最小值分别为

$$\begin{cases}\underline{F}_{1,m} \leq \underline{F}_1(x_1) \leq \underline{F}_{1,M}\\ \bar{F}_{1,m} \leq \bar{F}_1(x_1) \leq \bar{F}_{1,M}\\ \underline{F}_{1,m}' \leq \underline{F}_1'(x_1) \leq \underline{F}_{1,M}'\\ \bar{F}_{1,m}' \leq \bar{F}_1'(x_1) \leq \bar{F}_{1,M}'\end{cases},\begin{cases}|C_{1,1}(x_1)| \leq C_{1,M_1}\\ |C_{2,1}(x_1)| \leq C_{2,M_1}\\ |C_{3,1}(x_1)| \leq C_{3,M_1}\\ |C_{4,1}(x_1)| \leq C_{4,M_1}\end{cases}。\quad (21)$$

由式(4)可知

$$\begin{cases}G_1(\bar{x}_2) = \begin{cases}(1-\theta_1(x_2))\underline{F}_1(x_1) + \theta_1(x_2)\bar{F}_1(x_1) & x_2 \geq 0\\ (1-\theta_2(x_2))\underline{F}_1'(x_1) + \theta_2(x_2)\bar{F}_1'(x_1) & x_2 < 0\end{cases}\\ \Delta_1(\bar{x}_2) = \begin{cases}(1-\theta_1(x_2))C_{1,1}(x_1) + \theta_1(x_2)C_{2,1}(x_1) & x_2 \geq 0\\ (1-\theta_2(x_2))C_{3,1}(x_1) + \theta_2(x_2)C_{4,1}(x_1) & x_2 < 0\end{cases}\end{cases} \quad (22)$$

进一步可得

$$0 < G_{1,m} \leq G_1(\bar{x}_2) \leq G_{1,M} \quad (23)$$

$$0 \leq |\Delta_1(\bar{x}_2)| \leq C_1^* \quad (24)$$

$$\text{式中，}\begin{cases}G_{1,m} = \min\{\underline{F}_{1,m},\bar{F}_{1,m},\underline{F}_{1,m}',\bar{F}_{1,m}'\}\\ G_{1,M} = \max\{\underline{F}_{1,M},\bar{F}_{1,M},\underline{F}_{1,M}',\bar{F}_{1,M}'\}\\ C_1^* = \max\{|C_{1,M_1}| + |C_{2,M_1}|,|C_{3,M_1}| + |C_{4,M_1}|\}\end{cases}。$$

第 i 步($2\leq i\leq n-1$)　沿 $e_i = \boldsymbol{x}_i - \alpha_{if}$ 对 e_i 求导可得

$$\dot{e}_i = f_i(\bar{x}_i,x_{i+1}) + d_i(t) - \dot{\alpha}_{if} \quad (25)$$

将式(5)和式(12)代入式(25)可得

$$\dot{e}_i = F_i(\bar{x}_i,x_{i+1}) + f_i(\bar{x}_i,0) + d_i(t) - \dot{\alpha}_{if} = G_i(\bar{x}_{i+1})x_{i+1} + \Delta_i(\bar{x}_{i+1}) + \boldsymbol{\Theta}_i^{*\mathrm{T}}\boldsymbol{\xi}(\bar{x}_i) + \mu_i + d_i(t) - \dot{\alpha}_{if} \quad (26)$$

构造虚拟控制律 α_i 以及自适应律为

$$\alpha_i = -k_ie_i - \zeta_i\dot{\alpha}_{if}\tanh(e_i\dot{\alpha}_{if}/v_i) - \frac{\hat{\Phi}_ie_i}{2a_i^2}\boldsymbol{\xi}^{\mathrm{T}}(\bar{x}_i)\boldsymbol{\xi}(\bar{x}_i) - \hat{\theta}_i\tanh(e_i/v_i) \quad (27)$$

$$\begin{cases}\dot{\hat{\theta}}_i = \gamma_ie_i\tanh(e_i/v_i) - \sigma_i\gamma_i\hat{\theta}_i\\ \dot{\hat{\Phi}}_i = \dfrac{\beta_ie_i^2}{2a_i^2}\boldsymbol{\xi}^{\mathrm{T}}(\bar{x}_i)\boldsymbol{\xi}(\bar{x}_i) - \sigma_i\beta_i\hat{\Phi}_i\end{cases} \quad (28)$$

式中，$k_i>0$，$\beta_i>0$，$\gamma_i>0$，$\sigma_i>0$，$a_i>0$，$v_i>0$ 和 $\zeta_i \geq G_{i,m}^{-1}$ 都是设计参数。

将信号 α_i 通过一个一阶滤波器，滤波器的输出为 α_{i+1f}，时间常数为 τ_{i+1}，即

$$\tau_{i+1}\dot{\alpha}_{i+1f} + \alpha_{i+1f} = \alpha_i \quad \alpha_{i+1f}(0) = \alpha_i(0)\ 。\quad (29)$$

定义滤波器的输出误差为 $y_{i+1} = \alpha_{i+1f} - \alpha_i$，因此

$$\dot{y}_{i+1} = \dot{\alpha}_{i+1f} - \dot{\alpha}_i =$$

$$-\frac{y_{i+1}}{\tau_{i+1}}+\left(-\frac{\partial\alpha_i}{\partial e_i}\dot{e}_i-\frac{\partial\alpha_i}{\partial\hat{\boldsymbol{\Phi}}_i}\dot{\hat{\boldsymbol{\Phi}}}_i-\frac{\partial\alpha_i}{\partial\bar{\boldsymbol{x}}_i}\dot{\bar{\boldsymbol{x}}}_i-\frac{\partial\alpha_i}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i-\frac{\partial\alpha_i}{\partial\dot{\alpha}_{if}}\ddot{\alpha}_{if}\right)=$$

$$-\frac{y_{i+1}}{\tau_{i+1}}+B_{i+1}(\bar{\boldsymbol{e}}_{i+1},\bar{\boldsymbol{y}}_{i+1},\bar{\hat{\boldsymbol{\Phi}}}_i,\bar{\hat{\boldsymbol{\theta}}}_i,y_{\rm d},\dot{y}_{\rm d},\ddot{y}_{\rm d}) \quad (30)$$

式中，$\begin{cases}\bar{\boldsymbol{e}}_{i+1}=[e_1,e_2,\cdots,e_{i+1}]^{\rm T}\\ \bar{\boldsymbol{y}}_{i+1}=[y_2,\cdots,y_{i+1}]^{\rm T}\\ \bar{\hat{\boldsymbol{\Phi}}}_i=[\hat{\Phi}_1,\hat{\Phi}_2,\cdots,\hat{\Phi}_i]^{\rm T}\\ \bar{\hat{\boldsymbol{\theta}}}_i=[\hat{\theta}_1,\hat{\theta}_2,\cdots,\hat{\theta}_i]^{\rm T}\end{cases}$。

由 $x_{i+1}=e_{i+1}+\alpha_{i+1f}$ 和 $y_{i+1}=\alpha_{i+1f}-\alpha_i$，可得 $x_{i+1}=e_{i+1}+\alpha_i+y_{i+1}$。存在未知连续函数如下

$$\begin{cases}\underline{F}_i(\bar{\boldsymbol{x}}_i)=\kappa_{\underline{F}_i}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ \bar{F}_i(\bar{\boldsymbol{x}}_i)=\kappa_{\bar{F}_i}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ \underline{F}'_i(\bar{\boldsymbol{x}}_i)=\kappa_{\underline{F}'_i}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ \bar{F}'_i(\bar{\boldsymbol{x}}_i)=\kappa_{\bar{F}'_i}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ C_{1,i}(\bar{\boldsymbol{x}}_i)=\kappa_{C_{1,i}}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ C_{2,i}(\bar{\boldsymbol{x}}_i)=\kappa_{C_{2,i}}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ C_{3,i}(\bar{\boldsymbol{x}}_i)=\kappa_{C_{3,i}}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\\ C_{4,i}(\bar{\boldsymbol{x}}_i)=\kappa_{C_{4,i}}(\bar{\boldsymbol{e}}_i,\bar{\boldsymbol{y}}_i,\bar{\hat{\boldsymbol{\Phi}}}_{i-1},\bar{\hat{\boldsymbol{\theta}}}_{i-1},y_{\rm d},\dot{y}_{\rm d})\end{cases}。\quad (31)$$

定义如下紧集

$$\Pi_i:=\left\{\sum_{j=1}^{i}\left(\frac{G_{i,m}\tilde{\theta}_j^2}{\gamma_j}+\frac{G_{i,m}\tilde{\Phi}_j^2}{\gamma_j}+e_j^2\right)+\sum_{j=2}^{i}y_j^2\leqslant 2\omega\right\}。\quad (32)$$

从式(32)和假设2可知，函数 $\kappa_{\underline{F}_i}(\cdot)$，$\kappa_{\bar{F}_i}(\cdot)$，$\kappa_{\underline{F}'_i}(\cdot)$，$\kappa_{\bar{F}'_i}(\cdot)$，$\kappa_{C_{1i}}(\cdot)$，$\kappa_{C_{2i}}(\cdot)$，$\kappa_{C_{3i}}(\cdot)$ 和 $\kappa_{C_{4i}}(\cdot)$ 中的所有变量都包含在紧集 $\Pi_i\times\Pi_0$ 中。因此，它们在紧集 $\Pi_i\times\Pi_0$ 上存在如下最大值和最小值，即

$$\begin{cases}\underline{F}_{i,m}\leqslant\underline{F}_i(\bar{\boldsymbol{x}}_i)\leqslant\underline{F}_{i,M}\\ \bar{F}_{i,m}\leqslant\bar{F}_i(\bar{\boldsymbol{x}}_i)\leqslant\bar{F}_{i,M}\\ \underline{F}'_{i,m}\leqslant\underline{F}'_i(\bar{\boldsymbol{x}}_i)\leqslant\underline{F}'_{i,M}\\ \bar{F}'_{i,m}\leqslant\bar{F}'_i(\bar{\boldsymbol{x}}_i)\leqslant\bar{F}'_{i,M}\end{cases},\begin{cases}|C_{1i}(\bar{\boldsymbol{x}}_i)|\leqslant C_{1,M_i}\\ |C_{2i}(\bar{\boldsymbol{x}}_i)|\leqslant C_{2,M_i}\\ |C_{3i}(\bar{\boldsymbol{x}}_i)|\leqslant C_{3,M_i}\\ |C_{4i}(\bar{\boldsymbol{x}}_i)|\leqslant C_{4,M_i}\end{cases}。\quad (33)$$

注意由式(4)可得

$$0<G_{i,m}\leqslant G_i(\bar{\boldsymbol{x}}_{i+1})\leqslant G_{i,M} \quad (34)$$

$$0\leqslant|\Delta_i(\bar{\boldsymbol{x}}_{i+1})|\leqslant C_i^* \quad (35)$$

式中，$\begin{cases}G_{i,m}=\min\{\underline{F}_{i,m},\bar{F}_{i,m},\underline{F}'_{i,m},\bar{F}'_{i,m}\}\\ G_{i,M}=\max\{\underline{F}_{i,M},\bar{F}_{i,M},\underline{F}'_{i,M},\bar{F}'_{i,M}\}\\ C_i^*=\max\{|C_{1,M_i}|+|C_{2,M_i}|,|C_{3,M_i}|+|C_{4,M_i}|\}\end{cases}$。

第 n 步 沿 $e_n=x_n-\alpha_n$ 对 e_n 求导可得

$$\dot{e}_n=f_n(x,v(u))+d_n(t)-\dot{\alpha}_{nf} \quad (36)$$

将式(5)、式(12)代入式(36)可得

$$\begin{aligned}\dot{e}_n&=F_n(x,v(u))+f_n(x,0)+d_n(t)-\dot{\alpha}_{nf}=\\&G_n(\bar{\boldsymbol{x}}_{n+1})(ku+\varepsilon_u)+\Delta_n(\bar{\boldsymbol{x}}_{n+1})+\\&\boldsymbol{\Theta}_n^{*\rm T}\boldsymbol{\xi}(\boldsymbol{x})+\mu_n+d_n(t)-\dot{\alpha}_{nf}。\end{aligned}\quad (37)$$

构造实际控制器 u 以及自适应律为

$$\begin{aligned}u=&N(\varsigma)[G_{n,m}k_ne_n+\dot{\alpha}_{nf}\tanh(e_n\dot{\alpha}_{nf}/v_n)+\\&G_{n,m}\frac{\hat{\Phi}_ne_n}{2a_n^2}\boldsymbol{\xi}^{\rm T}(\boldsymbol{x})\boldsymbol{\xi}(\boldsymbol{x})+G_{n,m}\hat{\theta}_n\tanh(e_n/v_n)]\end{aligned}\quad (38)$$

$$\begin{aligned}\dot{\varsigma}=&G_{n,m}k_ne_n^2+e_n\dot{\alpha}_{nf}\tanh(e_n\dot{\alpha}_{nf}/v_n)+\\&G_{n,m}\frac{\hat{\Phi}_ne_n^2}{2a_n^2}\boldsymbol{\xi}^{\rm T}(\boldsymbol{x})\boldsymbol{\xi}(\boldsymbol{x})+e_nG_{n,m}\hat{\theta}_n\tanh(e_n/v_n)\end{aligned}\quad (39)$$

$$\begin{cases}\dot{\hat{\theta}}_n=\gamma_ne_n\tanh(e_n/v_n)-\sigma_n\gamma_n\hat{\theta}_n\\ \dot{\hat{\Phi}}_n=\dfrac{\beta_ne_n^2}{2a_n^2}\boldsymbol{\xi}^{\rm T}(\boldsymbol{x})\boldsymbol{\xi}(\boldsymbol{x})-\sigma_n\beta_n\hat{\Phi}_n\end{cases}\quad (40)$$

式中，$k_n>0$，$\beta_n>0$，$\gamma_n>0$，$\sigma_n>0$，$a_n>0$，$v_n>0$ 和 $\zeta_n\geqslant G_{n,m}^{-1}$ 均是设计参数。与第 i 步相似，存在

$$\begin{cases}\underline{F}_{n,m}\leqslant\underline{F}_n(\bar{\boldsymbol{x}}_n)\leqslant\underline{F}_{n,M}\\ \bar{F}_{n,m}\leqslant\bar{F}_n(\bar{\boldsymbol{x}}_n)\leqslant\bar{F}_{n,M}\\ \underline{F}'_{n,m}\leqslant\underline{F}'_n(\bar{\boldsymbol{x}}_n)\leqslant\underline{F}'_{n,M}\\ \bar{F}'_{n,m}\leqslant\bar{F}'_n(\bar{\boldsymbol{x}}_n)\leqslant\bar{F}'_{n,M}\end{cases},\begin{cases}|C_{1,n}(\bar{\boldsymbol{x}}_n)|\leqslant C_{1,M_n}\\ |C_{2,n}(\bar{\boldsymbol{x}}_n)|\leqslant C_{2,M_n}\\ |C_{3,n}(\bar{\boldsymbol{x}}_n)|\leqslant C_{3,M_n}\\ |C_{4,n}(\bar{\boldsymbol{x}}_n)|\leqslant C_{4,M_n}\end{cases}。\quad (41)$$

注意由式(4)可得

$$0<G_{n,m}\leqslant G_n(\bar{\boldsymbol{x}}_{n+1})\leqslant G_{n,M} \quad (42)$$

$$0\leqslant|\Delta_n(\bar{\boldsymbol{x}}_{n+1})|\leqslant C_n^* \quad (43)$$

式中，$\begin{cases}G_{n,m}=\min\{\underline{F}_{n,m},\bar{F}_{n,m},\underline{F}'_{n,m},\bar{F}'_{n,m}\}\\ G_{n,M}=\max\{\underline{F}_{n,M},\bar{F}_{n,M},\underline{F}'_{n,M},\bar{F}'_{n,M}\}\\ C_n^*=\max\{|C_{1,M_n}|+|C_{2,M_n}|,|C_{3,M_n}|+|C_{4,M_n}|\}\end{cases}$。

定理1 考虑式(1)一类纯反馈非线性系统，设计虚拟控制律 α_i 为式(16)、式(27)；实际控制器 u 为式(38)，以及自适应律为式(17)、式(28)和式(40)，使得闭环系统所有信号半全局一致终结有界，并且通过选择适当的设计参数，系统输出趋于给定参考轨迹的一个小邻域。

证明 定义 Lyapunov 函数为

$$V=\sum_{i=1}^{n}\frac{e_i^2}{2}+\sum_{i=1}^{n}\frac{G_{i,m}\tilde{\theta}_i^2}{2\gamma_i}+\sum_{i=1}^{n}\frac{G_{i,m}\tilde{\Phi}_i^2}{2\beta_i}+\frac{1}{2}\sum_{i=1}^{n-1}y_{i+1}^2 \quad (44)$$

对 V 求导可得

$$\dot{V}\leqslant\sum_{i=1}^{n-1}G_i(\bar{\boldsymbol{x}}_{i+1})x_{i+1}e_i+G_n(\bar{\boldsymbol{x}}_{n+1})kue_n+\sum_{i=1}^{n}|e_i|G_{i,m}\theta_i^*-$$

$$e_1\dot{y}_d-\sum_{i=2}^{n}e_i\dot{\alpha}_{if}+\sum_{i=1}^{n}\boldsymbol{\Theta}_i^{*\mathrm{T}}\boldsymbol{\xi}(\bar{\boldsymbol{x}}_i)e_i-\sum_{i=1}^{n}\frac{G_{i,m}\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i}-\sum_{i=1}^{n}\frac{G_{i,m}\tilde{\Phi}_i\dot{\hat{\Phi}}_i}{\beta_i}+\sum_{i=2}^{n}\left(-\frac{y_i^2}{\tau_i}+|y_i|B_i\right)\tag{45}$$

式中，$\theta_i^*=G_{i,m}^{-1}(\mu_i^*+C_i^*+d_i^*)$，$i=1,\cdots,n-1$，$\theta_n^*=G_{n,m}^{-1}(\mu_n^*+C_n^*+d_n^*)+\varepsilon_u^*$，由 Young 不等式可得

$$\dot{V}\leqslant\sum_{i=1}^{n-1}G_i(\bar{\boldsymbol{x}}_{i+1})x_{i+1}e_i+G_n(\bar{\boldsymbol{x}}_{n+1})kue_n+\sum_{i=1}^{n}|e_i|G_{i,m}\theta_i^*-e_1\dot{y}_d-\sum_{i=2}^{n}e_i\dot{\alpha}_{if}+\sum_{i=1}^{n}\frac{e_i^2\|\boldsymbol{\Theta}_i^*\|}{2a_i^2}\boldsymbol{\xi}^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)\xi(\bar{\boldsymbol{x}}_i)+\frac{a_i^2}{2}-\sum_{i=1}^{n}\frac{G_{i,m}\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i}-\sum_{i=1}^{n}\frac{G_{i,m}\tilde{\Phi}_i\dot{\hat{\Phi}}_i}{\beta_i}+\sum_{i=2}^{n}\left(-\frac{y_i^2}{\tau_i}+|y_i|B_i\right)\tag{46}$$

式中：$\Phi_i=G_{i,m}^{-1}\|\boldsymbol{\Theta}_i^*\|$；$a_i$ 是任意正常数。

代入控制律和自适应律，在式(46)的右边同时加减 $\dot{\varsigma}$ 并根据引理 2 可得

$$\dot{V}\leqslant-\sum_{i=1}^{n}k_iG_{i,m}e_i^2+G_{n,M}kN(\varsigma)\dot{\varsigma}+\dot{\varsigma}+\sum_{i=1}^{n}G_{i,m}\sigma_i(\tilde{\theta}_i\hat{\theta}_i+\tilde{\Phi}_n\hat{\Phi}_n)+\sum_{i=1}^{n-1}G_{i,M}(|e_{i+1}|+|y_{i+1}|)|e_i|+\sum_{i=2}^{n}\left(-\frac{y_i^2}{\tau_i}+|y_i|B_i\right)+\sum_{i=1}^{n}\left(\frac{a_i^2}{2}+0.278\,5G_{i,m}\theta_i^*v_i+0.278\,5v_i\right)。\tag{47}$$

由定义可知，$\Pi_i\in\mathbf{R}^{4i-1}$ 和 Π_0 是紧集。因此，$\Pi_i\times\Pi_0\in\mathbf{R}^{4i+2}$ 是紧集，由式(19)和式(30)可知，$B_{i+1}(\cdot)$ 是由紧集 $\Pi_i\times\Pi_0$ 中的变量 $\bar{\boldsymbol{e}}_{i+1}$，$\bar{\boldsymbol{y}}_{i+1}$，$\bar{\hat{\Phi}}_i$，$\bar{\hat{\theta}}_i$，$y_d$，$\dot{y}_d$ 和 $\ddot{y}_d$ 构成的连续函数，因此在紧集 $\Pi_i\times\Pi_0$ 上存在最大值 M_{i+1}，即 $|B_{i+1}|\leqslant M_{i+1}$，因此对 V 求导

$$\dot{V}\leqslant-\sum_{i=1}^{n}k_iG_{i,m}e_i^2+\sum_{i=2}^{n}\left(-\frac{y_i^2}{\tau_i}+|y_i|M_i\right)+G_{n,M}kN(\varsigma)\dot{\varsigma}+\dot{\varsigma}+\sum_{i=1}^{n}G_{i,m}\sigma_i(\tilde{\theta}_i\hat{\theta}_i+\tilde{\Phi}_n\hat{\Phi}_n)+\sum_{i=1}^{n-1}G_{i,M}(|e_{i+1}|+|y_{i+1}|)|e_i|+\sum_{i=1}^{n}\left(\frac{a_i^2}{2}+0.278\,5G_{i,m}\theta_i^*v_i+0.278\,5v_i\right)。\tag{48}$$

由 Young 不等式可得

$$\begin{cases}|y_i|M_i\leqslant0.5c_1y_i^2+0.5M_i^2/c_1\\G_{i,M}|y_{i+1}||e_i|\leqslant0.5G_{i,M}^2y_{i+1}^2c_2+0.5e_i^2/c_2\\G_{i,M}|e_{i+1}||e_i|\leqslant0.5G_{i,M}e_i^2+0.5G_{i,M}e_{i+1}^2\end{cases}\tag{49}$$

将式(49)代入式(48)可得

$$\dot{V}\leqslant-\left(k_1G_{1,m}-\frac{G_{1,M}}{2}-\frac{1}{2c_2}\right)e_1^2-\left(k_nG_{i,m}-\frac{G_{i,M}}{2}\right)e_n^2-\sum_{i=1}^{n-1}y_{i+1}^2\left(\frac{1}{\tau_{i+1}}-\frac{c_1}{2}-\frac{G_{i,M}^2c_2}{2}\right)-\frac{1}{2}\sum_{i=1}^{n}G_{i,m}\sigma_i(\tilde{\theta}_i^2+\tilde{\Phi}_i^2)+\sum_{i=2}^{n-1}\left(-\left(k_iG_{i,m}-G_{i,M}-\frac{1}{2c_2}\right)e_i^2\right)+C_1+G_{n,M}kN(\varsigma)\dot{\varsigma}+\dot{\varsigma}\tag{50}$$

式中，$C_1=\frac{1}{2}\sum_{i=1}^{n}G_{i,m}\sigma_i(\theta_i^{*2}+\Phi_i^2)+\sum_{i=1}^{n-1}\left(\frac{M_{i+1}^2}{2c_1}\right)+\sum_{i=1}^{n}\left(\frac{a_i^2}{2}+0.278\,5G_{i,m}\theta_i^*v_i+0.278\,5v_i\right)$。

选取设计参数为
$$\begin{cases}\frac{1}{\tau_{i+1}}\geqslant\frac{c_1}{2}+\frac{G_{i,M}^2c_2}{2}+\omega_1\\k_i\geqslant G_{i,m}^{-1}\left(G_{i,M}+\frac{1}{2c_2}+\omega_2\right)\\k_1\geqslant G_{1,m}^{-1}\left(\frac{G_{1,M}}{2}+\frac{1}{2c_2}+\omega_2\right)\\k_n\geqslant G_{i,m}^{-1}\left(\frac{G_{i,M}}{2}+\omega_2\right)\end{cases}$$
，其中，ω_1 和 ω_2 是任意正常数。因此

$$\dot{V}\leqslant-\sum_{i=1}^{n}\omega_2e_i^2-\frac{1}{2}\sum_{i=1}^{n}G_{i,m}\sigma_i(\tilde{\theta}_i^2+\tilde{\Phi}_i^2)-\sum_{i=1}^{n-1}\omega_1y_{i+1}^2+G_{n,M}kN(\varsigma)\dot{\varsigma}+\dot{\varsigma}+C_1\tag{51}$$

利用式(44)，可将式(51)改写为

$$\dot{V}\leqslant-C_2V+C_1+G_{n,M}kN(\varsigma)\dot{\varsigma}+\dot{\varsigma}\tag{52}$$

对式(52)两边积分，然后化简得

$$V(t)\leqslant(V(0)-C_1/C_2)\mathrm{e}^{-C_2t}+C_1/C_2+\mathrm{e}^{-C_2t}\int_0^t[G_{n,M}kN(\varsigma)+1]\dot{\varsigma}\,\mathrm{e}^{C_2\tau}\mathrm{d}\tau\leqslant V(0)+C_1/C_2+\int_0^t[G_{n,M}kN(\varsigma)+1]\dot{\varsigma}\,\mathrm{e}^{-C_2(t-\tau)}\mathrm{d}\tau\ 。\tag{53}$$

根据引理 3 可知 $V(t)$，$\varsigma(t)$ 及 $\int_0^tG_{n,M}kN(\varsigma)\dot{\varsigma}\mathrm{d}\tau$ 在区间 $[0,t_f)$ 上有界，由此，令

$$\int_0^t|G_{n,M}kN(\varsigma)+1|\dot{\varsigma}\,\mathrm{e}^{-C_2(t-\tau)}\mathrm{d}\tau\leqslant M\tag{54}$$

则由式(53)和式(54)可得

$$V(t)\leqslant V(0)+C_3+M\tag{55}$$

式中：$C_2=\min\limits_{i=1,\cdots,n}\{2\omega_1,2\omega_2,\sigma_i\gamma_i,\sigma_i\beta_i\}$；$C_3=C_1/C_2$。由式(55)可知 $V(t)$ 有界，从而闭环系统的所有信号 $\hat{\Phi}$，$\hat{\theta}$，e 和 ς 均为半全局一致终结有界。

由式(44)和式(55)可得

$$\lim_{t\to\infty}|e_1|\leqslant\lim_{t\to\infty}\sqrt{2V(t)}\leqslant\sqrt{2(V(0)+C_3+M)}。\tag{56}$$

因此，可以通过调节设计参数使得系统输出趋近于期望轨迹的一个小邻域内。

3 仿真分析

考虑如下纯反馈非线性系统

$$\begin{cases}\dot{x}_1=f_1(x_1,x_2)+d_1(t)\\\dot{x}_2=f_2(x,v(u))+d_2(t)\\y=x_1\end{cases}\tag{57}$$

式中：$f_1(x_1,x_2)=x_1+x_2+\frac{x_2^3}{5}$；$d_1(t)=0.2\sin t$；$d_2(t)=0.1\cos t$；非仿射函数$f_2(x,v(u))$模型为

$$f_2(x,v(u))=\begin{cases}x_1x_2 & -2.5<v(u)<1.5\\ x_1x_2+v(u)+\frac{(v(u))^3}{7} & \text{其他}\end{cases}。\tag{58}$$

由式(58)可以看出，$f_2(x,v(u))$关于u是不连续的。非线性死区模型为

$$v(u)=\begin{cases}(1+0.3\sin u)(u-0.5) & u\geqslant 0.5\\ 0 & -0.3<u<0.5\\ (0.8+0.2\cos u)(u+0.3) & u\leqslant -0.3\end{cases}。\tag{59}$$

期望轨迹$y_d=0.5(\sin t+\sin(0.5t))$。选择设计参数：$\beta_1=\beta_2=1$，$\gamma_1=\gamma_2=1$，$\sigma_1=\sigma_2=0.05$，$\tau_2=0.1$，$a_1=a_2=0.2$。选择 RBF 神经网络：$\xi^{\mathrm{T}}(x_1)$包含3个节点，并且中心均匀分配在区间[−2,2]上，宽度为2；$\xi^{\mathrm{T}}(x_2)$包含9个节点，并且中心均匀分配在区间[−2,2]×[−2,2]上，宽度为2。初始值设置为：$[x_1(0),x_2(0)]^{\mathrm{T}}=[0.5,1.8]^{\mathrm{T}}$，$\hat{\theta}_1(0)=\hat{\theta}_2(0)=0$和$\hat{\Phi}_1(0)=\hat{\Phi}_2(0)=0$。选取 Nussbaum 函数为$N(\varsigma)=\mathrm{e}^{\varsigma^2}\cos(\pi\varsigma/2)$。仿真结果如图1～图5所示。闭环系统所有信号半全局一致终结有界，并且跟踪误差可收敛到原点附近的一个小邻域内。

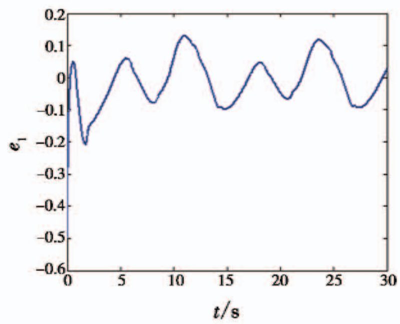

图1 跟踪误差e_1

Fig.1 Tracking error e_1

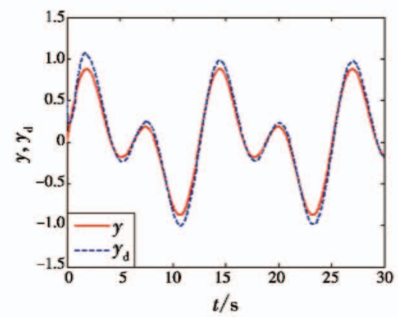

图2 期望轨迹y_d和系统输出y

Fig.2 Reference trajectory y_d and system output y

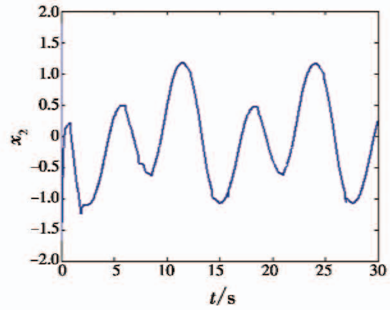

图3 系统状态变量x_2

Fig.3 System state variable x_2

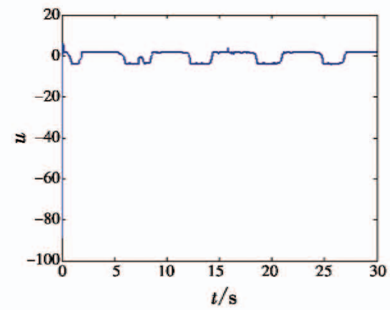

图4 控制输入u

Fig.4 Control input u

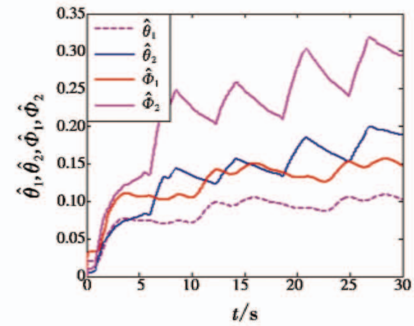

图5 自适应参数$\hat{\theta}_1$，$\hat{\theta}_2$，$\hat{\Phi}_1$和$\hat{\Phi}_2$

Fig.5 Adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\Phi}_1$ and $\hat{\Phi}_2$

4 结束语

本文研究了一类非仿射函数不连续的纯反馈非线性系统的跟踪控制问题。与以往文献相比，取消了非仿射函数的连续条件，可以被应用于更广泛的一类纯反馈系统。本文进一步研究的方向是在状态不可测条件下设计非线性系统的观测器。

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