Evaluation of the performance of linearly frequency modulated signals generated by heterodyning two free-running laser diodes*

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A method to evaluate the influence of the laser linewidth on the linearly frequency-modulated (LFM) signals generated by heterodyning two free-running laser diodes (LDs) is proposed. The Pearson correlation coefficient between the instantaneous frequency of the generated LFM signal and that of an ideal LFM signal is introduced to quantify the quality of the generated LFM signal. The closed-form solution of the correlation coefficient is given, which shows that the correlation coefficient is determined by the ratio of the LFM signal bandwidth to the square root of the total linewidth of the two LDs when the observation interval is fixed. Simulation results are also given, which proves the correctness of the theoretical results.

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Linearly-frequency modulated (LFM) signals are widely used in radar systems to simultaneously achieve large detection range and high range resolution[1]. The conventional electrical-based generation methods are usually lack of reconfigurability and limited in operating bandwidth and frequency due to the well-known electronic bottleneck. Microwave photonics[2,3], which uses photonic methods to realize microwave functions, has some unique advantages, such as low loss, large operating bandwidth, high frequency, good reconfigurability, and immunity to electromagnetic interference. In the past few years, many photonic-based methods for LFM signal generation have been demonstrated to deal with the problems encountered in the electrical domain. One method is based on space-to-time mapping[4] or frequency-to-time mapping[5], but the time duration of the generated LFM signal is usually very short, thus limiting its application in radar systems. Another photonic method for LFM signal generation is heterodyning two optical wavelengths at a photodetector (PD). To generate two phase-locked optical wavelengths, methods based on optical external modulation[6-8] and optical frequency comb[9-11] are good solutions. However, to generate high-frequency LFM signals, the modulation indices need to be very high. If the frequency of the LFM signal reaches the terahertz band, the LFM signal is difficult to be generated because of the limitation of the modulation bandwidth of the modulator and the order of the optical sidebands that can be generated. Furthermore, some methods in Refs.[6–11] need optical filtering to select the optical wavelengths that are beaten at the PD, which makes the system complicated, costly and hard to be tuned in a large frequency range.

Directly using two individual laser diodes (LDs) with narrow linewidths to generate the two optical wavelengths is a simple and economical solution, which can simultaneously generate high-frequency signals and lower the system cost. In modern coherent radar systems, the two optical wavelengths need to be phase-locked[12-14] to achieve good phase noise performance. Nevertheless, for some radar applications, where the phase noise performance is not that important, the complicated optical phase-locked loop can be removed. The problem is now how the linewidths of the optical wavelengths influence the performance of the generated LFM signals, which needs to be further studied to guide its applications in

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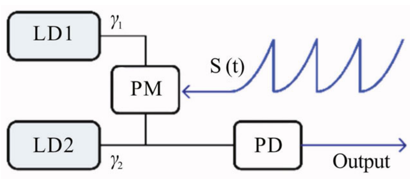
radar systems.

In this letter, the influence of the laser linewidth on the LFM signals generated by heterodyning two free-running LDs is analyzed and investigated. For the first time, the Pearson correlation coefficient between the instantaneous frequency of the generated LFM signal and that of an ideal LFM signal is introduced to quantify the quality of the generated LFM signal. The closed-form solution of the correlation coefficient is given, which shows that the correlation coefficient is determined by the ratio of the LFM signal bandwidth to the square root of the total linewidth of the two LDs when the observation interval is fixed.

Fig.1 shows the schematic diagram of the generation of LFM signals by heterodyning two free-running LDs. The two LDs have a central wavelength difference of ν_0. The optical signal from LD1 is phase modulated by a parabolic signal $S(t)$ in a phase modulator (PM), and then combined with the optical signal from LD2. The combined optical signal is detected in a PD to generate an LFM signal. Besides, to increase the time-bandwidth product, $S(t)$ can be replaced by a split parabolic signal[7].



LD: laser diode; PM: phase modulator; PD: photodetector

Fig.1 Schematic diagram of the generation of LFM signals by heterodyning two free-running LDs (γ_1 and γ_2 are the linewidths.)

Assuming the driving signal of the PM is $S(t)=V_\pi kt^2$, the generated LFM signal can be expressed as

$$E(t)\propto\cos[2\pi\nu_0 t+\pi kt^2+\phi_1(t)-\phi_2(t)], \tag{1}$$

where V_π is the half-wave voltage of the PM, $V_\pi k$ is the amplitude coefficient of the driving signal, and $\phi_1(t)$ and $\phi_2(t)$ represent the phase noise of the two LDs, respectively. If the two LDs are ideal, or the two LDs are locked in phase, an ideal LFM signal can be obtained with an instantaneous frequency of ν_0+kt. However, when the two LDs are not phase-locked, i.e., $\phi_1(t)-\phi_2(t)\neq 0$, the instantaneous frequency of the generated LFM signal can be expressed as

$$\nu(t)=\nu_0+kt+\frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\left[\phi_1(t)-\phi_2(t)\right]. \tag{2}$$

It is well-known that phase noise is the time-domain representation of linewidth. Eq.(2) indicates that the instantaneous frequency of the LFM signal is influenced by the linewidths of the two LDs if they are not phase-locked. It is known that the linewidth of the generated signal equals to the total linewidth of the two LDs[15]. Therefore, the performance of the generated LFM signal will be decreased due to the random variation in frequency caused by the laser linewidth. To evaluate the influence, the Pearson correlation coefficient R between the instantaneous frequency ν_Y of an ideal LFM signal and the instantaneous frequency ν_X of the generated LFM signal using two free-running LDs with certain linewidths is calculated, which is expressed as

$$R=\frac{\mathrm{cov}(\nu_X,\nu_Y)}{\sqrt{D(\nu_X)}\sqrt{D(\nu_Y)}}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}, \tag{3}$$

where n is the number of observations, x_i and y_i are the individual observations indexed with i, and $\bar{x}$ and $\bar{y}$ are the averages of observations of ν_X and ν_Y. ν_X and ν_Y have the following relationship

$$\nu_X=\nu_Y+\Delta\nu, \tag{4}$$

with

$$\Delta\nu=\frac{1}{2\pi}\cdot\frac{\mathrm{d}}{\mathrm{d}t}[\phi_1(t)-\phi_2(t)], \tag{5}$$

where $\Delta\nu$ is the frequency jitter of the LFM signal caused by the linewidths of the LDs, and $\Delta\nu(t)\ll\nu_Y$.

The phase jitter $\Delta\phi(t,\tau)$, i.e., the random phase change between t and $t+\tau$, is usually a zero-mean stationary Gaussian process with a variance $\sigma_{\Delta\phi}^2$[16], which can be expressed as

$$\sigma_{\Delta\phi}^2=2\pi\gamma\tau, \tag{6}$$

where γ is the full width at half maximum (*FWHM*) of the Lorentzian laser field spectrum of the LD. When the observation interval is τ, the relationship between the phase jitter and frequency jitter is

$$\Delta\nu(t,\tau)=\frac{\Delta\phi(t,\tau)}{2\pi\tau}. \tag{7}$$

Therefore, the frequency jitter $\Delta\nu(t,\tau)$ is also a zero-mean stationary Gaussian process, whose variance can be expressed as

$$\sigma_{\Delta\nu}^2=\frac{\gamma}{2\pi\tau}. \tag{8}$$

The correlation coefficient R in Eq.(3) can be simplified as

$$R=\frac{D(\nu_Y)+\mathrm{cov}(\Delta\nu,\nu_Y)}{\sqrt{D(\nu_Y)}\sqrt{D(\nu_Y)+\sigma_{\Delta\nu}^2+2\cdot\mathrm{cov}(\Delta\nu,\nu_Y)}}. \tag{9}$$

As can be seen from Eq.(9), when both $D(\nu_Y)$ and $\mathrm{cov}(\Delta\nu,\nu_Y)$ are known, the value of R can be obtained.

It is assumed that the central frequency and bandwidth of the LFM signal are f_0 and B, respectively, so $D(\nu_Y)=B^2/12$ can be directly calculated. The covariance $\mathrm{cov}(\Delta\nu,\nu_Y)$ describes the correlation between the theoretical frequency ν_Y and the frequency jitter $\Delta\nu$. It is known from Eqs.(7) and (8) that $\Delta\nu$ follows a Gaussian distribution with zero mean, and ν_Y is a deterministic signal. Therefore, ν_Y and $\Delta\nu$ are independent of each other, and the covariance $\mathrm{cov}(\Delta\nu,\nu_Y)$ is zero.

Based on the above analysis, the Pearson correlation coefficient R can be further simplified as

$$R = \frac{1}{\sqrt{1+\frac{6}{\pi}\cdot\frac{\gamma}{\tau B^2}}}. \tag{10}$$

As shown in Eq.(10), R is related to the observation interval τ, the linewidth γ, and the LFM signal bandwidth B. If the observation interval τ is fixed, the correlation coefficient R is determined by the ratio of the LFM signal bandwidth to the square root of the total linewidth of the two LDs. The correlation coefficient R decreases as the ratio decreases, which represents a decrease in correlation between the generated LFM signal and the ideal LFM signal. Therefore, the linewidths of the two LDs should be controlled in a certain range to guarantee an acceptable performance of the generated LFM signal.

Fig.2 shows the histogram of the correlation coefficient R simulated 50 000 times, when γ_1 and γ_2 are both set to 25 MHz and the generated LFM signals have a frequency sweeping range from 20 GHz to 21 GHz in 500 ns. The sampling rate of the system is 100 GHz and the observation interval τ is set to 100 ps. The solid and dashed lines in Fig.2 are the Gaussian fitting curve and the Lorentz fitting curve of this histogram, respectively. In the following content, the correlation coefficient is the average of multiple simulation results.

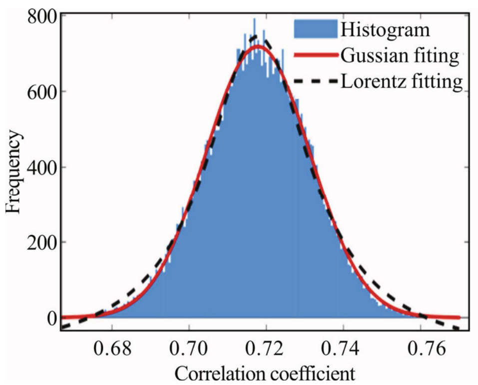


Fig.2 Histogram of the correlation coefficient and fitting curves

The influence of the total laser linewidth on the generated LFM signal is studied. The generated LFM signal has a central frequency of 15 GHz, a frequency sweeping range from 10 GHz to 20 GHz, and a chirp rate of 1 GHz/ns. As shown in Fig.3, the instantaneous frequency of the LFM signal is ideal when the total linewidth γ equals to 0. The corresponding pulse compression performance is also demonstrated. An ultra-narrow pulse is obtained, which can achieve very high resolution. When the linewidth γ increases to 20 MHz, the instantaneous frequency of the LFM signal has a small deterioration compared with the case with 0 linewidth. The correlation between the generated LFM signal and the ideal LFM signal is also calculated, and a small performance degradation is observed. If the linewidth γ increases to 0.2 GHz, we can roughly observe the linear frequency change. It can also be seen that side modes with much higher amplitude are generated in the correlation, which influences the resolution of the LFM signal. When the linewidth γ reaches to 2 GHz, it is difficult to distinguish the frequency variation characteristics of the signal, and the compressed pulse has many high-amplitude side modes, leading to a very wide *FWHM* and greatly affecting the performance of the radar system. The Pearson correlation coefficients of the four cases are 1, 0.996, 0.963, and 0.748, respectively.

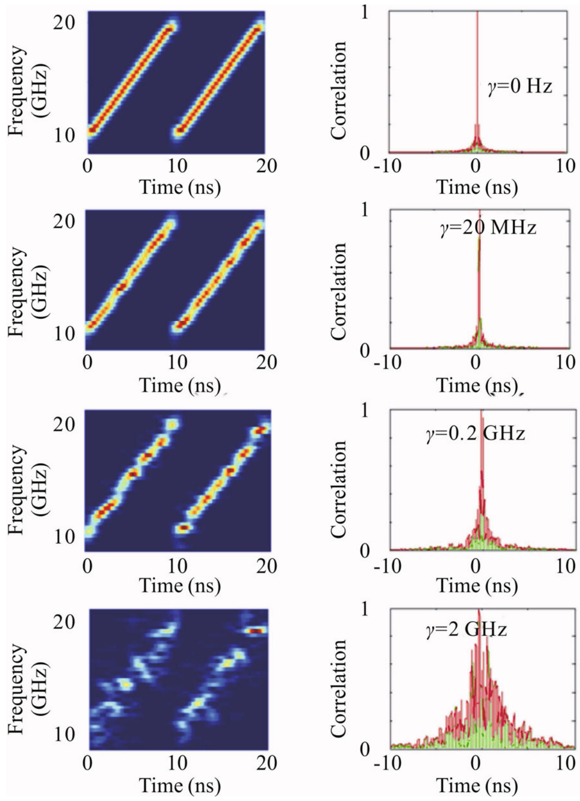


Fig.3 The instantaneous frequencies of the generated LFM signal, and the correlation between the generated LFM signal and the ideal LFM signal when the total linewidth is 0, 20 MHz, 0.2 GHz, and 2 GHz, respectively

The influence of the observation interval τ on the correlation coefficient is investigated. In this study, the LFM signal has a bandwidth of 10 GHz and a frequency sweeping range from 20 GHz to 30 GHz in 500 ns. Fig.4 gives the curves of the simulated correlation coefficients under different observation intervals when the total linewidth γ sweeps from 1 MHz to 100 GHz. It can be seen that when the observation interval τ increases from 50 ps to 200 ps, the correlation coefficient R also increases. When the linewidth is small, within 100 MHz, the correlation coefficients are very close to 1. As the linewidth increases, the three curves shown in Fig.4 begin to separate. When the linewidth is very large, these curves approach zero. The simulated curves show good agreement with the curves calculated by Eq.(10) when the linewidth γ is less than 5 GHz. In comparison, when the linewidth is greater than 5 GHz, the simulation results and the calculation curves are not consistent with each other. Here, the frequency range occupied by the LFM signal is 20 GHz to 30 GHz. If the linewidth γ is too large, a large part of the Lorentzian-shaped phase noise power spectrum of the generated LFM signal falls into the frequency range less than 0, which will affect the accuracy of the model we built in Eq.(10). In practice, the linewidth of the commercially available LDs can be less than several MHz, and even less than several kHz, so the Lorentzian-shaped phase noise power spectrum of the generated LFM signal can be considered almost all located in the frequency range greater than 0. The model we built in Eq.(10) is very accurate when the linewidth is small.

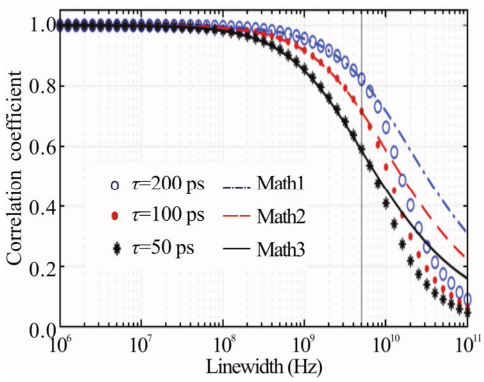


Fig.4 Correlation coefficient versus the total linewidth under different observation intervals

Then the influence of the bandwidth of the LFM signal on the correlation coefficient under different laser linewidths is studied with the results shown in Fig.5. The four LFM signals all have a time duration of 500 ns, and a starting frequency of 20 GHz. The stop frequencies of the four LFM signals are 22 GHz, 25 GHz, 30 GHz, and 40 GHz, which correspond to bandwidths of 2 GHz, 5 GHz, 10 GHz, and 20 GHz. In this study, the observation interval τ is set to 100 ps. It can be seen from Fig.5 that when the linewidth γ is fixed, the correlation coefficient R increases with the increase of the bandwidth of the LFM signal, which means the LFM signal with larger bandwidth is more tolerant of the influence from the laser linewidth. When the linewidth γ is less than 5 GHz, the simulated curves agree well with the theoretical lines calculated by Eq.(10). When the linewidth γ is greater than 5 GHz, the simulation results and the calculation curves are not consistent with each other. The reason is the same as we discussed in Fig.4.

From the above results, we know that the correlation coefficient is related to the observation interval τ, the bandwidth of the LFM signal B, and the total linewidth of the two LDs γ. As shown in Eq.(10), the correlation coefficient is determined by $B/\sqrt{\gamma}$ if τ is fixed. Fig.6 shows the correlation coefficients of the LFM signals when $B/\sqrt{\gamma}$ changes under different linewidth γ. The observation interval τ is set to 100 ps. The linewidths γ of the four cases are 1 kHz, 100 kHz, 10 MHz, and 1 GHz, and the starting frequencies of the four cases are all 20 GHz. The fifth curve is a theoretical curve obtained from Eq.(10). As can be seen, the five curves are consistent with each other, and the correlation coefficients are the same as long as the ratio $B/\sqrt{\gamma}$ is fixed, which means the quality of the generated LFM signal can be determined by $B/\sqrt{\gamma}$. For example, when $B/\sqrt{\gamma}$ equals to 1.9×10^5 $\mathrm{Hz}^{1/2}$, the correlation coefficients of the four simulation results and the theoretical result are all 0.8. With different linewidths, the bandwidths are 5.8 MHz, 58 MHz, 0.58 GHz, and 5.8 GHz, respectively. Therefore, in practical applications, the total linewidth of the lasers to be used and the bandwidth of the LFM signal can be jointly determined according to the required signal performance, i.e., the correlation coefficient.

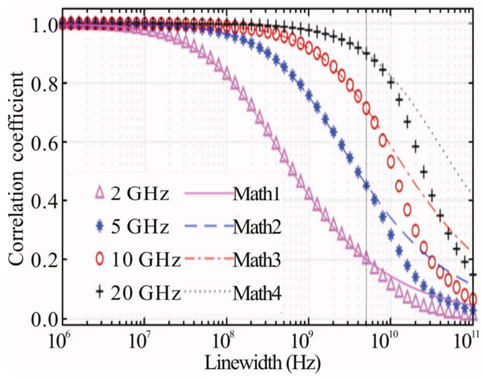


Fig.5 Correlation coefficient versus the total linewidth under different signal bandwidths

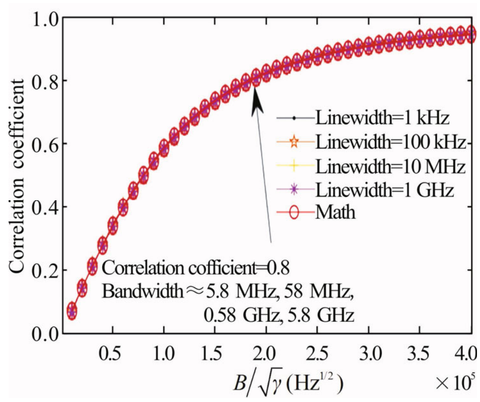


Fig.6 Correlation coefficient versus $B/\sqrt{\gamma}$ under different total linewidths

In conclusion, the influence of the laser linewidth on the LFM signals generated by heterodyning two free-running LDs is analyzed and investigated. The key significance of the work is that, for the first time, the performance of the LFM signal is evaluated by the Pearson correlation coefficient between the instantaneous frequency of the generated LFM signal and that of an ideal LFM signal. The closed-form solution of the correlation coefficient is given, which shows that the correlation coefficient is determined by the ratio of the LFM signal bandwidth to the square root of the total linewidth of the two LDs when the observation interval is fixed. The theory is verified by the simulation results. This work provides a solution to evaluate the performance of the LFM signals generated by heterodyning two free-running LDs, which can guide its application in radar systems.

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