Performance analysis of multi-hop parallel FSO system over double generalized gamma distribution considering two transmission beams*

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The performances of multi-hop parallel free space optical (FSO) communication system are investigated over double generalized gamma (double GG) distribution for plane and spherical waves considering path loss and pointing errors (PE). Specifically, the closed-form expressions of outage probability and average bit error rate (*ABER*) are derived with Meijer-G function and further confirmed by Monte Carlo (MC) simulation. Subsequently, the outage performances of this system are analyzed in detail with the influence of PE, turbulence strengths, structure parameters and weather conditions for plane and spherical waves. Moreover, cyclic coding is used in this work to further optimize system performance.

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In recent years, free space optical (FSO) communication has drawn considerable researchers' attention on account of its various unique features and advantages, such as large bandwidth, license-free spectrum, high date rate, low cost and inherent security, and thus it has been regarded as a promising optical wireless communication (OWC) technique in many fields[1,2]. Despite the outstanding features of FSO communication, the performance and availability of FSO links can be disrupted by atmospheric turbulence-induced fading, path loss and pointing errors (PE)[3]. To address these shortcomings, a lot of techniques have been introduced into FSO communication, including multiple-input multiple-output (MIMO) scheme[4-7] and relay-assisted transmission scheme[6,8,9]. In Ref.[4], the outage probability and the average bit error rate (*ABER*) of MIMO FSO system with maximal ratio combining (MRC) diversity technique over Gamma-Gamma (G-G) fading channels with generalized pointing errors were studied. The results showed that the MIMO FSO system performance with MRC scheme can be improved with the increase of the receiver aperture size. On the other hand, relay-assisted transmission scheme was first introduced into FSO communication in Ref.[10]. Moreover, this scheme has been proved that it is an effective method to extend coverage and mitigate the effects of fading[10]. Recently, a serial-parallel combined relay orthogonal frequency division multiplexing (OFDM) FSO communication system in which the relay protocol of the system is decode and forward (DF) was proposed over M distribution model[11]. Actually, the serial-parallel combined relay is a combination of serial (i.e., multi-hop transmission) and parallel relaying (i.e., cooperative diversity), also known as multi-hop parallel relaying, which is more suitable for practical applications[11,12].

In addition, many statistical channel models have been presented to characterize the probability density function (PDF) of atmospheric turbulence-induced fading. Among them, a unifying mathematical model named double generalized gamma (double GG) has been proposed[13]. This study showed that this novel model could demonstrate an excellent match to the simulation data over all the turbulence regimes. And it is further found that double GG model is clearly superior over previous models and is very generic since it contains some commonly-used fading models as special cases[13]. Some works over this new model has been done considering several effective mitigation techniques, such as multi-hop scheme[3], MIMO scheme[14], spatial diversity[15] and hybrid RF/FSO link[9,16]. In Ref.[3], the *ABER* performances of multi-hop FSO system considering the existence of path loss and PE over double GG channel model were studied for plane and spherical waves. However, no work has been reported on the performance of multi-hop parallel FSO system over double GG distributed turbulence channel considering the combined influence of turbulence-induced fading, PE and path loss for plane and spherical waves yet, to the best of our knowledge.

Our contribution in this work is summarized as follows:

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A multi-hop parallel FSO system is investigated over the unifying double GG fading channel with the influence of path loss and PE. Then, the closed-form expressions of outage probability and *ABER* are derived with the help of Meijer-G function and verified by Monte Carlo (MC) simulation. And the outage performances are analyzed in detail and the *ABER* performances are presented under different weather conditions for plane and spherical waves. Besides, the simple and effective cyclic coding is used to further improve *ABER* performance.

Fig.1 shows the structure of the multi-hop parallel FSO communication system. DF relaying and binary phase shift keying subcarrier intensity modulation (BPSK-SIM) are employed in this FSO system. The source node transmits the data bits to the destination node in the chosen cooperative path which is decided by the max–min path selection criterion[17]. Based on DF relaying method, only one relay is allowed to decode and retransmit the signal to the next node at one time. In addition, each of links are assumed as identically and independently distributed (i.i.d.)[3]. Therefore, the received electrical signal at the *j*th hop in the *i*th path can be expressed as

$$r_{i,j}=\xi I_{i,j}s_{i,j}+n_{i,j}, \tag{1}$$

where i=(1,…,M), j=(1,…,N). M and N are defined as structure parameters, ξ represents the detector responsivity, $I_{i,j}$ is the aggregated channel fading coefficient which is defined as $I_{i,j}=I_{i,j}^{l}I_{i,j}^{a}I_{i,j}^{p}$ in this work, where $I_{i,j}^{l}$ refers to path loss, $I_{i,j}^{a}$ is atmospheric turbulence-induced fading and $I_{i,j}^{p}$ is the fading due to PE. $s_{i,j}$ is the BPSK modulated transmitted signal with average power $P_{i,j}$ and $n_{i,j}$ is the signal-independent zero mean additive white Gaussian noise (AWGN) with variance σ_n^2 in point-to-point (PP) link. Considering intensity modulation with direct detection (IM/DD) technique, the instantaneous electrical signal-to-noise ratio (*SNR*) $\mu_{i,j}$ at the input of the optical receiver in each PP link can be given as[18]

$$\mu_{i,j}=\xi^2P_{i,j}^2I_{i,j}^2/2\sigma_n^2=\bar{\mu}_{i,j}I_{i,j}^2, \tag{2}$$

where $\bar{\mu}_{i,j}$ is defined as the average electrical *SNR*.

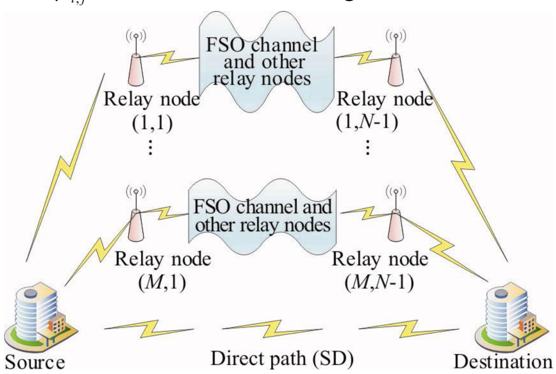


Fig.1 Structure of the multi-hop parallel FSO communication system

In this work, double GG distribution is used to describe the channel fading induced by atmospheric turbulence and the PDF of $I_{i,j}^{a}$ can be expressed as[13]

$$f_{I_{i,j}^{a}}\left(I_{i,j}^{a}\right)=\frac{k_{2_{i,j}}\alpha_{i,j}\beta_{i,j}^{\varphi_{1_{i,j}}-1/2}\alpha_{i,j}^{\varphi_{2_{i,j}}-1/2}\left(2\pi\right)^{1-\left(\alpha_{i,j}+\beta_{i,j}\right)/2}}{I_{i,j}^{a}\Gamma\left(\varphi_{1_{i,j}}\right)\Gamma\left(\varphi_{2_{i,j}}\right)}\times$$
$$G_{t_{i,j},0}^{0,t_{i,j}}\left[\frac{\alpha_{i,j}^{\alpha_{i,j}}\beta_{i,j}^{\beta_{i,j}}\Omega_{1_{i,j}}^{\beta_{i,j}}\Omega_{2_{i,j}}^{\alpha_{i,j}}}{{I_{i,j}^{a}}^{k_{2_{i,j}}\alpha_{i,j}}\varphi_{1_{i,j}}^{\beta_{i,j}}\varphi_{2_{i,j}}^{\alpha_{i,j}}}\middle|\begin{matrix}Q_{0_{i,j}}\\-\end{matrix}\right], \tag{3}$$

where $Q_{0_{i,j}}=\Delta\left(\beta_{i,j}:1-\varphi_{1_{i,j}}\right),\Delta\left(\alpha_{i,j}:1-\varphi_{2_{i,j}}\right)$, $\Gamma(\cdot)$ refers to the gamma function, $\varphi_{1_{i,j}}$ and $\varphi_{2_{i,j}}$ stand for shape parameters, k_1, k_2, Ω_1, Ω_2 refer to scale parameters, $\Delta(m, n)=n/m, (n+1)/m, \ldots, (n+m-1)/m$, $t_{i,j}=\alpha_{i,j}+\alpha_{i,j}$, $\alpha_{i,j}$ and $\alpha_{i,j}$ are positive integers which satisfy $\alpha_{i,j}/\beta_{i,j}=k_{1_{i,j}}/k_{2_{i,j}}$. $G_{p,q}^{m,n}[\cdot]$ is the Meijer-G function. For $k_i\to0$, $\varphi_i\to\infty$, the double GG distribution reduces to the LN model. For k_i=1, Ω_i=1, it coincides with GG model. For φ_i=1, it becomes Double-Weibull while for k_i=1, Ω_i=1, φ_2=1 it coincides with the K model[13].

According to the Beer-Lambert's Law, the path loss is conclusive and can be described as[19]

$$I_{i,j}^{l}=\exp\left(-\zeta z_{i,j}\right), \tag{4}$$

where ζ is the fixed attenuation coefficient for diverse weather scenarios at a wavelength of 1 550 nm, $z_{i,j}$ is the transmission distance of each link. Parameters of attenuation coefficient are selected from Ref.[19].

In terms of the effect of PE, the mathematical model is utilized in this work wherein the influence of beam-width, detector size and jitter variance are all taken into consideration, which is given by[16]

$$f_{I_{i,j}^{p}}\left(I_{i,j}^{p}\right)=\frac{\delta_{i,j}^2}{T_{0_{i,j}}^{\delta_{i,j}^2}}{I_{i,j}^{p}}^{\delta_{i,j}^2-1}\quad,0\le I_{i,j}^{p}\le T_{0_{i,j}}, \tag{5}$$

where $T_{0_{i,j}}=erf^2\left(\eta_{i,j}\right)$ is the fraction of the collected power when the instantaneous radial displacement between the beam centroid and the detector center equals zero, $\eta_{i,j}=\sqrt{\pi/2}b_{i,j}/w_{Z_{i,j}}$, $erf(\cdot)$ stands for the error function, $b_{i,j}$ refers to the aperture radius and $w_{z_{i,j}}$ represents the beam-width at the distance of $z_{i,j}$. Moreover, $\delta_{i,j}=w_{Zeq_{i,j}}/2\sigma_{s_{i,j}}$ is a measure of the severity of the PE effect, $\sigma_{s_{i,j}}$ is the jitter standard deviation, $w_{Zeq_{i,j}}^2=w_{Z_{i,j}}^2\sqrt{\pi}erf\left(\eta_{i,j}\right)\Big/\left(2\eta_{i,j}\exp\left(-\eta_{i,j}^2\right)\right)$.

On the basis of the previous analysis, the PDF of the aggregated channel fading coefficient can be calculated as[20]

$$f_{I_{i,j}}\left(I_{i,j}\right)=\int_0^{T_{0_{i,j}}}f_{I_{i,j}|I_{i,j}^{p}}\left(I_{i,j}\mid I_{i,j}^{p}\right)f_{I_{i,j}^{p}}\left(I_{i,j}^{p}\right)\mathrm{d}I_{i,j}^{p}=$$
$$\int_0^{T_{0_{i,j}}}\frac{f_{I_{i,j}^{a}}(I_{i,j}/I_{i,j}^{p}I_{i,j}^{l})}{I_{i,j}^{p}I_{i,j}^{l}}f_{I_{i,j}^{p}}\left(I_{i,j}^{p}\right)\mathrm{d}I_{i,j}^{p}=$$
$$\frac{\delta_{i,j}^2\alpha_{i,j}^{\varphi_{2_{i,j}}-1/2}\beta_{i,j}^{\varphi_{1_{i,j}}-1/2}\left(2\pi\right)^{1-\left(\alpha_{i,j}+\beta_{i,j}\right)/2}}{I_{i,j}\Gamma\left(\varphi_{1_{i,j}}\right)\Gamma\left(\varphi_{2_{i,j}}\right)}\times$$

$$G_{t_{i,j}+k_{2_{i,j}}\alpha_{i,j},k_{2_{i,j}}\alpha_{i,j}}^{0,\ t_{i,j}+k_{2_{i,j}}\alpha_{i,j}}\left[\left(\frac{T_{0_{i,j}}I_{i,j}^{l}}{I_{i,j}}\right)^{k_{2_{i,j}}\alpha_{i,j}}\times\right.$$
$$\left.\frac{\alpha_{i,j}^{\alpha_{i,j}}\beta_{i,j}^{\beta_{i,j}}\Omega_{1_{i,j}}^{\beta_{i,j}}\Omega_{2_{i,j}}^{\alpha_{i,j}}}{\varphi_{1_{i,j}}^{\beta_{i,j}}\varphi_{2_{i,j}}^{\alpha_{i,j}}}\middle|\begin{matrix}\Delta\left(k_{2_{i,j}}\alpha_{i,j}:1-\delta_{i,j}^2\right),Q_{0_{i,j}}\\ \Delta\left(k_{2_{i,j}}\alpha_{i,j}:-\delta_{i,j}^2\right)\end{matrix}\right]. \tag{6}$$

Thereby, with the aid of Ref.[20] along with the relationship of $f_{\mu_{i,j}}(\mu_{i,j})=f_{I_{i,j}}(\sqrt{\mu_{i,j}/\bar{\mu}_{i,j}})/2\sqrt{\mu_{i,j}\bar{\mu}_{i,j}}$, Eq.(6) can be simplified and the PDF of $\mu_{i,j}$ can be obtained as

$$f_{\mu_{i,j}}\left(\mu_{i,j}\right)=\frac{\delta_{i,j}^2\alpha_{i,j}^{\varphi_{2_{i,j}}-1/2}\beta_{i,j}^{\varphi_{1_{i,j}}-1/2}\left(2\pi\right)^{1-\left(\alpha_{i,j}+\beta_{i,j}\right)/2}}{2\mu_{i,j}\Gamma\left(\varphi_{1_{i,j}}\right)\Gamma\left(\varphi_{2_{i,j}}\right)}\times$$
$$G_{1,t_{i,j}+1}^{t_{i,j}+1,0}\left[\frac{\varphi_{1_{i,j}}^{\beta_{i,j}}\varphi_{2_{i,j}}^{\alpha_{i,j}}\left(T_{0_{i,j}}I_{i,j}^{l}\right)^{-k_{2_{i,j}}\alpha_{i,j}}}{\alpha_{i,j}^{\alpha_{i,j}}\beta_{i,j}^{\beta_{i,j}}\Omega_{1_{i,j}}^{\beta_{i,j}}\Omega_{2_{i,j}}^{\alpha_{i,j}}}\times\right.$$
$$\left.\left(\frac{\mu_{i,j}}{\bar{\mu}_{i,j}}\right)^{\frac{k_{2_{i,j}}\alpha_{i,j}}{2}}\middle|\begin{matrix}1+\dfrac{\delta_{i,j}^2}{k_{2_{i,j}}\alpha_{i,j}}\\ \dfrac{\delta_{i,j}^2}{k_{2_{i,j}}\alpha_{i,j}},1-Q_{0_{i,j}}\end{matrix}\right]. \tag{7}$$

Hence, the cumulative distribution function (CDF) of $\mu_{i,j}$ can be calculated by[20]

$$F_{\mu_{i,j}}\left(\mu_{i,j}\right)=\int_0^{\mu_{i,j}}f_{\mu_{i,j}}\left(\mu_{i,j}\right)\mathrm{d}\mu_{i,j}=$$
$$\frac{\delta_{i,j}^2\alpha_{i,j}^{\varphi_{2_{i,j}}-1/2}\beta_{i,j}^{\varphi_{1_{i,j}}-1/2}\left(2\pi\right)^{1-\left(\alpha_{i,j}+\beta_{i,j}\right)}2^{\left(\varphi_{1_{i,j}}+\varphi_{2_{i,j}}-2\right)}}{\Gamma\left(\varphi_{1_{i,j}}\right)\Gamma\left(\varphi_{2_{i,j}}\right)k_{2_{i,j}}\alpha_{i,j}}\times$$
$$G_{3,2t_{i,j}+3}^{2t_{i,j}+2,1}\left[\left(\frac{\varphi_{1_{i,j}}^{\beta_{i,j}}\varphi_{2_{i,j}}^{\alpha_{i,j}}\left(T_{0_{i,j}}I_{i,j}^{l}\right)^{-k_{2_{i,j}}\alpha_{i,j}}2^{-(\alpha_{i,j}+\beta_{i,j})}}{\alpha_{i,j}^{\alpha_{i,j}}\beta_{i,j}^{\beta_{i,j}}\Omega_{1_{i,j}}^{\beta_{i,j}}\Omega_{2_{i,j}}^{\alpha_{i,j}}}\right)^2\times\right.$$
$$\left.\left(\frac{\mu_{i,j}}{\bar{\mu}_{i,j}}\right)^{k_{2_{i,j}}\alpha_{i,j}}\middle|\begin{matrix}1,\Delta\left(2:\dfrac{k_{2_{i,j}}\alpha_{i,j}+\delta_{i,j}^2}{k_{2_{i,j}}\alpha_{i,j}}\right)\\ \Delta\left(2:\dfrac{\delta_{i,j}^2}{k_{2_{i,j}}\alpha_{i,j}},1-Q_{0_{i,j}}\right),0\end{matrix}\right]. \tag{8}$$

In this system, the max-min path selection criterion is utilized, which means that the best relay is selected based on the highest value of the minimum of source-to-relay, relay-to-relay, and relay-to-destination *SNR*[17]. And it is assumed that all the links have the same average *SNR*, i.e., $\bar{\mu}_{i,j}=\bar{\mu}_{s,d}=\bar{\mu}$. Here, $\bar{\mu}_{s,d}$ is the average *SNR* of the direct path. Considering that all the links are statistically independent of each other, the CDF of the equivalent *SNR* of this system μ_{equ} can be achieved as[21]

$$F_{\mu_{\mathrm{equ}}}\left(\mu\right)=F_{\mu_{s,d}}\left(\mu_{s,d}\right)\times\prod_{i=1}^{M}\left[1-\prod_{j=1}^{N}\left(1-F_{\mu_{i,j}}\left(\mu_{i,j}\right)\right)\right], \tag{9}$$

where $F_{\mu_{s,d}}\left(\mu_{s,d}\right)$ is the CDF of the instantaneous SNR of the SD link $\mu_{s,d}$ and $F_{\mu_{s,d}}\left(\mu_{s,d}\right)=F_{\mu_{i,j}}\left(\mu_{s,d}\right)$.

As is known, outage probability is defined as the probability that the end-to-end output *SNR* falls below a specified threshold μ_{th}[8]. Thus, with the help of Eqs.(2), (8) and (9), the analytical expression for outage probability of the multi-hop parallel FSO system can be achieved as

$$P_{\mathrm{out}}=P\left(\mu_{\mathrm{equ}}<\mu_{\mathrm{th}}\right)=P\left(\mu\le\sqrt{\mu_{\mathrm{th}}/\mu_{\mathrm{equ}}}\right)=$$
$$F_{I_{\mathrm{equ}}}\left(\sqrt{\mu_{\mathrm{th}}/\mu_{\mathrm{equ}}}\right)=\Psi_{s,d}\times G_{3,2t_{s,d}+3}^{2t_{s,d}+2,1}\left[\mu_n^{-k_{2_{s,d}}\alpha_{s,d}}\times\right.$$
$$\left.Z_{s,d}\middle|\begin{matrix}1,\Delta\left(2:\dfrac{k_{2_{s,d}}\alpha_{s,d}+\delta_{s,d}^2}{k_{2_{s,d}}\alpha_{s,d}}\right)\\ \Delta\left(2:\dfrac{\delta_{s,d}^2}{k_{2_{s,d}}\alpha_{s,d}},1-Q_{0_{s,d}}\right),0\end{matrix}\right]\times$$
$$\prod_{i=1}^{M}\left[1-\prod_{j=1}^{N}\left(1-\Psi_{i,j}\times G_{3,2t_{i,j}+3}^{2t_{i,j}+2,1}\left[Z_{i,j}\times\right.\right.\right.$$
$$\left.\left.\left.\mu_n^{-k_{2_{i,j}}\alpha_{i,j}}\middle|\begin{matrix}1,\Delta\left(2:\dfrac{k_{2_{i,j}}\alpha_{i,j}+\delta_{i,j}^2}{k_{2_{i,j}}\alpha_{i,j}}\right)\\ \Delta\left(2:\dfrac{\delta_{i,j}^2}{k_{2_{i,j}}\alpha_{i,j}},1-Q_{0_{i,j}}\right),0\end{matrix}\right]\right)\right], \tag{10}$$

where $\mu_{\mathrm{n}}=\mu_{\mathrm{equ}}/\mu_{\mathrm{th}}$ is the normalized *SNR*, $F_{I_{\mathrm{equ}}}\left(I\right)$ is the CDF of end-to-end light intensity I_{equ} in this FSO system,

$$\Psi_{s,d}=\frac{\delta_{s,d}^2\alpha_{s,d}^{\varphi_{2_{s,d}}-1/2}\beta_{s,d}^{\varphi_{1_{s,d}}-1/2}\left(2\pi\right)^{1-\left(\alpha_{s,d}+\beta_{s,d}\right)}2^{\left(\varphi_{1_{s,d}}+\varphi_{2_{s,d}}-2\right)}}{\Gamma\left(\varphi_{1_{s,d}}\right)\Gamma\left(\varphi_{2_{s,d}}\right)k_{2_{s,d}}\alpha_{s,d}}, \tag{11}$$

$$Z_{s,d}=\left(\frac{\varphi_{1_{s,d}}^{\beta_{s,d}}\varphi_{2_{s,d}}^{\alpha_{s,d}}\left(T_{0_{s,d}}I_{s,d}^{l}\right)^{-k_{2_{s,d}}\alpha_{s,d}}2^{-(\alpha_{s,d}+\beta_{s,d})}}{\alpha_{s,d}^{\alpha_{s,d}}\beta_{s,d}^{\beta_{s,d}}\Omega_{1_{s,d}}^{\beta_{s,d}}\Omega_{2_{s,d}}^{\alpha_{s,d}}}\right)^2, \tag{12}$$

$$\Psi_{i,j}=\frac{\delta_{i,j}^2\alpha_{i,j}^{\varphi_{2_{i,j}}-1/2}\beta_{i,j}^{\varphi_{1_{i,j}}-1/2}\left(2\pi\right)^{1-\left(\alpha_{i,j}+\beta_{i,j}\right)}2^{\left(\varphi_{1_{i,j}}+\varphi_{2_{i,j}}-2\right)}}{\Gamma\left(\varphi_{1_{i,j}}\right)\Gamma\left(\varphi_{2_{i,j}}\right)k_{2_{i,j}}\alpha_{i,j}}, \tag{13}$$

$$Z_{i,j}=\left(\frac{\varphi_{1_{i,j}}^{\beta_{i,j}}\varphi_{2_{i,j}}^{\alpha_{i,j}}\left(T_{0_{i,j}}I_{i,j}^{l}\right)^{-k_{2_{i,j}}\alpha_{i,j}}2^{-(\alpha_{i,j}+\beta_{i,j})}}{\alpha_{i,j}^{\alpha_{i,j}}\beta_{i,j}^{\beta_{i,j}}\Omega_{1_{i,j}}^{\beta_{i,j}}\Omega_{2_{i,j}}^{\alpha_{i,j}}}\right)^2. \tag{14}$$

For BPSK-SIM, the *ABER* of the multi-hop parallel system can be can be given as[17]

$$P_{\mathrm{e}}=\int_0^{\infty}\frac{1}{2\sqrt{\pi}}\mu^{-1/2}\mathrm{e}^{-\mu}F_{\mu_{\mathrm{equ}}}(\mu)\mathrm{d}\mu. \tag{15}$$

With the help of Gauss-Laguerre quadrature (GLQ) rule[22], the *ABER* over double GG aggregated fading channels considering PE and path loss can be approximately derived as

$$P_{\mathrm{e}}\approx\frac{1}{2\sqrt{\pi}}\sum_{m=1}^{n}\Phi_m F_{\mu_{\mathrm{equ}}}(\chi_m), \tag{16}$$

where χ_m is the *m*th root of the GLQ polynomial $L_n^{-1/2}\left(\chi\right)$, and the corresponding weight is given as

$$\Phi_m=\frac{\Gamma\left(n+1/2\right)\chi_m}{n!\left(n+1\right)^2\left[L_n^{-1/2}\left(\chi_m\right)\right]^2}. \tag{17}$$

The outage and *ABER* performances are evaluated for two transmission beams in detail on the basis of Eqs.(10) and (16). MC simulation is adopted to confirm the correctness of the derived theoretical expression wherein the acceptance rejection method is used to generate random values obeying double GG distribution and the inverse transform method is adopted to generate random values from PE. The distance in each link is fixed to 1 km. Parameters of double GG distribution are selected from Ref.[13]. Besides, the (15, 5) cyclic coding is utilized in this part to further improve the FSO system performance.

Fig.2 presents the outage performances of the FSO system over double GG distribution for different degrees of turbulence severity and normalized jitters for two waves. It is obviously seen that the theoretical curves have an excellent fit to MC simulation data, which reflects the correctness of the achieved outage probability expressions. Then, as can be seen, the outage probability is increasing with the increase of turbulence severity for both waves and it will increase sharply if considering PE. Besides, the outage performance degrades with the increase of the normalized jitter σ_s/a at a fixed normalized beam-width w_z/a for these two waves. This is because the impact of severity of PE will get stronger with the increasing normalized jitter. Furthermore, the superiority of double GG is more obvious for spherical wave. Giving an example, at a fixed *SNR* of 54 dB without PE under strong turbulence condition, the outage probability for plane wave is 3.93×10^{-6}, while it is 2.45×10^{-6} for spherical wave.

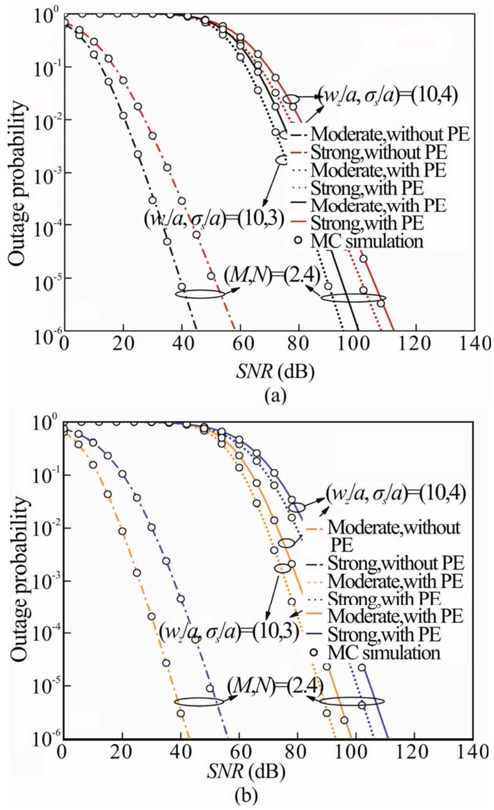


Fig.2 The outage probability versus *SNR* of FSO system under different turbulence strength and normalized jitters for (a) plane wave and (b) spherical wave

The outage performances of the studied FSO system for two waves are illustrated in Fig.3 under moderate turbulence condition. As is observed, the increasing number of parallel paths *M* at a fixed *N* can improve the outage performance for both two waves. Additionally, for these two waves, the variation of structure parameters has a more apparent impact on the scenario of considering PE. That is to say, setting structure parameters reasonably is an effective technique to improve outage performance of FSO system. For example, at a targeted outage probability of 10^{-6} for plane wave propagation, the required *SNR* for (M, N)=(2, 8) without PE is about 48.5 dB while it is approximately 27.2 dB for (M, N)=(5, 3). It is obviously found that the latter one can save about 21.3 dB *SNR* gain. At given values of w_z/a=10 and σ_s/a=4, the needed *SNR* for (M, N)=(2, 8) with PE is approximately 103.9 dB while it is about 81.3 dB for (M, N)=(5, 3). Thus, the latter one can save about 22.6 dB *SNR* gain, which is smaller than 27.2 dB.

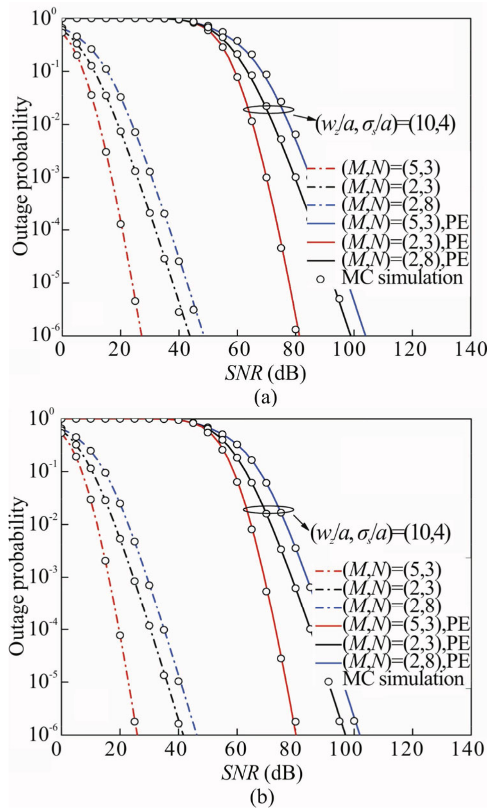


Fig.3 The outage probability versus *SNR* of FSO system under moderate turbulence condition and different structure parameters for (a) plane wave and (b) spherical wave

Fig.4 shows the outage performances of this FSO system under different weather conditions for plane and spherical waves, respectively. Specifically, the value of outage probability is the least under very clear condition and increases sharply in the presence of fog at a given *SNR* with the same structure parameters for two waves. In addition, the outage probability for spherical wave is slightly smaller than that of plane wave under the same transmission condition. For instance, when *SNR* is 72 dB, the outage probability for plane wave under drizzle weather considering PE is 1.7×10^{-3} while it is 1.2×10^{-3} for spherical wave.

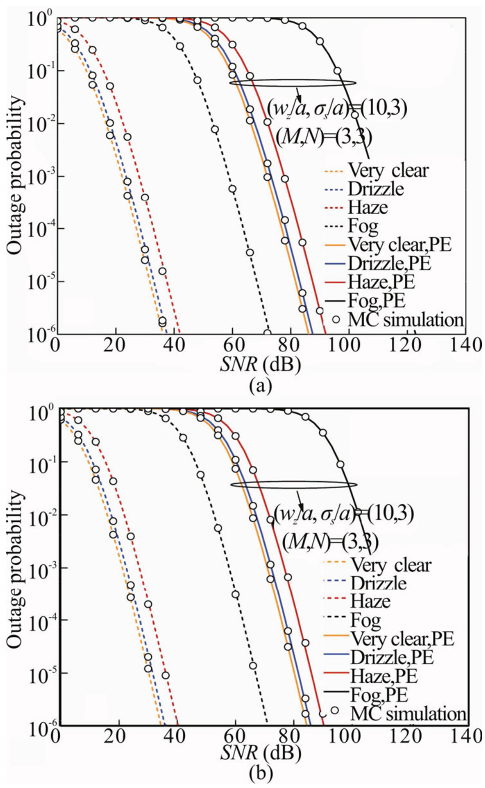
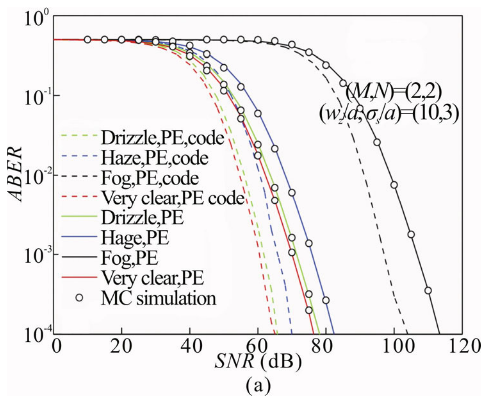


Fig.4 The outage probability versus *SNR* of FSO system considering path loss for (a) plane wave and (b) spherical wave

Fig.5 plots the *ABER*s of multi-hop parallel FSO system against *SNR* over aggregated double GG distribution under different weather conditions with and without cyclic coding for plane and spherical waves. It can be found that the analytical results have an excellent agreement with MC simulation data, which further manifests the validity of our theoretical derivation. It is obviously seen that the *ABER* performance has the same behavior with outage performance for these two waves when weather condition varies. Moreover, in terms of *ABER* performance, the superiority for spherical wave over double GG distribution is more distinct than plane wave. Last but not least, the *ABER* performance can be improved by using cyclic codes. For example, for plane wave, at a fixed SNR of 100 dB, the *ABER* without cyclic coding is 7.56×10^{-3}, while the *ABER* with cyclic coding is 3×10^{-4}, which is an order of magnitude lower than the former.



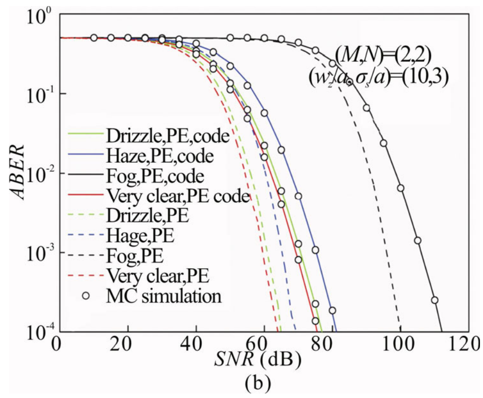


Fig.5 The *ABER* versus *SNR* of FSO system considering path loss and PE with and without cyclic coding for (a) plane and (b) spherical waves

In summary, the outage and ABER performances of multi-hop parallel FSO system with DF relaying for plane and spherical waves are investigated over double GG distribution considering path loss and PE. Based on the best path selection scheme, the closed-form outage probability and ABER expressions are derived with the help of Meijer-G function and confirmed by MC simulation. The results show that for these two waves, at a fixed link length, the outage performance of this FSO system can be affected by different degrees of turbulence severity, the existence of PE and various weather conditions. However, outage performance can be enhanced by increasing the number of parallel paths reasonably and the ABER performance can be improved by using cyclic coding. In addition, it is worth mentioning that the superiority of double GG is more obvious for spherical wave for any condition in the multi-hop parallel FSO system.

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