

# Influence of surface optical phonon on the electronic surface states in wurtzite group-III nitride ternary mixed crystals\*

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An intermediate-coupling variational method is presented to investigate the surface electron states in wurtzite  $A_xB_{1-x}N$  ( $A, B=Al, Ga$  and  $In$ ) ternary mixed crystals (TMCs). Corresponding effective Hamiltonian are derived by considering the surface-optical-phonon (SO-phonon) influence and anisotropic structural effect. The surface-state energies of electron, the coupling constants and the average penetrating depths of the electronic surface-state wave functions have been numerical computed as a function of the composition  $x$  and the surface potential  $V_0$  for the wurtzite  $Al_xGa_{1-x}N$ ,  $Al_xIn_{1-x}N$  and  $In_xGa_{1-x}N$ , respectively. The results show that the surface-state levels of electron are reduced with the increasing of the composition  $x$  in wurtzite  $A_xB_{1-x}N$ . It is also found that the electron-surface-optical-phonon (e-SO-p) coupling lowers the surface-state energies of electron and the shifts of the electronic surface-state energy level in the wurtzite  $Al_xGa_{1-x}N$  and  $Al_xIn_{1-x}N$  increase with the increasing of the composition  $x$ . However, in the wurtzite  $In_xGa_{1-x}N$ , the case is contrary. The influence of the e-SO-p interaction on the surface electron states can not be neglected in wurtzite  $A_xB_{1-x}N$ .

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The wide band-gap III-V compounds of semiconductors have become vital materials for their application in the field of the optoelectronic devices in the spectral region of green, blue and ultraviolet<sup>[1,2]</sup>. TMCs can play important roles in the modern electronics. Their properties can be varied by changing their composition. Vast researches involve the lattice dynamics of TMCs materials have been conducted in experimentally and theoretically. The III-nitride (III-N) TMCs, such as  $Al_xGa_{1-x}N$ ,  $Al_xIn_{1-x}N$  and  $In_xGa_{1-x}N$ , have got particular interest in recent years. The polar III-N TMCs have a direct band gap over a wide range that minimum gap is 1.97 eV for  $InN$ , the maximum value is 6.20 eV for  $AlN$  and an intermediate value is 3.39 eV for  $GaN$  in whole range of composition. The III-N TMCs provide more flexible opportunities for continuous layers in heterostructures and quantum wells with desirable lattice constants and band offsets, which permits for potential application of the III-N compounds in light-emitting and laser diodes<sup>[3,4]</sup>. In the  $A_xB_{1-x}N$  ( $A, B=Al, Ga$  and  $In$ ) TMCs, phonon modes and electron-phonon (e-p) coupling have been studied theoretically within the framework of modified random-element-isodisplacement (MREI) model<sup>[5-7]</sup>. The

nitride semiconductors appear two natural crystal structures which hexagonal and cubic structure and display many unusual properties in III-V compounds. Due to the lower symmetries of the wurtzite phase, it has more complicated phonon dynamics and carrier-phonon interactions comparing the zinc-blende one. The wurtzite nitride crystals have a significant anisotropic influence on the effective mass of polaron compared with the other III-V semiconductors. The phonon branches are more distinct in the wurtzite nitride semiconductors and their TMCs. Over the past decades, the wurtzite nitride semiconductors have appeared as a matter of broad experimental and theoretical interests about their polaronic property, electronic effective mass, interface-phonon and e-p scatter<sup>[8-10]</sup>.

The intrinsic electronic surface-states in polar crystals have attracted particular investigation by experimental and theoretical scientists. Their energy levels, state densities and existence conditions have been published<sup>[11-13]</sup>. It is recognized that the termination of the lattice give rise to the intrinsic surface electron states whose wave functions are localized in the near of the surface and decay rapidly inside the materials. As we known that the e-p

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interaction influences the electronic characteristics obviously and produces polaronic states in polar crystals. Furthermore, the features of the e-p scatter in semi-infinite material are quite differ from those in bulk one duo to the presence of surface. Surface modes of phonon appears and play an important role in determining the polaronic properties around a surface. The nearly-free-electron approximation (NFEA) and the variational approach were used to treat the influences of the e-p interaction and hydrostatic pressure on intrinsic surface-states in III-N compound semiconductors<sup>[14,15]</sup>. The results show that the e-p interaction lowers the surface-state energy levels and the SO-phonon contribution on the surface-state of electron is dominant, in specially, for the semiconductors which has strong e-p coupling and wide band gap. Many scientists researched the characteristics of polaron in the GaN, AlN and InN. However, in wurtzite  $A_xB_{1-x}N$  ( $A, B=Al, Ga$  and  $In$ ), the influence of the e-p interaction on the electronic surface-state is not mentioned in the previous investigations<sup>[16,17]</sup>. For wurtzite  $A_xB_{1-x}N$ , in researching the features of the electronic surface-states, it is necessary to account the influence of the e-SO-p coupling and structural anisotropy.

In this article, we have investigate the influence of the e-p interaction on the surface electron state in wurtzite III-N TMCs by using a variational method within the framework of the dielectric continuum model<sup>[18]</sup>. The influence of structural anisotropy and e-SO-p interaction are considered. An effective Hamiltonian for this e-p system introduced by Lee-Low-Pines (LLP)-like treatment<sup>[19]</sup>. A variational calculation for the electronic surface-state energies, shifts of the electronic surface-state energy level, coupling constants and average penetrating depths of the electronic surface-state wave functions in wurtzite III-N TMCs was performed. The numerical computation results for the wurtzite  $Al_xGa_{1-x}N$ ,  $Al_xIn_{1-x}N$  and  $In_xGa_{1-x}N$  are given as function of the composition  $x$  and discussed.

Assuming a semi-infinite semiconductor system, in which the positive-half space ( $z \geq 0$ ) is occupying with TMCs  $A_xB_{1-x}N$ , while the negative-half region ( $z \leq 0$ ) is the vacuum. The materials have the translational symmetry in the surface plane and the symmetry has been broken along the direction perpendicular to the surface. We take the anisotropic axis as the  $z$  axis and denote its perpendicular direction as  $\perp$ . An electron moves in the material and couples with lattice vibrations. Using the NFEA, such an e-p interaction Hamiltonian can be presented as<sup>[11]</sup>

$$H = H_e + H_{ph} + H_{e-p}, \quad (1)$$

where  $H_e$  is the free electronic Hamiltonian and is given in the follow form

$$H_e = \frac{p_{\perp}^2}{2m_{\perp}^*} + \frac{p_z^2}{2m_{0z}} + V(z), \quad (2)$$

where  $p_{\perp}$  and  $p_z$  are the components of the electronic momentum in the  $x$ - $y$  plane and  $z$  orientation, and  $m_{\perp}^*$  is the electronic band mass in the two band model with rest mass  $m_{0z}$ , respectively. The one-dimensional pseudo-potential  $V(z)$  describing the potential experienced by

the electron in the  $z$  orientation can be written as<sup>[14]</sup>

$$V(z) = \begin{cases} -2V_1 \cos(2\pi z/c), & z \geq 0 \\ V_0, & z \leq 0 \end{cases}, \quad (3)$$

where  $c$  is the lattice constant of the polar crystal in  $z$  orientation,  $V_1 = E_g/2$  ( $E_g$  refers to the forbidden band gap of material) and  $V_0$  are the vacuum energy levels in the two band models, respectively.  $H_{ph}$  in Eq.(1) is the SO-phonon field Hamiltonian and can be expressed as

$$H_{ph} = H_{SO} = \sum_k \hbar \omega_{SO} a_k^{\dagger} a_k. \quad (4)$$

We only taking the SO-phonon effect into account for the e-p coupling. The last term in Eq.(1), the e-p interaction one, can be written as

$$H_{e-ph} = H_{e-SO} = \sum_k \left( F_s \frac{e^{-D_s|k|}}{k^{1/2}} e^{ik \cdot \rho} a_k + h.c. \right), \quad (5)$$

with

$$F_s = \left[ \frac{4\pi e^2 \hbar}{S \omega_{SO}} C_s \right]^{1/2}, \quad (6)$$

$$C_s = \sqrt{(\omega_{SO}^4 - B_L^2 \omega_{SO}^2 + \omega_{LT}^2 \omega_{ST}^2)^3 (\omega_{SO}^4 - B_L^2 \omega_{SO}^2 + \omega_{\perp L}^2 \omega_{\perp L}^2) / \{ \sqrt{\varepsilon_{\perp}^{\infty} \varepsilon_z^{\infty}} [(B_L^2 - B_L^2) \omega_{SO}^4 + 2(\omega_{\perp L}^2 \omega_{\perp L}^2 - \omega_{LT}^2 \omega_{ST}^2) \omega_{SO}^2 + B_L^2 \omega_{LT}^2 \omega_{ST}^2 - B_L^2 \omega_{\perp L}^2 \omega_{\perp L}^2] \}}, \quad (7)$$

$$D_s = \begin{cases} \left[ \frac{\varepsilon_{\perp}^{\infty} (\omega_{SO}^2 - \omega_{ST}^2) (\omega_{SO}^2 - \omega_{\perp L}^2)}{\varepsilon_z^{\infty} (\omega_{SO}^2 - \omega_{LT}^2) (\omega_{SO}^2 - \omega_{\perp L}^2)} \right]^{1/2}, & z \geq 0 \\ 1, & z \leq 0 \end{cases}, \quad (8)$$

$$\omega_{SO}^2 = \frac{\varepsilon_{\perp}^{\infty} \varepsilon_z^{\infty} B_L^2 - B_L^2}{2(\varepsilon_{\perp}^{\infty} \varepsilon_z^{\infty} - 1)} - \frac{\sqrt{[\varepsilon_{\perp}^{\infty} \varepsilon_z^{\infty} (\omega_{\perp L}^2 - \omega_{\perp L}^2) - (\omega_{\perp L}^2 - \omega_{ST}^2)]^2 + 4\varepsilon_{\perp}^{\infty} \varepsilon_z^{\infty} (\omega_{\perp L}^2 - \omega_{\perp L}^2) (\omega_{\perp L}^2 - \omega_{ST}^2)}}{2(\varepsilon_{\perp}^{\infty} \varepsilon_z^{\infty} - 1)}, \quad (9)$$

$$B_j^2 = \omega_{Lj}^2 + \omega_{Tj}^2, \quad j = L, T. \quad (10)$$

In Eqs.(4) and (5),  $a_k^{\dagger}$  and  $a_k$  represents the creation and annihilation operators of a SO-phonon with wave vector  $\mathbf{k}$  and frequency  $\omega_{SO}$ ,  $S$  is the surface area of the lattice, and  $\mathbf{r} = (\rho, z)$ , where  $\rho$  and  $z$  are the  $x$ - $y$  plane and  $z$  components of the electron coordinates, respectively.

In order to simplify the computation for the influence of the e-SO-p scatter, we making two unitary transformations<sup>[14]</sup>. The first unitary transformation  $U_1$  is to eliminate the  $\rho$  and given by

$$U_1 = \exp \left( -i \sum_k a_k^{\dagger} a_k \mathbf{k} \cdot \rho \right), \quad (11)$$

$$H_1 = U_1^{-1} H U_1 =$$

$$\left( \frac{p_{\perp} - \sum_k \hbar k a_k^{\dagger} a_k}{2m^*} \right)^2 + \frac{p_z^2}{2m_0} + V(Z) + \sum_k \hbar \omega_{SO} a_k^{\dagger} a_k + \sum_k \left( F_s \frac{e^{-D_s|k|}}{k^{1/2}} a_k + h.c. \right), \quad (12)$$

where  $\rho$  of the electron disappears.

We taking the second unitary transformation  $U_2$  form is

$$U_2 = \exp\left(\sum_k a_k^\dagger f_k - a_k f_k^*\right). \quad (13)$$

Under the condition of the lower temperature limit for a slow-moving electron, we perform a variational treatment to solve the Hamiltonian  $H_1$  on the trial state

$$|\psi\rangle = U_2 |0\rangle |\phi_\lambda(z)\rangle, \quad (14)$$

where  $|0\rangle$  describes the zero-SO-phonon state and the  $|\phi_\lambda(z)\rangle$  is the trial wave function in the  $z$  orientation for an electron, whose variational parameter is  $\lambda$ .

The corresponding total variational energy of the e-p coupling system has obtained by following routes

$$E_v = \langle \psi | H_1 | \psi \rangle = \langle \phi_\lambda(z) | H^* | \phi_\lambda(z) \rangle, \quad (15)$$

with

$$H^* = \langle 0 | U_2^{-1} H_1 U_2 | 0 \rangle = \frac{p_\perp^2}{2m_\perp^*} + \frac{p_z^2}{2m_{0z}} + V(z) + \sum_k \left( F_s \frac{e^{-D_s|k|}}{k^{1/2}} f_k + h.c. \right) + \frac{\hbar^2}{2m_\perp^*} \left( \sum_k f_k^2 k \right)^2 + \sum_k f_k^2 \left( \hbar\omega_{so} - \frac{\hbar\mathbf{k} \cdot \mathbf{p}_\perp}{m_\perp^*} + \frac{\hbar^2 k^2}{2m_\perp^*} \right), \quad (16)$$

where  $H^*$  represents a one-dimensional and two-band model Hamiltonian, including the e-p interaction and is named the effective polaron Hamiltonian. The displacement amplitude  $f_k$  and its conjugate  $f_k^*$  in Eq.(16) can be determined by using a variational treatment similar to that used by LLP method in the bulk-polaron problems<sup>[19]</sup> and requires

$$\frac{\partial H^*}{\partial f_k} = \frac{\partial H^*}{\partial f_k^*} = 0. \quad (17)$$

Without losing of generality in the following, we can put  $p_\perp = 0$  since we concentrated only on the surface electronic state and the electron  $z$ -orientation motion and in the  $x$ - $y$  plane is inconsequential. Hence, the effective Hamiltonian  $H^*$  of such an e-p system can be simplified to follows form

$$H^* = \frac{p_z^2}{2m_{0z}} + V(z) + V_{\text{eff}}(z), \quad (18)$$

with

$$V_{\text{eff}}(z) = -\alpha_s \hbar\omega_{so} \beta(z), \quad (19)$$

where  $V_{\text{eff}}(z)$  is an effective and  $z$ -orientation potential,  $\alpha_s$  is the e-SO-p coupling constant and defined as<sup>[14]</sup>

$$\alpha_s = \frac{4e^2}{\hbar\omega_{so}^2} \sqrt{\frac{m_\perp^*}{2\hbar\omega_{so}}} C_s, \quad (20)$$

where  $C_s$  and  $\omega_{so}$  have been defined in Eqs.(7) and (9), respectively.

The optical phonons exhibit one mode behavior in wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$  according to the result in the MREI model<sup>[7]</sup>. In this case, we can as-

suming the influence of the value of the composition  $x$  is a linear independence on these parameters in the lattice constant, band gap, the electron effective mass, LO/TO-phonon frequency and the optical dielectric constant. The parameters of TMCs can be obtained from binary parameters using a linear interpolation

$$T_{A_x B_{1-x} N} = x T_{AN} + (1-x) T_{BN}, \quad (21)$$

where  $T_{A_x B_{1-x} N}$  denotes the lattice constant, band gap, the electron effective mass, LO/TO-phonon frequency and the optical dielectric constant,  $T_{AN}$  and  $T_{BN}$  are the corresponding parameters in the binary crystals AN and BN, respectively.

Now let's start from  $H^*$  in Eq.(18) to compute the electronic surface-state energy level. The corresponding variational energy is given by

$$E_v = \langle \phi_\lambda(z) | H^* | \phi_\lambda(z) \rangle, \quad (22)$$

where the trial wave function  $|\phi_\lambda(z)\rangle$  for the polaronic surface state can be chosen to be the following form<sup>[11]</sup>

$$|\phi_\lambda(z)\rangle = \begin{cases} A e^{-\lambda_1 z} \cos(\pi z/c + \lambda_2), & z \geq 0 \\ B e^{\lambda_2 z}, & z \leq 0 \end{cases}, \quad (23)$$

where  $A$  and  $B$  are the normalization constants of the wave function  $|\phi_\lambda(z)\rangle$ , and treat  $\lambda_1$  and  $\lambda_2$  as variational parameters to seek the surface electron state energy.

Inserting Eqs.(18) and (23) into Eq.(22), we can obtain the expectation value of  $H^*$ <sup>[14]</sup>

$$E_v = \frac{\hbar^2 A^2 \pi^2}{8m_{0z} c^2 \lambda_1} - \frac{V_0 A^2 \cos^2 \lambda_2}{2 \left( \lambda_1 + \frac{\pi}{c} \tan \lambda_2 \right)} - E_{e-so} - \frac{V_1 A^2}{4} \times \left( \frac{\cos 2\lambda_2}{\lambda_1} + \frac{2\lambda_1}{\lambda_1^2 + \frac{\pi^2}{c^2}} + \frac{\lambda_1 \cos 2\lambda_2 - \frac{2\pi}{c} \sin 2\lambda_2}{\lambda_1^2 + \frac{4\pi^2}{c^2}} \right), \quad (24)$$

where  $E_{e-so} = \alpha_s \hbar\omega_{so} A^2 (I + g \cos^2 \lambda_2)$  is the e-SO-p coupling contribution to the electronic surface state energy,  $I$  and  $g$  are defined alike to that in Ref.[14].

The average penetrating depth of the electronic surface-state wave function in the material is defined by

$$d = \langle \phi_\lambda(z) | z | \phi_\lambda(z) \rangle = \frac{A^2 \left( \lambda_1^2 - \frac{\pi^2}{c^2} \right) \cos 2\lambda_2 - \frac{2\pi A^2 \lambda_1}{c} \sin 2\lambda_2}{8\lambda_1^2 + 8 \left( \lambda_1^2 + \frac{\pi^2}{c^2} \right)^2}. \quad (25)$$

In Eqs.(24) and (25), the variational parameters  $\lambda_i$  ( $i=1, 2$ ) are determined by

$$\frac{\partial E_v}{\partial \lambda_i} = 0, \quad i = 1, 2. \quad (26)$$

The complexity of Eq.(26) gives rise to necessity of numerical solutions to seek the minimum value of  $E_v$ , i.e., the electronic surface-state energy  $E_s$ .

Using a variational approach we have numerically computed the electronic surface-state energies, coupling

constants and the average penetrating depths of the electronic surface-state wave functions as function of the composition  $x$  and surface potential  $V_0$  in the wurtzite TMCs, such as  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$ , respectively. The parameters used in the calculations are listed in Tab.1—3<sup>[20,21]</sup> and the results are plotted in Figs.1—5. For the sake of understanding the effect of structural anisotropy, we have also calculated the same items in the zinc-blende structure.

**Tab.1 The optical phonon energies of materials used in calculations measured in meV**

| Material | $\hbar\omega_{z\perp}$ | $\hbar\omega_{z\parallel}$ | $\hbar\omega_{\perp\parallel}$ | $\hbar\omega_{\perp\perp}$ |
|----------|------------------------|----------------------------|--------------------------------|----------------------------|
| GaN      | 91.10                  | 66.06                      | 91.97                          | 69.53                      |
| AlN      | 110.68                 | 71.10                      | 113.54                         | 83.42                      |
| InN      | 72.63                  | 54.91                      | 73.75                          | 57.88                      |

**Tab.2 The dielectric constants of materials used in calculations**

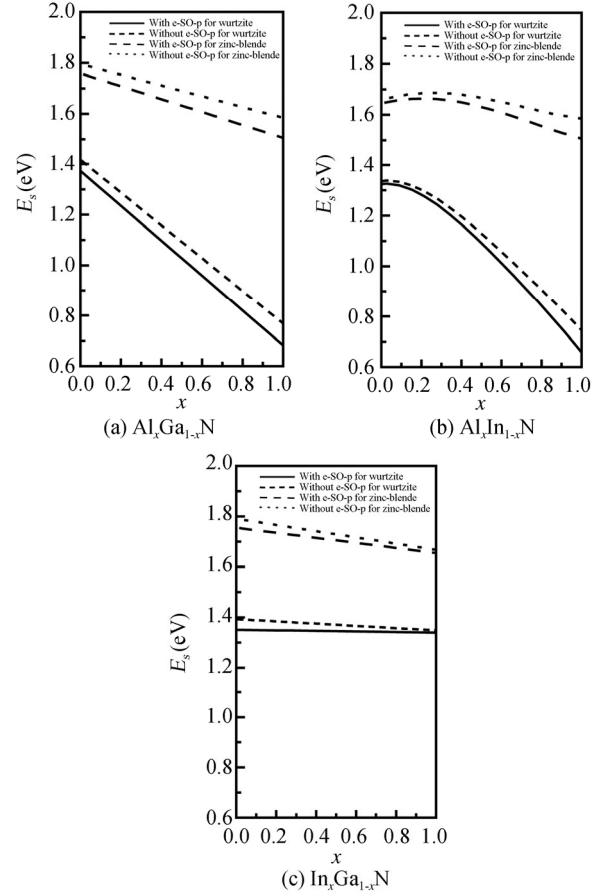
| Material | $\epsilon_z^0$ | $\epsilon_{\perp}^0$ | $\epsilon_z^{\infty} = \epsilon_{\perp}^{\infty}$ |
|----------|----------------|----------------------|---|
| GaN      | 10.18          | 9.36                 | 5.35  |
| AlN      | 11.72          | 8.97                 | 4.84  |
| InN      | 14.70          | 13.64                | 8.40  |

**Tab.3 The lattice constants, band gaps and effective masses of electrons of materials, where  $m^*$  is in the electron rest mass  $m_0$**

| Material | $c$ (nm) | $E_g$ (meV) | $m_{\perp}^* = m_{0z}^*$ |
|----------|----------|-------------|--------------------------|
| GaN      | 0.518    | 3390        | 0.20                     |
| AlN      | 0.4982   | 6200        | 0.30                     |
| InN      | 0.5760   | 1970        | 0.12                     |

To clearly understand the influence of the e-SO-p coupling on the surface-states of electron, in Fig.1, we have plotted the electronic surface-state energy levels  $E_s$  with and without the SO-phonon influence as a function of the composition  $x$  with  $V_0=5.0$  eV for the wurtzite and zinc-blende  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$ , respectively<sup>[11]</sup>. Fig.1 shows that  $E_s$  are almost linear decreases with the increasing of the composition  $x$  for  $\text{Al}_x\text{Ga}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$ , and are non-monotonous for  $\text{Al}_x\text{In}_{1-x}\text{N}$ . The surface-state energy levels  $E_s$  with e-SO-p interaction are in the range of 1.342 eV to 0.644 eV, 1.327 eV to 0.645 eV and 1.342 eV to 1.328 eV for wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$  while the composition  $x$  increases from 0 to 1 and the net decrease is 52.0%, 51.4% and 1.0%, respectively. The values of the  $E_s$  without e-p coupling varies from 1.386 eV to 0.732 eV, from 1.338 eV to 0.733 eV and from 1.386 eV to 1.339 eV for wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$ , respectively. It also gives that  $E_s$  for wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$  are less than that for zinc-blende cases. The electronic surface-state energy levels  $E_s$  with the SO-phonon contribution in wurtzite materials are

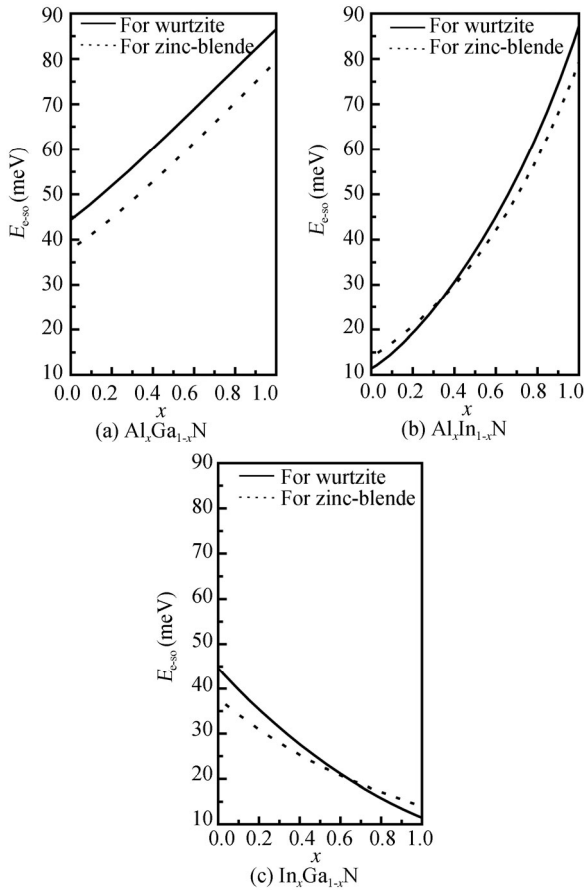
lower than that for corresponding zinc-blende one for 475 meV, 381 meV and 387 meV for  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $\text{In}_x\text{Ga}_{1-x}\text{N}$  at  $x=0.20$ . It is follows that the broader the forbidden band gap or the stronger the e-p interaction, the lower are the electronic surface-state energy levels.



**Fig.1 The electronic surface state energy levels  $E_s$  with and without e-SO-p interactions for wurtzite structures, and with and without e-SO-p interactions for zinc-blende structures as functions of the composition  $x$  for several III-N TMCs**

In Fig.2, we have illustrated the shifts of the electronic surface-state energy level  $E_{e\text{-SO}}$  as function of the composition  $x$  caused by the e-SO-p interaction with  $V_0=5.0$  eV. Fig.2 shows that the e-SO-p interaction reduced the electronic surface-state energy levels for all computed  $\text{A}_x\text{B}_{1-x}\text{N}$ . In the other word, the electronic surface-state energy levels with the e-SO-p interaction are always less than that without SO-phonon effect. The two curves of the  $E_s$  with and without the influence of the e-SO-p interaction are divided distinctly from each other. The shifts of the electronic surface-state energy levels  $E_{e\text{-SO}}$  are decades of meV for all aforementioned materials.  $E_{e\text{-SO}}$  is 52.129 meV, 19.071 meV and 35.493 meV for wurtzite  $\text{Al}_{0.20}\text{Ga}_{0.80}\text{N}$ ,  $\text{Al}_{0.20}\text{In}_{0.80}\text{N}$  and  $\text{In}_{0.20}\text{Ga}_{0.80}\text{N}$ , respectively. The shift of the electronic surface-state energy levels  $E_{e\text{-SO}}$  is related to SO-phonon energy  $\hbar\omega_{\text{SO}}$  and e-SO-p coupling constant  $\alpha_s$ . The SO-phonon energy and e-SO-p coupling constant are increased with the in-

creasing of the composition, for the wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$  and  $\text{Al}_x\text{In}_{1-x}\text{N}$ , respectively. It caused the increasing of  $E_{e\text{-SO}}$  with the composition  $x$ , as Fig.2(a) and (b) shown, for the wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$  and  $\text{Al}_x\text{In}_{1-x}\text{N}$ , respectively. At  $x=0.0$ , the shifts of the electronic surface-state energy levels are  $E_{e\text{-SO}}=44.456$  meV and  $11.224$  meV, and at  $x=1.0$ , they increase to  $87.927$  meV and  $88.359$  meV for wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$  and  $\text{Al}_x\text{In}_{1-x}\text{N}$ , respectively. For the wurtzite  $\text{In}_x\text{Ga}_{1-x}\text{N}$ , the case is contrary, that  $E_{e\text{-SO}}$  is decrease with the increasing of the composition  $x$  as the Fig.2(c) shows. It is due to the decrease of the SO-phonon energy and e-SO-p coupling constant with the increasing of the composition. The shifts are  $E_{e\text{-SO}}=44.456$  meV and  $11.075$  meV at  $x=0.0$  and  $1.0$ , respectively. It indicates that the stronger e-p coupling is, the greater the shift of the electronic surface state energy level.



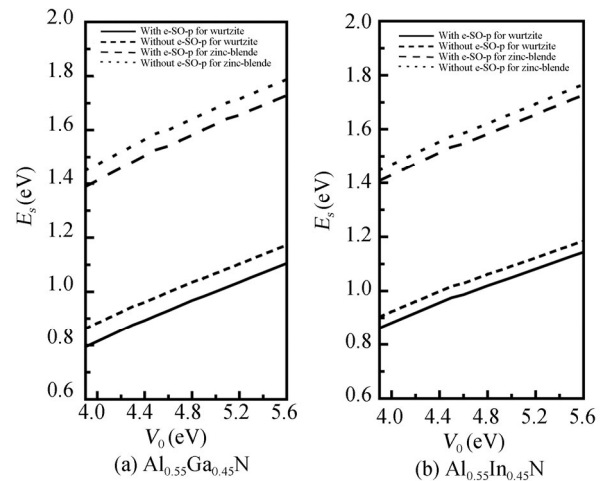
**Fig.2 The shifts of the electronic surface state energy levels  $E_{e\text{-SO}}$  for wurtzite and zinc-blende structures as functions of the composition  $x$  for several III-N TMCs**

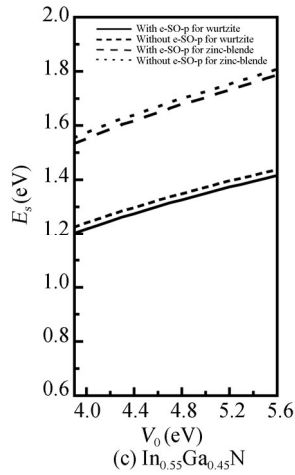
We have also numerical computed the electronic surface-state energy levels  $E_s$  as function of  $V_0$ . We have chosen the value of  $V_0$  to vary in a reasonable range  $3.9\text{--}5.6$  eV<sup>[11]</sup>. The electronic surface-state energy levels and its shifts are shown in Figs.3 and 4 as function of the surface potential  $V_0$  in the numerical computations, respectively. Fig.3 shows that the electronic surface-state energy levels  $E_s$  are linear increase with the increasing of  $V_0$  at the composition  $x=0.55$  for all calculated TMCs. It

is also found, in Fig.3(a) and (b), that two curves of the surface-state energy level are separated distinctly from each other for  $\text{Al}_{0.55}\text{Ga}_{0.45}\text{N}$  and  $\text{Al}_{0.55}\text{In}_{0.45}\text{N}$ . However, the corresponding curves in Fig.3(c) very close to each other for the  $\text{In}_{0.55}\text{Ga}_{0.45}\text{N}$ . It is clear that the e-p interaction lowers the electronic surface-state energy levels. The shifts of the electronic surface-state energy level  $E_{e\text{-SO}}$  caused by the e-SO-phonon interaction in the surface of material are determined mainly by the SO-phonon energy and e-SO-phonon coupling constant. Hence,  $E_{e\text{-SO}}$  are almost independent of surface potential  $V_0$  as Fig.4 shows. The shift of the surface state energy level  $E_{e\text{-SO}}$  is  $67.637$  meV,  $41.428$  meV and  $22.445$  meV for wurtzite  $\text{Al}_{0.55}\text{Ga}_{0.45}\text{N}$ ,  $\text{Al}_{0.55}\text{In}_{0.45}\text{N}$  and  $\text{In}_{0.55}\text{Ga}_{0.45}\text{N}$  at  $V_0=5.0$  eV, respectively.

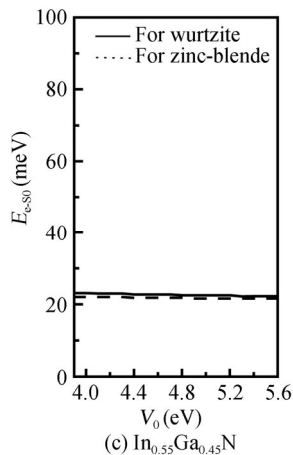
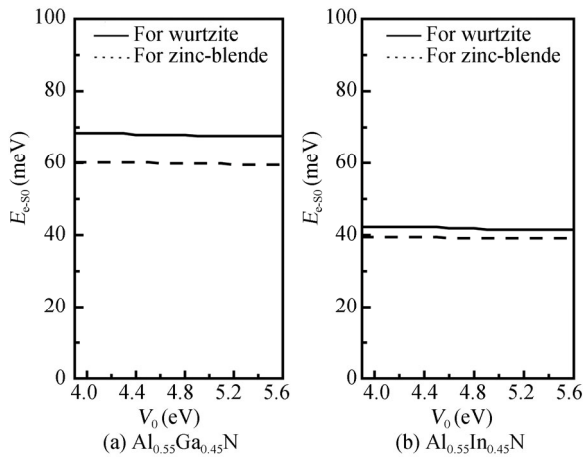
Fig.5 gives the e-SO-p coupling constants  $\alpha_s$  as function of the composition  $x$ , where  $V_0=5.0$  eV. It is seen that there are monotonous changes of the e-SO-p coupling constant  $\alpha_s$  with the composition  $x$  for all calculated materials. The e-SO-p coupling constant  $\alpha_s$  is increase with the increasing of the composition  $x$  for  $\text{Al}_x\text{Ga}_{1-x}\text{N}$  and  $\text{Al}_x\text{In}_{1-x}\text{N}$ . Nevertheless, for  $\text{In}_x\text{Ga}_{1-x}\text{N}$ ,  $\alpha_s$  is become weak with the increasing of the composition  $x$ . It is also found that the e-SO-p coupling constants  $\alpha_s$  in wurtzite materials is larger than that in zinc-blende structures in the whole range of the composition  $x$ .  $\alpha_s$  equals to  $0.758$ ,  $0.851$ ,  $0.497$ ,  $0.743$ ,  $0.818$  and  $0.441$  for the wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ , wurtzite  $\text{Al}_x\text{In}_{1-x}\text{N}$ , wurtzite  $\text{In}_x\text{Ga}_{1-x}\text{N}$ , zinc-blende  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ , zinc-blende  $\text{Al}_x\text{In}_{1-x}\text{N}$  and zinc-blende  $\text{In}_x\text{Ga}_{1-x}\text{N}$  at  $x=0.55$  and with  $V_0=5.0$  eV, respectively. The value of the e-SO-p coupling constant  $\alpha_s$  gets the corresponding numerical value that in the end material AN and BN at  $x=0.0$  and  $x=1.0$ .

We also computed the average penetrating depths of the electronic surface-state wave function  $d$  as function of the composition  $x$ . The result shows that the values of  $d$  are within the scope of  $0.094\text{--}0.070$  nm for wurtzite  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ,  $0.134\text{--}0.069$  nm for wurtzite  $\text{Al}_x\text{In}_{1-x}\text{N}$  and  $0.094\text{--}0.138$  nm for wurtzite  $\text{In}_x\text{Ga}_{1-x}\text{N}$  in the entire range of composition  $x$ , respectively. Tab.4 shows the average penetrating depths  $d$  for aforesaid materials at  $x=0.55$  and  $V_0=5.0$  eV. The average penetrating depth is less than the lattice constant of material. It reveals that the wave function of the electronic surface-state is localizes near the surface.

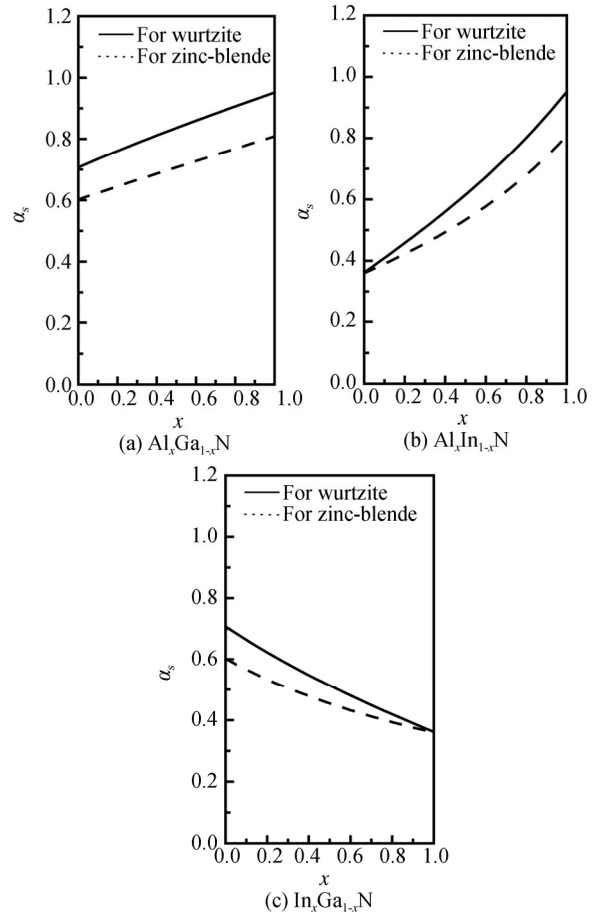




**Fig.3** The electronic surface state energy levels  $E_s$  with and without e-SO-p interactions for wurtzite structures, and with and without e-SO-p interactions for zinc-blende structures as functions of the surface potential  $V_0$  for several III-N TMCs



**Fig.4** The shifts of the electronic surface state energy levels  $E_{e-so}$  for wurtzite and zinc-blende structures as functions of the surface potential  $V_0$  for several III-N TMCs



**Fig.5** The e-SO-p coupling constants  $\alpha_s$  for wurtzite and zinc-blende structures as functions of the composition  $x$  for several III-N TMCs

**Tab.4** The average penetrating depth of the electronic surface state wave function  $d$  and lattice constant  $c$  for wurtzite  $Al_xGa_{1-x}N$ ,  $Al_xIn_{1-x}N$  and  $In_xGa_{1-x}N$

| Material | $Al_{0.55}Ga_{0.45}N$ | $Al_{0.55}In_{0.45}N$ | $In_{0.55}Ga_{0.45}N$ |
|----------|-----------------------|-----------------------|-----------------------|
| $d$ (nm) | 0.080                 | 0.090                 | 0.108                 |
| $c$ (nm) | 0.507                 | 0.533                 | 0.550                 |

In summary, a variational treatment is presented to calculate the electronic surface-state energy level in wurtzite group-III nitride TMCs. The influence of the SO phonon on the electronic surface-state are obtained and discussed for  $Al_xGa_{1-x}N$ ,  $Al_xIn_{1-x}N$  and  $In_xGa_{1-x}N$ , respectively. The numerical results show that the electronic surface-state energy decrease with the increasing of the composition  $x$  in wurtzite III-N TMCs. It is also found that the electronic surface-state energy levels in wurtzite structural material are less than that in the zinc-blende counterparts. It is indicated that the e-SO-p interaction lowers the electronic surface-state energy levels. Hence the influence of the e-SO-p interaction and structural anisotropy on the electronic surface-state in wurtzite

III-N TMCs can not be neglected, especially for the semiconductors which have wide band gap and strong e-p coupling.

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