An improved LMS algorithm for mode demultiplexing in frequency domain*

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In order to solve the problem that the traditional frequency domain least mean square (FD-LMS) algorithm will lose efficacy with the increase of differential mode group delay (*DMGD*) when the algorithm is used for demultiplexing of the 6×6 mode division multiplexing (MDM) system, an improved FD-LMS demultiplexing algorithm is proposed. By improving the error signal calculation method, the convergence performance of the output signal of the equalization filter is improved, and the steady-state error of the algorithm is reduced. Besides, the equalization performance of the traditional FD-LMS algorithm is compared with the improved FD-LMS algorithm. Simulation results show that the improved FD-LMS algorithm has great advantage over the traditional FD-LMS algorithm in demultiplexing performance on the premise that the computation complexity does not significantly increase. The optical signal to noise ratio (*OSNR*) penalty of the improved FD-LMS algorithm is 2.6 dB lower than that of traditional FD-LMS algorithm at a transmission distance of 80 km with *DMGD* is 50 ps/km.

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With the rapid development of bandwidth-consuming services represented by large data centers, the Internet of things and mobile Internet, the demand for bandwidth of optical fiber communication network in nowadays information society has reached an unprecedented height[1]. The wide application of technology such as time division multiplexing[2], wavelength division multiplexing[3], polarization division multiplexing (PDM)[4,5] and high dimensional and high order modulation make the capacity of single-mode fiber approach the theoretical limit gradually[6]. In order to cope with the predictable bandwidth crisis of optical communication network, a new capacity expansion technology, that is mode division multiplexing (MDM) technology based on few-mode fiber (FMF)[7], emerges at the right moment. This technology takes advantage of the orthogonality of the modes in FMF and uses the modes as channels to form a multi-input multi-output (MIMO) system, so as to expand the capacity of a single fiber[8]. However, in the MDM system based on FMF, the transmission performance of the system is seriously affected by impairments such as mode coupling (MC) and differential mode group delay (*DMGD*)[9-11]. At the same time, the combined effects of MC and *DMGD* complicates the demultiplexing of MDM systems[12].

MIMO equalization of the received signals can effectively compensate for MC and *DMGD* in the MDM system and restore the source signals from the mixed signals[13]. There are currently two kinds of algorithm for mode demultiplexing: time domain equalization algorithm and frequency domain equalization algorithm. The computation complexity of time domain equalization algorithm is terrible, while the frequency domain equalization algorithm transform the signals from time domain to frequency domain to processing through efficient FFT/IFFT, which makes complex convolution operation in time domain into multiply operation in frequency domain, greatly reduces the computation complexity. Nonetheless, frequency domain equalization algorithm will lose efficacy when *DMGD* increases to a certain extent.

In this paper, an improved frequency domain least mean square (FD-LMS) algorithm based on traditional FD-LMS algorithm, the representative algorithm of frequency domain equalization algorithm, is proposed. The algorithm constructs the deviation of the modulus between the output signal and the ideal signal, so that the real part and the imaginary part of the output signal of the equalization filter converge on two circles. Simulation results show that the improved FD-LMS algorithm has great advantage over the traditional FD-LMS algorithm in demultiplexing performance on the premise that the computation complexity does not increase significantly.

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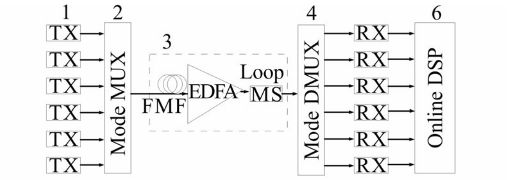
A 6×6 mode division multiplexing simulation system is set up. The structure diagram is shown in Fig.1. Modules 1—6 are signal transmitting module, mode multiplexer (MUX), few-mode fiber transmission link, mode demultiplexer (DMUX), coherent receiver module and digital signal processing (DSP) module. In this system, the fundamental mode LP01 mode of three-mode fiber, the two degenerate modes (LP11a and LP11b) of high order mode LP11 mode and their corresponding polarization modes (LP01*x*, LP01*y*, LP11a*x*, LP11a*y*, LP11b*x* and LP11b*y*) in *x* and *y* directions are used as independent channels to transmit data. At the data receiving end, the mixed signals are separated by mode demultiplexer and polarization demultiplexer, and Gaussian white noise is added to set the optical signal-to-noise ratio (*OSNR*) of the system, and then the optical signals are converted into electrical signals by coherent receivers. Finally, the DSP module is used to demultiplex the received signals and restore the source signals[14].



EDFA:erbium doped fiber amplifier; MS: mode scrambler

Fig.1 Simulation setup

The principle diagram of FD-LMS equalization algorithm is shown in Fig.2[15]. First, convert the received serial signals $x_{1,2}(n)$ into parallel signals in order to transform it into block data for subsequent FFT/IFFT processing. Then, FFT is used to transform the block data from time domain to frequency domain, and overlapping reservation method is used to complete the convolution operation between the received signals and the frequency domain equalizer. In order to facilitate calculation, the overlap coefficient is generally selected as 0.5. After dividing the data into blocks according to the overlapping reservation method, the cyclic convolution can be calculated by multiplying the block signals with the frequency domain filter directly. Finally, IFFT is performed on the frequency domain signal after equalization to get the estimated signals $y_{1,2}(n)$.

The error signal $\boldsymbol{e}_1(p)$ is obtained by subtracting the training sequence $d_1(p)$ from the recovered signal $\hat{y}_1(p)$:

$$\boldsymbol{e}_1(p)=(d_1(p)-\hat{y}_1(p))\exp\left(\mathrm{j}\phi_1(p)\right), \tag{1}$$

where $\phi_1(p)$ is the laser phase fluctuations estimated. Update the tap coefficient of the filter with $\boldsymbol{e}_1(p)$:

$$W_{ij}(p)=W_{ij}(p-1)+\mu E_i(p)\odot X_j{}^*(p), \tag{2}$$

where μ is the step size of the algorithm, $E_i(p)$ is the p-th block error signal of the i-th mode, $X_j{}^*(p)$ is the p-th data block in frequency domain of the j-th mode received signal, $\odot$ is Hadamard product that the corresponding elements of two matrices are multiplied one by one.

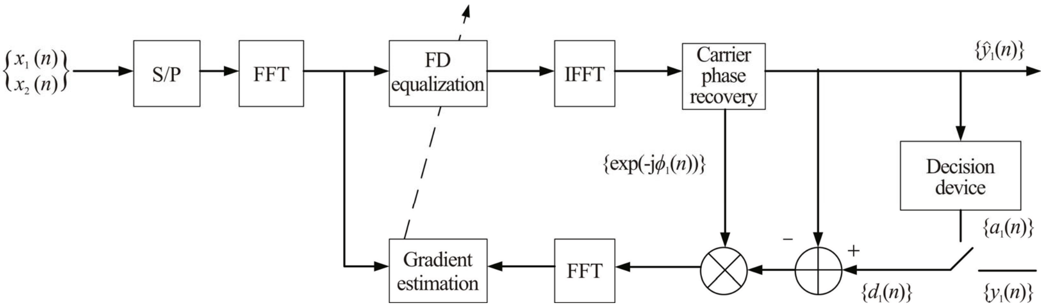


Fig.2 Frequency domain equalization principle of a single mode channel

According to the theoretical basis and characteristics of FD-LMS algorithm, we show the principle of the improved error signal as follows: by constructing the modulus value deviation between the output signal and the ideal signal, the real part and the imaginary part of the output signal of the equalization filter converge on two circles with radius R_{r1} and R_{i1} respectively. The radii of convergence R_{r1} and R_{i1} are obtained by calculating the amplitude and phase statistical characteristics of the received signal. Because the statistical characteristics of the real part and the imaginary part of the received signal are considered at the same time, the algorithm can utilize the amplitude and phase information of the signal in the meantime, in addition to equalization of signal amplitude, phase rotation can also be corrected, thus the steady-state error of the algorithm is reduced.

The improved error signal formula is as follows:

$$\begin{cases} \boldsymbol{e}_{r1}(p)=[\boldsymbol{R}_{r1}^2-\left|\Re(\hat{y}_1(p))\right|^2]\odot\Re(\hat{y}_1(p)) \\ \boldsymbol{e}_{i1}(p)=[\boldsymbol{R}_{i1}^2-\left|\Im(\hat{y}_1(p))\right|^2]\odot\Im(\hat{y}_1(p)) \end{cases}, \tag{3}$$

$$\boldsymbol{e}_1(p)=e_{r1}(p)+\mathrm{j}\cdot e_{i1}(p), \tag{4}$$

$$R_{r1}^2=\frac{E[\Re(s_1)^4]}{E[\Re(s_1)^2]}, R_{i1}^2=\frac{E[\Im(s_1)^4]}{E[\Im(s_1)^2]}, \tag{5}$$

where $\Re(\bullet)$ and $\Im(\bullet)$ represent the functions of real part and imaginary part respectively, R_{r1}^2 and R_i^2 represent the statistical modulus of the in-phase component and the orthogonal component of the source signal s_1 respectively, $\boldsymbol{R}_{r1}$ and $\boldsymbol{R}_{i1}$ are column vectors of length L composed of all elements with values of R_{r1} and R_{i1} respectively.

Set the total length of *DMGD* in the system as τ_{DMGD}, the code element period of the signal as T, the number of code element as $Q=\frac{\tau_{DMGD}}{T}$, the oversampling ratio of the receiver as R, the filter length as $N=RQ$, and the number of modes transmitted in the system as D. In order to get the information of $N/2$ symbols in one mode, the number of FFT/IFFT needed is $4+2D$, including 2 times of FFT in the input part, a pair of FFT/IFFT in the cycle update process and $2D$ times of FFT/IFFT needed for gradient constraint in the gradient estimation module. The number of complex multiplication required for each FFT/IFFT is $N\log_2(N)/2$. The synthesis of the final signal needs ND times complex multiplication, and the update of the odd and even two-way tap also needs ND times complex multiplication. Therefore, based on the complexity of the above steps, the number of complex multiplication required for each symbol of the traditional FD-LMS algorithm is:

$$C_{F1}=(4+2D)\log_2(N)+4D. \tag{6}$$

The number of complex multiplication required for each symbol of the improved FD-LMS algorithm is:

$$C_{F2}=(4+2D)\log_2(N)+4D+4. \tag{7}$$

In order to compare the demultiplexing performance of traditional FD-LMS algorithm and improved FD-LMS algorithm, a 6×6 MDM simulation system is set up, and its simulation parameters are shown in Tab.1.

Tab.1 Parameters for simulation

Parameter	Value
Fiber length	80 km
Fiber loss	0.2 dB/km
Fiber dispersion (LP01)	20 ps/nm/km
Fiber dispersion (LP11)	21 ps/nm/km
G_{offset}	6 dB
Coupling factor	0.1

Fig.3(a) and (b) are the signal constellation diagrams before demultiplexing when the *DMGD* is 25 ps/km and 50 pm/km, respectively. It can be seen that the distortion of signals is serious and the received signals needs to be demultiplexed at this time. Fig.4(a) and (b) show the signal constellation diagrams after demultiplexing with the traditional FD-LMS algorithm and the improved FD-LMS algorithm respectively when the *DMGD* is 25 ps/km. Fig.5(a) and (b) show the signal constellation diagrams after demultiplexing with the traditional FD-LMS algorithm and the improved FD-LMS algorithm respectively when the *DMGD* is 50 ps/km.

By comparing Fig.4(a) and (b), Fig.5(a) and (b), it can be seen that the signal aggregation phenomenon of the improved FD-LMS algorithm after demultiplexing is more obvious, and it has better demultiplexing performance compared with the traditional FD-LMS algorithm.

In order to further reflect the demultiplexing performance of the improved FD-LMS algorithm, Fig.6 shows the *BER* comparison curves of the two FD-LMS algorithms when *DMGD*=25 ps/km and *DMGD*=50 ps/km respectively.

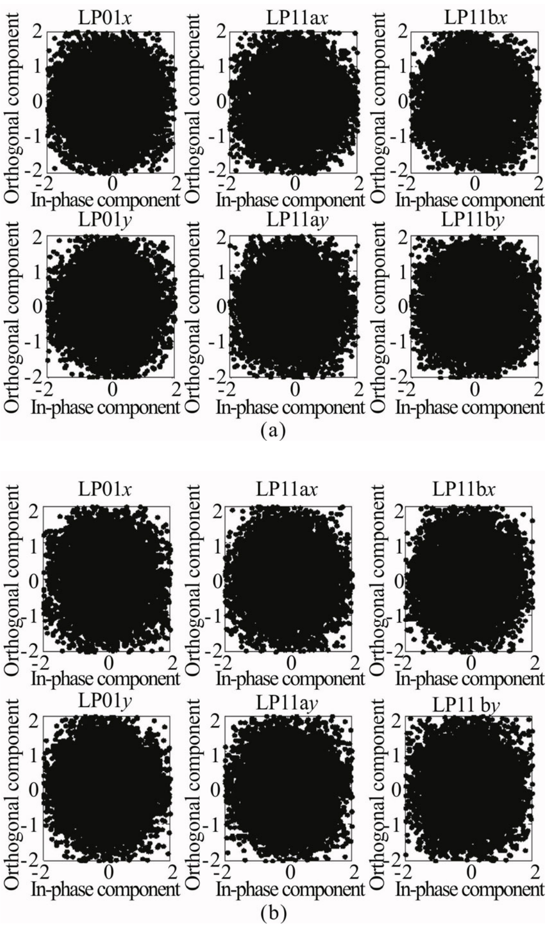
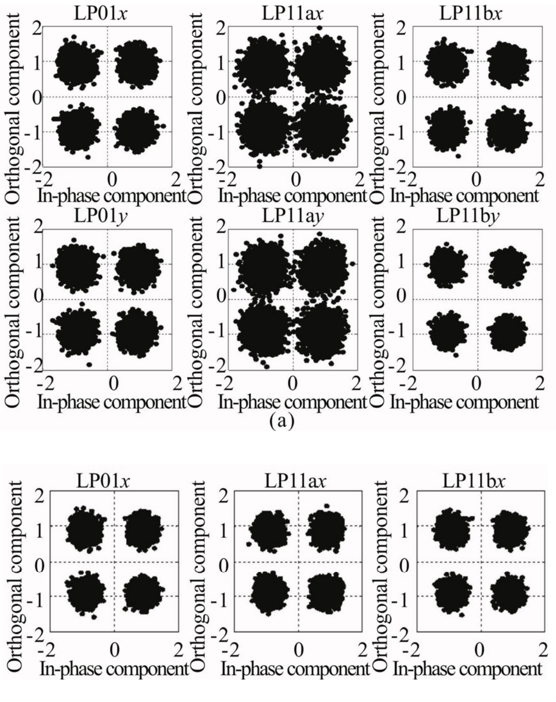


Fig.3 Signal constellation diagrams before demultiplexing: (a) *DMGD*=25 ps/km; (b) *DMGD*=50 ps/km



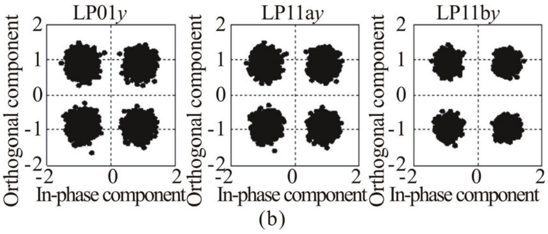


Fig.4 Signal constellation diagrams after demultiplexing when *DMGD*=25 ps/km: (a) Traditional FD-LMS; (b) Improved FD-LMS

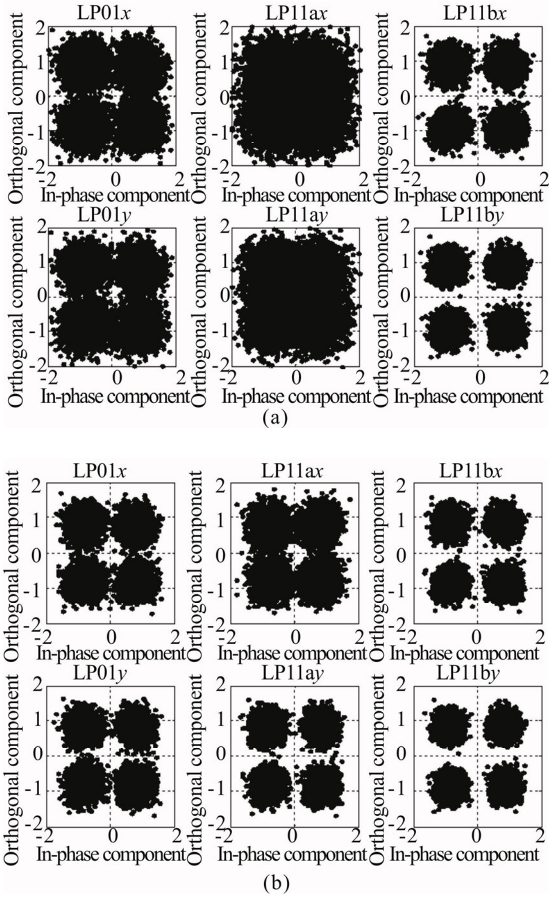


Fig.5 Signal constellation diagrams after demultiplexing when *DMGD*=50 ps/km: (a) Traditional FD-LMS; (b) Improved FD-LMS

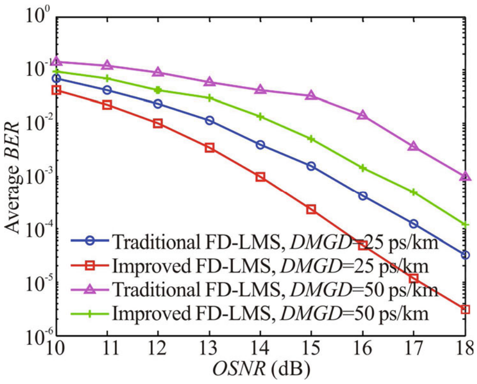


Fig.6 *BER* vs. *OSNR*

In order to measure the complexity of the two FD-LMS demultiplexing algorithms, Fig.7 shows the curves of the computational complexity of the two algorithms changing with *DMGD*.

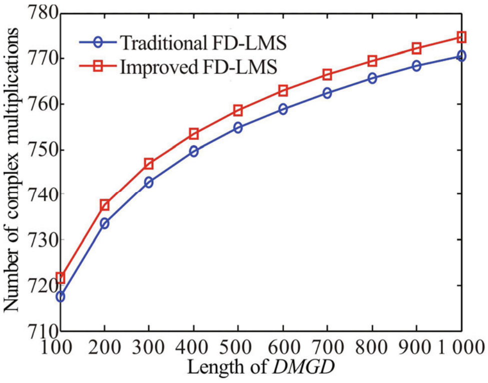


Fig.7 Complexity comparison

In this work, in order to solve the problem that the traditional FD-LMS algorithm will lose efficacy with the increase of *DMGD* when the algorithm is used for demultiplexing of the 6×6 MDM system, we use the improved FD-LMS equalization algorithm to demultiplex the system, and the equalization performance of the improved FD-LMS algorithm is compared with the traditional FD-LMS algorithm. Simulation results show that the improved FD-LMS algorithm has great advantage over the traditional FD-LMS algorithm in demultiplexing performance. However, the computation complexity of improved FD-LMS algorithm does not increase significantly. In the future, we plan to use an offline digital signal processing algorithm based on the lattice reduction to compensate for the damage in the MDM system.

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