Dynamical behavior and propagation characteristic of partially coherent sinh-Airy beams in oceanic turbulence*

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(Received 24 November 2019; Revised 17 February 2020)

By introducing the hyperbolic sine function to Airy beam, the dynamic behavior and propagation characteristics of partially coherent sinh-Airy beams in oceanic turbulence are studied using approximate analytical intensity expression. The influence of sinh modulation parameter, coherence length and ocean parameters on intensity evolution, beam width and kurtosis parameter is mainly discussed. The results show that a non-zero sinh modulation parameter presents not only the insensitivity to oceanic turbulence, but a smaller beam width. Furthermore, it also improves kurtosis parameter. These findings bring advantages in signal reception for long distance. In addition, a larger relative intensity of temperature or salinity fluctuations, mean square temperature dissipation rate, or a smaller dissipation rate of turbulence kinetic energy is more liable to increase beam width, or decrease intensity and kurtosis parameter of partially coherent sinh-Airy beams. The results provide an opportunity for improving signal reception of underwater communication or target detection by Airy beams or their groups.

Document code: A **Article ID:** 1673-1905(2021)01-0059-6

DOI https://doi.org/10.1007/s11801-021-9203-9

Turbulence is a typical phenomenon associated with chaotic behavior of fluid motion in ocean, flowing stream, smoke rising from a chimney or atmosphere[1]. Owing to the random or irregularity of physical process in turbulence, statistical average method in mathematical models is generally used for describing the essential characteristics of turbulent motion[2]. Two types of turbulence have attracted extensive attention owing to the fact that their effects are crucial in exploring how laser beams propagate. One is oceanic turbulence, which arises from the mixing effect between temperature and salinity fluctuation in the upper oceanic water. The other common type, atmospheric turbulence, refers to the chaotic changes in refractive index resulting from random fluctuation of temperature, pressure and flow velocity in atmosphere[3].

Laser propagations in the ocean and atmosphere are of great interest because of their potential application in optical communication, tracking system and target detection[4]. Especially in the area of underwater communication, propagation dynamics of some laser beams, including stochastic beams[5], partially coherent beams[6] and vector beams[7], have been studied extensively in oceanic turbulence. Based on the oceanic turbulence power spectrum proposed by Nikishov[3], the coherence and polarization behaviors of stochastic beams propagating in oceanic turbulence were dealt with by Korotkova et al[5]. Huang and Deng et al[8] investigated Strehl ratio and power-in-the-bucket of optical coherence lattices in oceanic turbulence. The intensity, gradient force and beam width of rotating elliptical Gaussian beams in oceanic and atmospheric turbulence were further discussed by Zhang et al[7].

It is a known fact that a non-diffracting beam with lower coherence may be used as a better information carrier due to its advantage in reducing turbulence-induced degradation. As typical examples of non-diffracting beams, Airy beams and their groups present a better ability to reduce turbulence effect[9]. Recently, cosh-Airy beams are also got some attention owing to the fact that cosh factor may be regarded as the superposition of two different decay factors[10], and it improves the ability to reduce turbulence effect in atmosphere[11]. However, the hyperbolic sine function of sinh(Ωx) represents the difference between two exponential factors with opposite signs, where the sinh modulation parameter of Ω can be also considered as a decay factor in another way. What would happen in turbulence if we introduce the hyperbolic sine function to partially coherent Airy beams?

The motive of this paper goes to study the effect of sinh modulation parameter, coherence length and oceanic

* This work has been supported by the Natural Science Foundation of Sichuan Education Committee (No.17ZA0072).
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parameters on dynamical behavior and propagation characteristic of partially coherent sinh-Airy beams and to explore the evolution of some key parameters. By introducing the hyperbolic sine function to partially coherent Airy beams, the resulting beams improve the ability in resisting oceanic turbulence-induced degradation. Our work can be applicable for underwater communication or target detection by Airy beams or their groups.

Introducing the hyperbolic sine function to Airy optical field, the electric field of sinh-Airy beams at the source plane z=0 in the Cartesian coordinate system can be described as

$$E\left(x_1,0\right)=E_0 Ai\left(x/x_0\right)\exp\left[ax/x_0\right]\sinh\left(\Omega x\right), \qquad (1)$$

where $Ai(\cdot)$ is the Airy function, a is decay factor, E_0 is amplitude factor determined by the input power P, x_0 is transverse scale, and Ω represents sinh modulation parameter related to the hyperbolic sine function. For partially coherent sinh-Airy beams with Schell correlation term, the cross-spectral density function at the source plane z=0 is given as[12]

$$W^{(0)}(x_1,x_2,0)=E\left(x_1,0\right)E^*\left(x_2,0\right)\exp\left[-\frac{\left(x_1-x_2\right)^2}{\sigma^2}\right], \qquad (2)$$

where the asterisk is complex conjugation, and σ is the coherence length.

Taking the method of the extended Huygens-Fresnel integral[13], the cross-spectral density of partially coherent sinh-Airy beams at the receiver plane is determined by[2]

$$\begin{aligned}&W\left(x_1,x_2,z\right)=\frac{k}{2\pi z}\iint \mathrm{d}x_1'\mathrm{d}x_2' W^{(0)}\left(x_1,x_2,z\right)\times\\&\left\langle\exp\left[\psi\left(x_1',x_1\right)+\psi\left(x_2',x_2\right)\right]\right\rangle_m\times\\&\exp\left\{\frac{\mathrm{i}k}{2z}\left[\left(x_1^{\,2}-x_2^{\,2}\right)-\right.\right.\\&\left.\left.2\left(x_1x_1'-x_2x_2'\right)+\left(x_1'^2-x_2'^2\right)\right]\right\},\end{aligned} \qquad (3)$$

where the wave number k=2π/λ related to wave length λ, and $\langle\cdot\rangle_m$ is the statistical average of the turbulent medium statistics, which is approximately given by

$$\exp\left[-\frac{\left(x_1'-x_2'\right)^2+\left(x_1'-x'\right)\left(x_1-x_2\right)+\left(x_1-x_2\right)^2}{\rho_s^2}\right]. \qquad (4)$$

For the oceanic turbulence[14],

$$\rho_s=\left(\frac{3}{\pi^2k^2z\int_0^\infty\kappa^3\varphi_n\left(\kappa\right)\mathrm{d}\kappa}\right)^{1/2}, \qquad (5)$$

where κ is the spatial wave number, and $\varphi_n(\kappa)$ represents the power spectrum of the ocean refractive index fluctuation composed of temperature and salinity fluctuations, which can be written as

$$\begin{aligned}&\varphi_n\left(\kappa\right)=0.388\times10^{-8}\varepsilon^{-1/3}\kappa^{-11/3}\left[1+\right.\\&\left.2.35\left(\kappa\eta\right)^{2/3}\right]f\left(\kappa,\tau,\chi_T\right),\end{aligned} \qquad (6)$$

$$\begin{cases}f\left(\kappa,\tau,\chi_T\right)=\dfrac{\chi_T}{\tau^2}\left(\omega^2\mathrm{e}^{-A_T\delta}+\mathrm{e}^{-A_S\delta}-2\tau\mathrm{e}^{-A_{TS}\delta}\right)\\A_T=1.863\times10^{-2}, A_S=1.9\times10^{-4}, A_{TS}=9.41\times10^{-3}\\\delta=8.284\left(\kappa\eta\right)^{4/3}+12.978\left(\kappa\eta\right)^2\end{cases}. \qquad (7)$$

In Eqs.(6)—(7), ε indicates dissipation rate of turbulence kinetic energy in unit mass liquid, and its value is found to be in the range of 10^{-10} m^2/s^3 to 10^{-1} m^2/s^3. η refers to Kolmogorov inner scale, and its value is fixed as one centimeter. τ is relative intensity of temperature and salinity fluctuations and its range is between –5 to 0. When the value of τ is equal to zero indicates that ocean turbulence is mainly induced by salinity, and the temperature-induced turbulence becomes dominant for τ=–5. χ_T is the mean square temperature dissipation rate with the range of 10^{-10} K^2/s to 10^{-4} K^2/s. The value of χ_T approaches to 10^{-4} K^2/s, which means that the seawater has a stronger temperature gradient force and a greater turbulent energy[6]. The combination effects between τ, χ_T and ε determine the magnitude of oceanic turbulence, and a strong turbulence can be acquired by an increase in τ, χ_T or a decrease in ε.

Substituting from Eqs.(4)—(7) into Eq.(3) and letting x_1=x_2=x, after tedious integral calculations, an approximate analytical expression of averaged intensity for partially coherent sinh-Airy beams at the receiver plane is obtained as

$$I\left(x,z\right)=W_1+W_2-W_3-W_4, \qquad (8)$$

where the component of averaged intensity

$$\begin{aligned}&W_j=\frac{\left|E_0\right|^2}{4}\exp\left[\mathrm{i}\left(\frac{2}{3}\theta^3+\frac{2}{3}\beta^3-\theta Q_j-\beta K_j\right)+J_j\right]\times\\&Ai\left(Q_j-\theta^2\right)Ai\left(K_j-\beta^2\right),\left(j=1,2,3,4\right)\end{aligned} \qquad (9)$$

$$J_{1,2}=\frac{-\mathrm{i}\xi z\left(\mu\pm2\Omega\right)}{k}+\frac{z^2\left(\mu\pm2\Omega\right)^2}{k^2p^2}, \qquad (10)$$

$$J_{3,4}=\frac{\mathrm{i}z\mu\left(-\xi\pm\Omega\right)}{k}+\frac{z^2\mu^2}{k^2p^2}, \qquad (11)$$

$$Q_{1,2}=\frac{2z^2\left(\mu\pm2\Omega\right)}{k^2p^2x_0}-\frac{\mathrm{i}z\left(2\xi+\mu\pm2\Omega\right)}{2kx_0}, \qquad (12)$$

$$Q_{3,4}=\frac{2z^2\mu}{k^2p^2x_0}+\frac{2\mathrm{i}z\left(-\xi\pm\Omega\right)-\mathrm{i}z\mu}{2kx_0}, \qquad (13)$$

$$K_{1,2}=\frac{2z^2\left(\mu\pm2\Omega\right)}{k^2p^2x_0}+\frac{\mathrm{i}z\left(-2\xi+\mu\pm2\Omega\right)}{2kx_0}, \qquad (14)$$

$$K_{3,4}=\frac{2z^2\mu}{k^2p^2x_0}+\frac{2\mathrm{i}z\left(-\xi\pm\Omega\right)+\mathrm{i}z\mu}{2kx_0}, \qquad (15)$$

$$\theta=\frac{\mathrm{i}z^2}{k^2p^2x_0}+\frac{z}{2kx_0^{\,2}},\quad \beta=\frac{\mathrm{i}z^2}{k^2p^2x_0}-\frac{z}{2kx_0^{\,2}}, \qquad (16)$$

$$\mu=\frac{2a}{x_0},\quad \xi=\frac{\mathrm{i}kx}{z},\quad p=1\Big/\sqrt{\frac{1}{\sigma^2}+\frac{1}{\rho_s^{\,2}}}. \qquad (17)$$

In Eqs.(10)—(15), for the subscript j=1 or 3, the sign

"±" in J_j, Q_j and K_j are positive, and other cases are negative. From Eqs.(8)—(17), one can find that averaged intensity is associated with sinh modulation parameter Ω, coherence length σ and oceanic parameters (i.e. τ, χ_T and ε).

In oceanic turbulence, some propagation parameters such as beam width and kurtosis parameter are important for studying how to reduce turbulence-induced degradation. Based on the mean-squared method, beam width is defined by[15]

$$w=2\sqrt{\frac{\int x^2 I(x,z)\,\mathrm{d}x}{\int I(x,z)\,\mathrm{d}x}}\,. \tag{18}$$

Kurtosis parameter K_x describes the beam sharpness, which is defined as [16]

$$K_x=\frac{\langle x^4\rangle}{\langle x^2\rangle^2},\ \langle x^n\rangle=\frac{\int_{-\infty}^{\infty} x^n I(x,z)\,\mathrm{d}x}{\int_{-\infty}^{\infty} I(x,z)\,\mathrm{d}x}\,. \tag{19}$$

In general, a larger kurtosis parameter is corresponding to a more sharp intensity profile. According to critical values of K_x=3, intensity profile can be divided into leptokurtic, perfectly normal and platykurtic distributions, respectively.

Based on the intensity expression and the definitions of some key parameters, dynamical behavior and propagation characteristic in oceanic turbulence will be analyzed in the next sections. Unless specifically described in the caption of the Figs, the related parameters are fixed to λ=633 nm, a=0.1, x_0=1 mm, P=1 W, η=10^{-2} m, τ=−4, χ_T=10^{-9} K^2/s, ε=10^{-3} m^2/s^3, σ=9 mm and Ω=50 m^{-1}.

Based on the expression of averaged intensity, we choose one of Airy functions, e.g. $Ai(Q_j-\theta^2)$ in Eq.(9) for analyzing the dynamical behavior of partially coherent sinh-Airy beams in oceanic turbulence. The Airy function follows polynomials trajectory described by

$$x=-\frac{1}{k^4p^4x_0^3}z^4+\frac{p^2-16ax_0^2-16x_0^3\Omega}{4k^2p^2x_0^3}z^2\,. \tag{20}$$

The corresponding kinematics equation in turbulence is given by

$$g=\frac{\mathrm{d}^2x}{\mathrm{d}z^2}=-\frac{12}{k^4p^4x_0^3}z^2+\frac{1}{2k^2x_0^3}-\frac{8a}{k^2p^2x_0}-\frac{8\Omega}{k^2p^2}\,, \tag{21}$$

where g plays a role of acceleration of the resulting beam. Its acceleration in turbulence is a variable value with sinh modulation parameter, coherence length, oceanic parameters and propagation distance z, which determines a complicated behavior in turbulence rather than a traditional parabola in free space[17]. When the turbulence approaches to infinitesimal (namely, $\rho_s\to\infty$) and the coherence length goes to infinity of $\sigma\to\infty$, the averaged intensity in Eqs.(8)—(17) can be transformed into that of a fully coherent sinh-Airy beam propagating in free space, and the acceleration in Eq.(21) also reduces to a constant $g=1/2k^2x_0^3$, which is consistent with the Ref.[17] in free space. Further, dynamical behaviors of other Airy functions in Eq. (9) are also dealt with by using the similar method.

Taking $Ai(Q_j-\theta^2)$ function as an example, Fig.1 depicts the dynamical behavior of partially coherent sinh-Airy beams in oceanic turbulence, and the corresponding acceleration and trajectory of fully coherent sinh-Airy beams in free space are also compared in Fig.1(a). One can see that sinh modulation parameter, coherence length and oceanic turbulence have obvious influence on dynamical behavior. The acceleration gradually changes from positive to negative value in turbulence propagation as shown in Fig.1(a1), which results in the behavior is not a traditional parabolic curve, but a complicated trajectory in Fig.1(a2). The negative acceleration is further accentuated by decreasing coherence length in Fig.1(b1). The acceleration is less affected by sinh modulation parameter, but in Fig.1(c2) the trajectory difference between two sinh modulation parameters (e.g. Ω=1 m^{-1} and Ω=90 m^{-1}) gradually decreases with propagation.

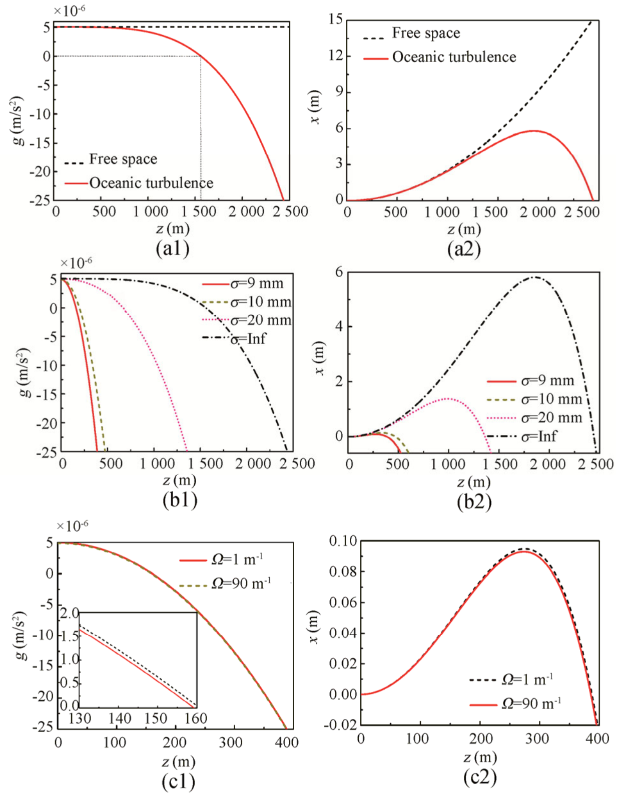

Fig.1 Dynamical behavior of partially coherent sinh-Airy beams in oceanic turbulence: (a) Ω=50 m^{-1}; (b) Ω=50 m^{-1}; (c) σ=9 mm

As aforementioned, the acceleration resulting from Airy function is a variable value related to sinh modulation parameter Ω, coherence length σ, oceanic parameters (i.e. relative intensity of temperature and salinity fluctuations τ, dissipation rate of turbulence kinetic energy in unit mass liquid ε and the mean square temperature dissipation rate χ_T) and propagation distance z. To provide more insight into the issue, intensity evolution and some key parameters, e.g. beam width and kurtosis parameters in oceanic turbulence are illustrated by numerical examples in this section.

Fig.2 depicts intensity evolution of partially coherent sinh-Airy beams propagating in the oceanic turbulence for different Ω and σ, where the effect of sinh modulation parameter on intensity in side lobes also marked by red ellipses. The beam spreading in a lower coherence is obviously larger than that in a higher coherence, which results in the rapid decay of intensity. As the coherence length increases, the main lobes not only present focusing properties, show the ability to travel a longer distance along a ballistic trajectory. The behavior can be attributed to the fact that the negative acceleration described by Eq.(21) further weakens the intensity trajectory in the existence of turbulence and coherence. In addition, a larger Ω can lead to the appearance of more side lobes marked by red ellipses. It is widely known that for the Airy beams and their groups the Poynting vector of side lobes gradually turn towards the direction of main lobes in propagation[18], which indicates that a larger Ω make it possible to improve intensity or energy of main lobe during propagation.

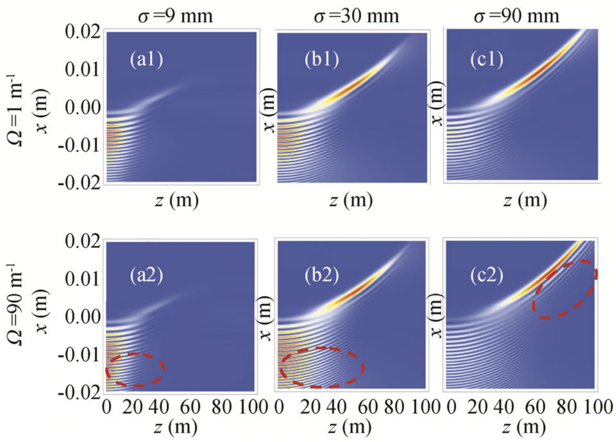

Fig.2 Intensity evolution of partially coherent sinh-Airy beams propagating in the oceanic turbulence for different coherence lengths *σ* and sinh modulation parameter *Ω*

To further investigate the effect of sinh modulation parameter Ω on the intensity evolution, Fig.3 shows the dependence of intensity profile in the oceanic turbulence on sinh modulation parameter Ω, and plots the peak intensity of I_p. When the beam propagates a short distance in turbulence, e.g. z=20 m, the peak intensity in a smaller Ω is larger than that in a larger Ω, but its peak will be surpassed with an increasing of distance, e.g. z=100 m. In Fig.3(b), the peak intensity initially decrease at z<30 m, then increase to exceed its initial value, and finally its peak intensity decrease to a lower intensity with a further propagation. In the region of z>95 m, one can note that the peak intensity of Ω=1 m^{-1} at z=100 m is eventually exceeded by that of Ω=90 m^{-1}. It may be the result of a larger Ω carrying more side-lobes, and these side-lobes transfer more energy to main lobes.

Fig.4 illustrates the effect of sinh parameters and coherent lengths on beam width of partially coherent sinh-Airy in oceanic turbulence. The beam width is symmetric about the axis of Ω=0, and its width decreases with the increase of the absolute value of Ω as shown in Fig.4(a). Especially for Ω=0 corresponding to the partially coherent Airy beam, the width reaches its maximal value, which prompts us to investigate how to reduce oceanic turbulence-induced degeneration by adjusting sinh parameters Ω and coherent length σ.

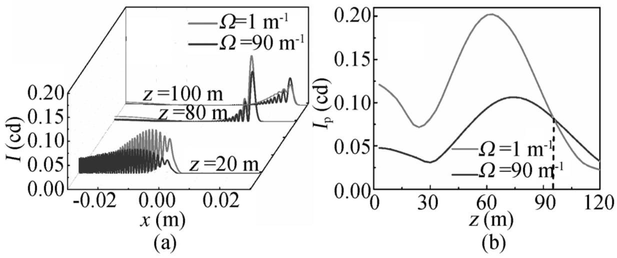

Fig.3 The dependence of (a) intensity *I* and (b) peak intensity I_p on sinh modulation parameters *Ω* in oceanic propagation

It is well known that a lower coherence generally possesses the ability to reduce turbulence effect, but it also leads to a larger beam width (as shown in Fig.4(b)), which is inconvenient for signal reception for long distance. One can clearly see that a larger Ω (e.g. Ω=90 m^{-1}) in Fig.4(c) shows not only a smaller beam width, but little difference of width between the turbulence and free space, which means that beam spreading in a larger Ω is less sensitive to oceanic turbulence than that in a smaller Ω. More importantly, it keeps its width for long distance.

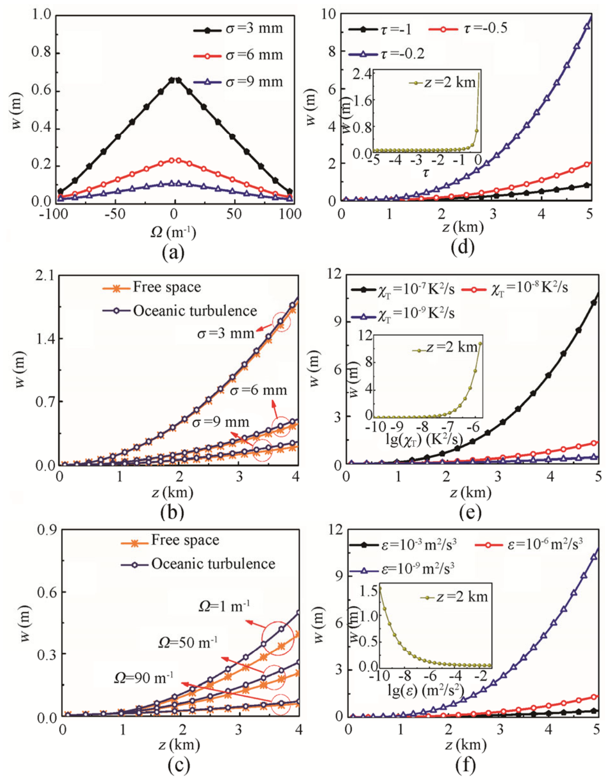

Fig.4 Beam width *w* of partially coherent sinh-Airy beams in oceanic turbulence versus coherent lengths *σ*, sinh modulation parameters *Ω*, *τ*, χ_T and *ε*: (a) *z*=2 km; (b) *Ω*=50 m^{-1}; (c) *σ*=9 mm

Fig.4(d)—(f) give further analysis for restraining beam spreading from the view of oceanic parameters. one can find that a larger τ or χ_T, or a smaller ε leads to the increase of beam width. At the receiver plane of z=2 km in sub-graphs of Fig.4(d)—(f), a slight increment in range of $-1<\tau<0$ or $\chi_T>10^{-7}$ K^2/s result in a significant increase in the beam width, and a reduction in 10^{-10} m^2/s^3<ε<10^{-6} m^2/s^3 also increase its width. It means that stronger oceanic turbulence is caused by salinity-induced fluctuations, stronger temperature gradient strength or weaker dissipation rate of turbulence kinetic energy, which play major roles in evolution of intensity and beam width.

Based on the definition of kurtosis parameter in Eq.(19), the dependence of kurtosis parameter of partially coherent sinh-Airy beams on sinh modulation parameter, coherence length and oceanic parameters in oceanic turbulence is plotted in Fig.5. For different coherence, kurtosis parameter K_x both shows symmetric structures about Ω=0. A distinct concave structure of K_x is found in a lower coherence, and its bottom can be lifted by an increase of coherence. As the further increase in coherence, the structure becomes a convex shape. One can clearly see that the enhancement of kurtosis parameter is accompanied by an increase in coherence, and for a lower coherence the value of K_x in Ω=0 is smaller than that in $\Omega\neq0$, however for a higher coherence the K_x in Ω=0 reaches maximal value. The results indicate that the sharpness of partially coherent sinh-Airy beams can be modified by sinh modulation parameters under the condition of different coherences. From Fig.5(b)—(d), one can find that there exists an oscillation mode of K_x in turbulence. With the increase of propagation distance, K_x gradually decreases and tends to a stable value. In oceanic propagation a larger τ or χ_T, or a smaller ε tends to decrease the beam sharpness.

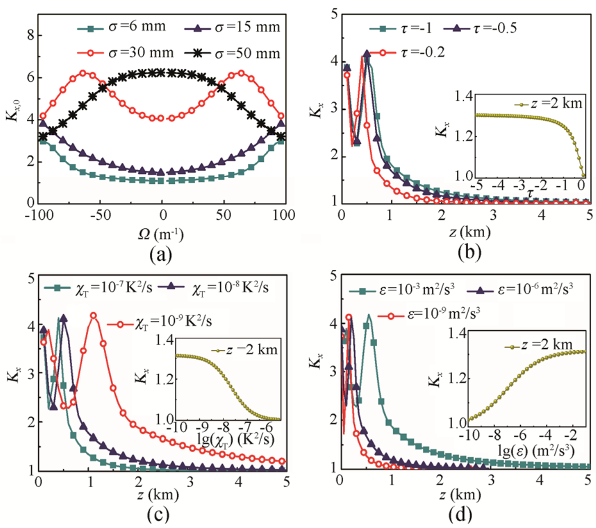

Fig.5 The dependence of kurtosis parameters K_x on sinh modulation parameter Ω, coherence length σ and oceanic parameters: (a) z=2 km; (b),(c),(d) σ=9 mm, Ω=50 m^{-1}

Based on the extended Huygens-Fresnel integral formula, an approximate analytical expression of averaged intensity of a partially coherent sinh-Airy beam passing through oceanic turbulence is obtained, which can be used to analysis dynamical behavior in turbulence. The Airy factor in intensity expression shows that its acceleration in oceanic turbulence is not fixed, but a variable value with sinh parameters Ω, coherence length σ, turbulence strength and propagation distance z. The acceleration determines the travel trajectory is a complicated curve rather than a traditional parabola.

The effect of sinh modulation parameter Ω, coherence length σ and oceanic parameters on intensity evolution, beam width and kurtosis parameter in turbulence is investigated. The results show that a larger Ω can overcome its intensity disadvantage at input plane, and gradually exceed that intensity in a smaller Ω with an increasing of propagation distance. A larger Ω not only presents a stronger ability in less sensitivity to oceanic turbulence, but also possesses a smaller beam width, which also brings convenience in signal reception for long distance.

Kurtosis parameter K_x shows symmetric structures about Ω=0. For a lower coherence, a distinct concave structure of K_x is found, and its bottom can be lifted by an increase of coherence. The enhancement of Kurtosis parameter K_x is accompanied by the increase in sinh modulation parameter and coherence length. For a lower coherence the value K_x in $\Omega\neq0$ is larger than that in Ω=0, however for a higher coherence the value of K_x in Ω=0 is maximal. The results indicate that the sharpness of partially coherent sinh-Airy beams can be modified by sinh modulation parameters under the condition of different coherences. In addition, a larger τ or χ_T, or a smaller ε is more liable to increase beam width, and decrease kurtosis parameter of partially coherent sinh-Airy beams. A study in consideration of time-harmonic field will be carried out for further work.

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