

A novel fiber Bragg grating vibration sensor with double equal-strength cantilever beams*

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In practice, since the measurement environments are usually complex, the electromagnetic interference problem has been an important issue for sensor applications, and the fiber sensor can overcome it effectively. This paper proposes a new type of vibration sensor based on fiber Bragg grating (FBG). The vibration sensor with double equal strength cantilever beams structure can further improve the measurement stability. The physical model of FBG vibration sensor was established, and the structural parameters of the double equal-strength cantilever beams were designed and optimized by finite element analysis. Both simulation and experimental tests are included in this paper to illustrate the low-frequency vibration performance of the designed vibration sensor. Among, the sensitivity of the sensor was $0.0240 \text{ pm}/(\text{m}\cdot\text{s}^{-2})$, the natural frequency was 185 Hz, and the linearity was obtained, which further verify the stability and reliability of the sensor vibration frequency measurement.

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Along with the rapid development of vibration testing technology, it's urgent to design and develop a matched vibration sensor with higher precision and well performance to meet the needs of the development^[1-3]. Fiber grating, a new passive device, has been developed rapidly in recent years. It can be used to avoid the problems caused by traditional electromagnetic sensors, such as lower reliability, shorter service life and limited measuring distance. Instead, it can be applied to the wide range of measuring objects; and it can be used to measure the object with multiple parameters precisely by the special structured-design and packing technology due to its advantages of faster response speed, higher sensitivity, better security and electrical insulation. Therefore, it will be widely applied in the field of vibration testing.

Teng li et al^[4] developed a fiber grating vibration sensor based on differential structure, which expand the measurement bandwidth of the sensor. N. Basumallick^[5] put a polyimide layer to the cantilever beam surface, and pasted a fiber grating layer on it. And they found that the sensitivity was improved, to be exact, twice as much as the original one, when the inherent frequency is not changed. D. M. Karabacak et al^[6] designed a fiber Bragg grating (FBG) vibration high speed monitoring system to realize vibration measurement in any unfriendly environment.

Feng Fu et al^[7] presented a compact and ultrasensitive bio-molecule sensor based on fiber specklegram. Chen Wang et al^[8] presented a precision non-contact displacement sensor based on the near-field property of specklegrams. Atsushi Wada et al^[9] proposed A fast interrogation method using a sinusoidal modulated laser diode for a fiber Fabry-Perot interferometric sensor consisting of Bragg gratings (FBG-FPI). Wang Yun et al^[10] designed a Bragg grating two-way acceleration sensor based on a circular flexible hinge. But the sensitivity of the above sensors is not high, and the response speed is not fast.

In this paper, we designed a dual-intensity cantilever beam fiber grating vibration frequency sensor based on the fiber grating vibration sensing principle. The finite element analysis of the mechanical model of the vibration sensor is performed to obtain its simulation performance index, and to provide a reliable reference for optimizing the design of fiber grating vibration sensors; to vibration experimental platform was set up of fiber grating vibration sensors to verify the natural frequency, sensitivity, linearity, and stability of the designed vibration sensor. That explains the reliability of the sensor.

Based on the fiber grating coupled mode theory, the Bragg equation can be expressed as

$$\lambda_B = 2n_{\text{eff}}\Lambda, \quad (1)$$

where λ_B and n_{eff} denote the FBG center wavelength and

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the effective refractive index, respectively, and Λ represents the grating period. Λ and n_{eff} are affected by the external environment of the FBG, resulting in a drift of the center wavelength λ_B . When the FBG is stressed, it will be compressed or stretched, causing the grating period Λ to change. According to the optical effect of the optical fiber, the effective refractive index n_{eff} will change as the FBG is stressed^[11].

Only light waves can be reflected by gratings, and the rest of the projection spectrum is not affected. Temperature and strain are physical parameters that FBG can directly sensitize, and they will cause Λ and n_{eff} to change. Temperature affects λ_B by thermo-optic effect and thermal expansion effect, while strain affects λ_B by elasto-optic effect and grating period change. The drift of λ_B with strain and temperature is^[12]

$$\Delta\lambda_B = (\alpha + \zeta)\lambda_B\Delta T + (1 - P_e)\lambda_B\varepsilon = K_T\Delta T + K_e\varepsilon, \quad (2)$$

where α and ζ denote the thermal expansion coefficient and thermo-optic coefficient of the optical fiber, P_e is the effective elasto-optic coefficient of the optical fiber, ε is the axial strain, K_T and K_e are the temperature and strain sensitivity coefficients of FBG, respectively.

At constant temperature, i.e. $\Delta T=0$, the FBG is only subjected to axial stress using

$$\frac{\Delta\lambda_B}{\lambda_B} = (1 - P_e)\varepsilon \quad \text{or} \quad \Delta\lambda_B = K_e\varepsilon, \quad (3)$$

where ε represents the dynamic strain of the fiber grating under the external vibration signal. For an FBG, the effective light-emission coefficient is a constant, and if the ambient temperature remains constant, K_e will be a constant, which indicates the FBG center-wavelength offset satisfies the requirements and its axial strain linearity is improved.

When the FBG vibration sensor is subjected to an external vibration signal, the sensor can convert the measured vibration parameter into the dynamic strain amount of the grating axis. It means the FBG reflection spectrum will have the periodic variation drift whose frequency is the same as that of the vibration signal. Therefore, vibration signal sensing is achieved by detecting the periodic drift of the center wavelength of the fiber grating.

Neglecting the cross-sensitivity between strain and temperature, according to Eq.(3), the wavelength drift can be expressed as

$$\Delta\lambda_B = \lambda_B(1 - P_e)\varepsilon. \quad (4)$$

It can be obtained that there is a linear relationship between the wavelength variation $\Delta\lambda_B$ and the axial strain ε . First, measure the central wavelength variation rule and then use the vibration sensor response function to conduct the vibration signal sensing. That is to say, during the vibration sensing process, the center wavelength variation data of the fiber grating is collected, and after the Fourier transform, the reflected vibration frequency value is the frequency value to be measured.

The cantilever beam is the most direct and simple inertial mechanism, and the structure of the equal-strength cantilever beam has developed into one of the typical

structures in FBG vibration sensing field. The basic working principle is that the force strain on the surface of cantilever beam is fixed on the surface of the cantilever beam, which makes the FBG center wavelength drift. Because the structure is equal to the isosceles triangle, the different position strain of the center line is the same, which avoids the chirp phenomenon caused by unevenness of the strain distribution in each position of FBG. Considering the inertia force, the strength beam of single arm is not strong, and the rotation and deflection are easy to appear. In this paper, the sensor is designed to adopt double equal-strength cantilever beams structure, which theoretically can avoid transverse vibration interference, improve working band, and obtain more abundant vibration signals.

The mathematical model of a double equal-strength cantilever beams FBG vibration sensor can be simplified to a single-degree-of-freedom vibration system with second-order as shown in Fig.1.

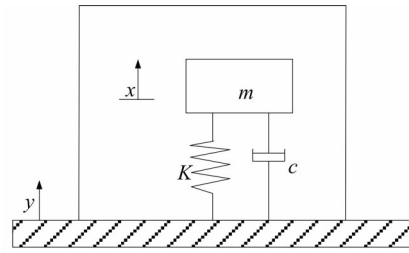


Fig.1 Schematic diagram of sensor operation

In Fig.1, c and K denote the damping and the equivalent spring rate of the cantilever beam of equal strength. m is the vibration mass; x is the fixed coordinate of the space, and y is the coordinate of the casing body. The whole mechanism is made up vibration with the base. At the same time, the relative displacement y occurs in the housing and the mass. Then, for the y coordinate system, the mass motion equation can be expressed as

$$\frac{d^2y}{dt^2} + 2\xi\omega_0 \frac{dy}{dt} + \omega_0^2 y = -a_g, \quad (5)$$

where a_g denotes the acceleration of the housing in the x coordinate system with the movement of the pedestal. $\omega_0 = \sqrt{K/m}$ is the natural angular frequency of the cantilever beam and the mass as a whole. The damping ratio is $\xi = \frac{c}{c_c} = c / (2\sqrt{mK})$, and c_c is the critical damping.

When the force F acts on the top of the equal-strength cantilever beam, the axial strains on the upper and lower surfaces are evenly distributed. The size can be expressed as

$$\varepsilon = \frac{6FL}{EBh^2}. \quad (6)$$

The equivalent spring rate k of a cantilever beam is

$$k = \frac{Bh^3}{6L^3} E, \quad (7)$$

where L represents the length of the beam, h is the

thickness of the beam, E represents the Young's modulus of the beam, and B is the width of the beam.

The natural angular frequency of the cantilever beam sensor can be expressed as

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{Bh^3 E}{6L^3 m}}, \quad (8)$$

where m represents the equivalent inertial mass of the cantilever beam sensor.

The relation between FBG central wavelength variation and vibration acceleration can be expressed as

$$\frac{\Delta\lambda_B}{\lambda_B} = (1 - P_c) \frac{6Lm}{Bh^2 E} a. \quad (9)$$

From Eq.(9), the sensitivity of the FBG vibration sensor is expressed as

$$S = \frac{\Delta\lambda_B}{a} = (1 - P_c) \frac{6Lm}{Bh^2 E} \lambda_B, \quad (10)$$

where P_c is the effective light-emission coefficient of the fiber, and λ_B represents the center wavelength of the FBG.

When the vibrator vibrates from the middle equilibrium position to the maximum amplitude position, the grating strain reaches its maximum and the wavelength drifts to the extreme point. When the vibrator returns to the intermediate equilibrium position, the center wavelength of the grating turns to the initial value, and the rear vibrator moves upward again. The grating will be in a contracted state. When the vibrator moves to the maximum amplitude position, the center wavelength of the fiber will change again to the extremum point. The vibrator will return to the middle equilibrium position and the grating center wavelength will turns to the initial value. Therefore, as the time domain curve of the center wavelength is changed by Fourier transform, its reflected frequency value is that of the measured vibration frequency.

Fig.2 shows the structure of double equal-strength cantilever beams, where both the dimensions and the structures of the upper and the lower beams are symmetrical. One end of the cantilever beam is fixed to the mass, while the other is fixed to the vibrating body. Glue the FBG to the surface of the cantilever beam using AB glue, and stick the FBG to the central axis of the surface of the cantilever beam as closely as possible.

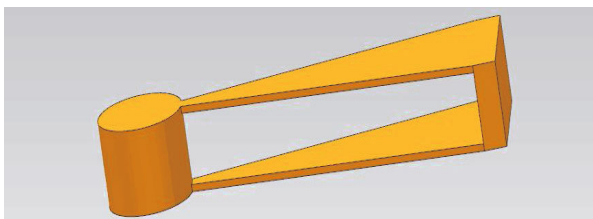


Fig.2 Sensor model

In the process of designing the sensor, material selection is very important because it affects the performance

of the sensor-related parameters directly. Therefore, it's necessary to consider the parameters, structure size as well as manufacture technology of the vibration sensor in order to avoid negative effects as much as possible.

The elastic element has to be characterized with a certain strength, better recovery and elasticity to ensure its reliability, stability and accuracy of the elastic element. The corrosion resistance of the selected material has to be very high to meet the needs of FBG vibration sensor and its complex detection environment. Tab.1 shows the performance characteristics of the materials commonly used in several industrial productions.

Tab.1 Performance characteristics of common materials

Name	Line expansion coefficient	Yield strength (MPa)	Elastic modulus (GPa)
45# steel	11	360	200
Cast steel	-	350	175
Bronze	16.6	1 250	131
304#	17.3	205	194
Hard alloy	23	280	72
Carbon	12	335—410	206

In general, as we can see, it's more suitable to use hard aluminum alloy in producing non-elastic components due to its lower density and higher strength. 304# stainless steel is characterized with high corrosion resistance, small expansion coefficient, high yield strength, and large elastic modulus. Even though alloy steel is suitable in producing high precision elastic elements, it's not easy to be processed. Therefore, 304# stainless steel is selected to be the sensor material based on the characteristics and availability of the listed materials. Then, we analyzed the sensitivity and natural frequency of the designed sensor.

Specific sensor structure parameters are shown in Tab.2.

Tab.2 Sensor structure parameters

No.	Parameter	Unit	Value
1	Cantilever width	mm	20
2	Cantilever thickness	mm	1
3	Mass radius	mm	7.5
4	Block height	mm	12
5	Cantilever length	mm	64
6	Elastic modulus	GPa	194

The finite element analysis software was used to simulate and analyze the double equal-strength cantilever beams FBG vibration sensor designed to verify the accuracy of the design parameters of the sensor and the reliability of the acceleration mechanic model. The

natural frequency is one of the important parameters of the vibration sensor. According to Eq.(10), the magnitude of the natural frequency of the cantilever beam structure is closely related to the equivalent inertial mass and stiffness of the elastic element. Therefore, it's critical to determine and calculate the parameters.

Within the elastic range, the stiffness represents the proportional coefficient of the load between the mechanical structure and the displacement proportional to it. That is the force required to cause the displacement per unit, which can be represented by

$$k=F/x_z. \tag{11}$$

As Cantilever beam belongs to an elastic element, its stiffness can be calculated by Eq.(11), where F represents the magnitude of the force exerted by the free end of the cantilever beam, and x_z denotes the longitudinal displacement magnitude of the free end of the cantilever beam.

A simulation model of the double equal-strength cantilever beams FBG vibration frequency sensor is established in the finite element simulation software. The static analysis module is selected. As shown in Fig.3(a), the fixed constraint is applied to the base surface of the cantilever beam and 1 N force is added to the positive axis of the free end of the cantilever beam while neglecting the effect of gravity. Based on above analysis, the free end of the cantilever beam, where is applied to the axial displacement, and the simulation result is shown in Fig.3(b). The result shows that the displacement of the free end of the cantilever beam is 0.040 895 mm.

The equivalent spring rate of an isotropic cantilever beam k can be expressed as follows:

$$k = \frac{BT^3}{6L^3} E, \tag{12}$$

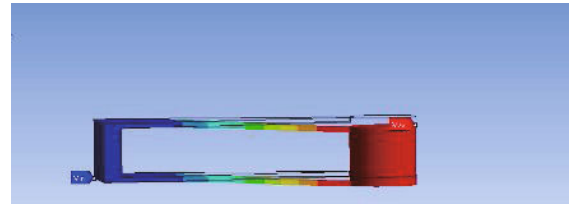
where L , T , E , and B denote the length of the beam, the thickness of the beam and the Young's modulus of the beam, and the width of the beam, respectively.

By Eq.(12), the theoretical calculation result of the cantilever structure stiffness is $24.379\ 72 \times 10^3$ N/m. The simulation result of the cantilever beam stiffness obtained by Eq.(11) is $24.452\ 87 \times 10^3$ N/m, we can see that the error between theoretical and simulation error is 0.3%, which verifies the reliability of the sensor model.

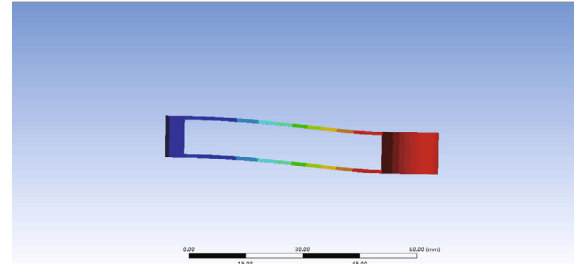
Modal analysis is commonly used in mechanical vibration to analyze properties of mechanical structures. The modal analysis module-Modal was used in the finite element software to obtain the simulation results. The sensor modal simulation results are shown in Fig.3(c). And we find that the natural frequency of the cantilever structure is 185.18 Hz.



(a) Loading calculation for cantilever stiffness



(b) Z-axis displacement of free end of cantilever beam



(c) Natural frequency simulation of cantilever beams

Fig.3 Sensor simulation results

The natural frequency of an equal strength cantilever beam sensor can be expressed as

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{BT^3 E}{6L^3 m}}, \tag{13}$$

where m is the equivalent inertial mass of the cantilever sensor.

Eq.(13) shows that the natural frequency of the cantilever sensor is related to the equivalent inertial mass. The equivalent inertial mass of the cantilever sensor is usually replaced by that of the free end mass block of the cantilever beam. However, there is a big difference between the obtained natural frequency value and that of the simulation. When the mass of the cantilever sensor is lighter and smaller, the mass of the beam must be taken into consideration. And based on the relevant principle of mechanical vibration, the equivalent inertial mass of the cantilever beam of equal strength is

$$m=0.24 \times m_l + m_k, \tag{14}$$

where m_l is the mass of the cantilever beam and m_k is the mass of the mass. According to Eqs.(13) and (14), the natural frequency of the designed cantilever beam vibration sensor is 183.97 Hz, and the error between the simulation results is only 0.66%. Therefore, when the mass quality can not be ignored, Eq.(14) should be used to convert the equivalent inertial mass and further calculate the natural frequency of the sensor.

Based on fiber-optic F-P filtering demodulation method, WaveCapture series high-speed fiber grating demodulation module and DLS light source are used in the custom demodulator to better the system demodulation. Its resolution is 1 pm.

Fig.4 shows the schematic diagram of the sensor demodulation system. A certain band of light is emitted by a broadband light source. After the fiber grating vibration frequency sensor, the reflected light of the measured vibration signal enters the tunable FP filter demodulation module to generate a vibration sensor to change the central wavelength. The demodulation module transmits the

demodulated signal to the host computer via the USB interface bus. After data analysis and processing by the upper computer, the measured parameters value of the system is obtained.

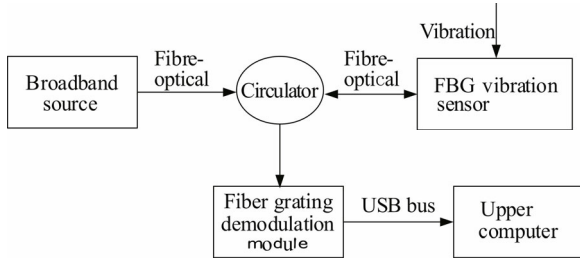
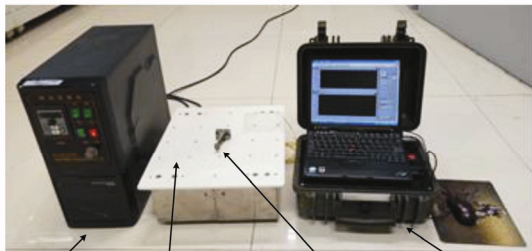


Fig.4 Schematic diagram of sensor demodulation system

The designed fiber grating vibration sensor is fixed on the vibration table to ensure that the beam surface and the vibration direction are perpendicular to each other, and the sensor connector is connected to the demodulation device, as shown in Fig.5. The center wavelength of FBG is 1 558 nm. Then, the vibration frequency is measured.



(a) FBG sensor physical map

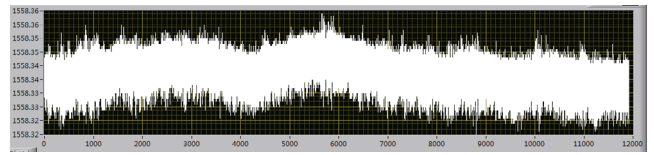


(b) Vibration experiment diagram

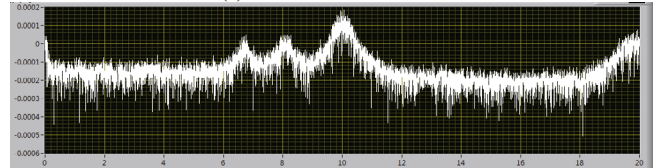
Fig.5 Vibration table experiment

Set the output frequency to 10 Hz, collect the data, and use the demodulator and “the demodulation signal processing system based on LabVIEW” to obtain the FBG vibration sensor measurement frequency of 10.052 Hz, as shown in Fig.6. It can be seen from the results in Fig.6 that temperature has an impact on the measurement results, mainly including two aspects, one is the temperature sensitivity of the FBG, and the other is the change of the double equal-strength spiral beams with temperature. However, the temperature has a very small effect on the measurement results. During the vibration process, the temperature of the cantilever beam will change stepwise, which will affect the center wavelength of the fiber grating. The method of the system to solve the temperature

cross-sensitivity problem is: Fourier processing the data through analysis and calculation software, thereby greatly reducing the influence of temperature on the measurement results.



(a) Time domain curve



(b) Frequency domain curve

Fig.6 Experimental results of vibration frequency measurement

Similarly, adjust the output frequency of the vibration table from 10 Hz to 310 Hz, the step length 20 Hz; read the frequency value measured by the corresponding FBG vibration frequency sensor. As shown in Tab.3, f_z represents the frequency value of the vibration table, f_G represents the measurement frequency value of the FBG vibration frequency sensor.

Tab.3 Vibration frequency measurement data of the vibration table

f_z (Hz)	f_G (Hz)	f_z (Hz)	f_G (Hz)	f_z (Hz)	f_G (Hz)
10	10.052	90	90.221	170	171.024
30	30.013	110	110.013	190	189.896
50	49.839	130	129.915	210	209.797
70	70.124	150	150.162	230	230.241

It can be seen that the designed double equal-strength cantilever beams FBG vibration sensor is characterized with high measurement accuracy, which can meet the requirements of high-precision measurement of vibration frequency.

Adjust the output frequency of the vibration table to maintain to 15 Hz and make it constant; adjust the vibration acceleration value 1.0 m/s² to 10.0 m/s² by using the acceleration sensor and its step length is approximately 1.0 m/s²; monitor and record the wavelength output signal of the FBG vibration sensor corresponding to the vibration acceleration using the demodulation equipment. Furthermore, the drift of FBG central wavelength of vibration sensor corresponding to different vibration acceleration can be obtained under the condition of constant vibration frequency, and the variation law between the vibration acceleration and the drift of FBG central wavelength can also be obtained.

The sensitivity of the FBG vibration sensor at a frequency

of 15 Hz was obtained by the fitting process, as shown in Fig.7.

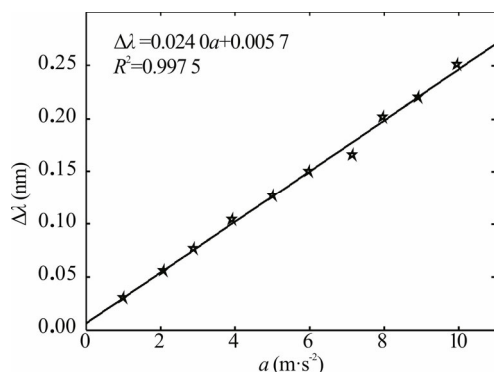


Fig.7 Linear fitting curve of sensor sensitivity at 15 Hz

It can be seen that the amount of change in the center wavelength of the FBG linearly increases as the acceleration increases. The least squares method is used to linearly fit the measurement data, and the function between the center wavelength change $\Delta\lambda$ and the acceleration a can be expressed as

$$\Delta\lambda = 0.024\ 0a + 0.005\ 7. \tag{15}$$

The linear fit is about 0.997 5. From Eq.(15), we find the sensitivity of the vibration sensor is 0.024 0 nm/(m·s⁻²), and it has better data fitting degree. Therefore, the developed FBG vibration sensor has a good sensitivity.

Following the same procedure, we measure the corresponding center wavelength drift when the corresponding center wavelength drift is at 25 Hz and 35 Hz, respectively. We take the magnitude of the acceleration value as the abscissa, the drift of the center wavelength at the frequency of 25 Hz and 35 Hz as the ordinate, and then make the corresponding numerical fitting curves, as shown in Figs.8 and 9.

The above fitting results show that the sensor sensitivities were 0.024 0 nm/(m·s⁻²), 0.023 5 nm/(m·s⁻²), and 0.024 3 nm/(m·s⁻²) at 15 Hz, 25 Hz, and 35 Hz, respectively; and the linearity is 0.997 5, 0.996 9 and 0.995 9. The average sensitivity is 0.024 0 nm/(m·s⁻²), and the sensitivity error is about 2%, which indicates this sensor has better linearity.

As one of the most important characteristics of the sensor, its natural frequency can be obtained based on its frequency response characteristics. In the experiment, we set the vibration acceleration of the vibration table to 5 m/s², and make the output frequency of the control vibration table from 5 Hz to 305 Hz with the steps of 20 Hz. In order to further measure the natural frequency of the sensor accurately, we refer to the simulation results discussed above. Select multiple frequency points near the 185 Hz position to measure and obtain the FBG center wavelength drift at different frequencies.

The frequency response characteristic curve of the sensor is shown in Fig.10. We find that the offset of the

central wavelength of the vibration sensor tend to increase when the frequency is around 150 Hz. When it reaches around 165 Hz, the offset jumps suddenly; when it is around 185 Hz, the deviation reach the maximum amount of displacement, which has a resonance effect, and in this case, the natural frequency of this sensor can be approximately 185 Hz. Since it is close to 185.18 Hz obtained by simulation analysis, the reliability of the proposed sensor is verified. According to the fitting curve, the upper limit of the frequency that the designed sensor can measure is about 250 Hz.

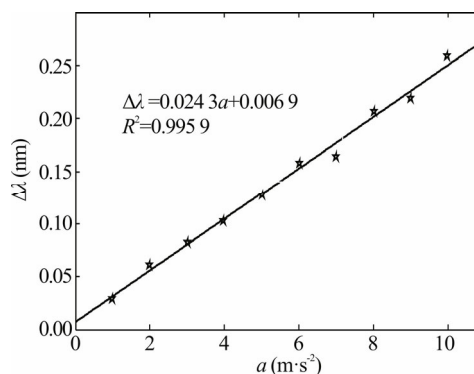


Fig.8 Linear fitting curve of sensor sensitivity at 25 Hz

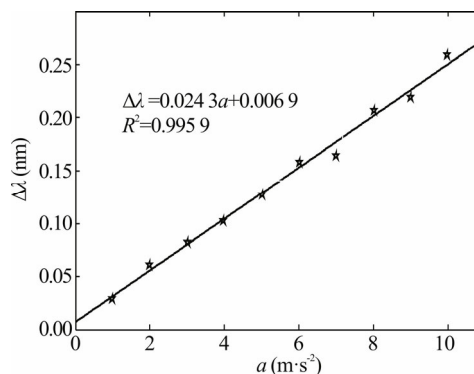


Fig.9 Linear fitting curve of sensor sensitivity at 35 Hz

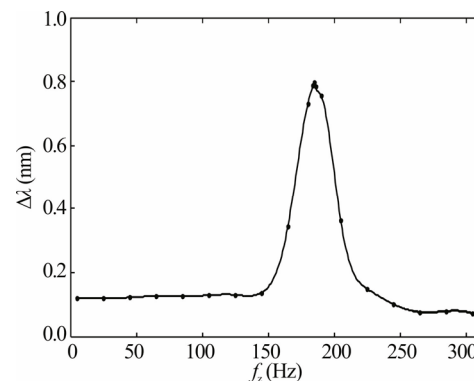


Fig.10 Fitting curve of sensor frequency response

In this paper, we design a novel double equal-strength cantilever beams FBG vibration sensor. The simulation and experimental results show that the design of FBG vibration sensor is characterized with high sensitivity,

good linearity, strong horizontal anti-interference, and high measurement accuracy of vibration frequency. Therefore, it can be utilized to measure mechanical equipment vibration frequency with higher reliability, which in turn, will guarantee its failure monitoring and fault diagnosis.

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