

# Performance improvement of a self-heterodyne detection BOTDR system employing broad-band laser\*

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The self-heterodyne detection Brillouin optical time domain reflectometer (BOTDR) system using broad-band laser is proposed to reduce coherent Rayleigh noise and improve the system performance. Compared with the system with narrow-band laser, the stimulated Brillouin scattering (SBS) threshold can be improved by about 3 dB. The experimental results of the narrow-band laser measurements for three times independently and the broad-band laser measurement for one time are compared. The root-mean-square (RMS) errors of Brillouin linewidth for two systems with narrow-band laser and broad-band laser are 6.9 MHz and 2.7 MHz, respectively, and the RMS errors of temperature for the heated fiber are about 1.3 °C and 0.7 °C. With the broad-band laser, signal-to-noise ratio (SNR) of the unheated fiber is approximately equivalent to that of the integrated three independent Brillouin signals for the narrow-band laser, and the results are believed to be beneficial for performance improvement and measurement time reduction.

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Distributed optical fiber sensor based on Brillouin scattering was first proposed by Farahi et al in 1989<sup>[1]</sup>, and has attracted intense interest due to the potential application in monitoring large civil structures, power cables, oil and gas pipelines and geological disaster<sup>[2,3]</sup>. One of the subjects mainly investigated is Brillouin optical time domain reflectometer (BOTDR) for single-fiber-ended measurement only and simple implementation. The backscattered spontaneous light is used to measure the round-trip time of optical pulses entering into the sensing fiber and the optical intensity at each sampling point. By analyzing the measured round-trip time and the spontaneous Brillouin intensity, the distributed physical quantities, such as temperature and strain, can be obtained<sup>[4,5]</sup>.

Some of the most popular methods to detect the spontaneous Brillouin intensity and frequency shift involve the optical filter detector, including Fabry-Perot interferometer<sup>[4]</sup>, Mach-Zahnder interferometer<sup>[6]</sup>, and high-precision fiber Bragg grating filter<sup>[7]</sup>. Other detection methods are based on the optical coherent detection, including the local heterodyne detection of reference light and Brillouin scattering light<sup>[8]</sup>, and the self-heterodyne detection of Rayleigh and Brillouin

scattering lights that dispenses with the extra reference light, simplifying the heterodyne detection system<sup>[9,10]</sup>. However, in the self-heterodyne detection BOTDR system, the beat signal is produced by Rayleigh scattering beating with the Stokes and anti-Stokes Brillouin scattering, that coherent Rayleigh noise (CRN) is inherent to backscattered Rayleigh radiation<sup>[11]</sup>, which can degrade the signal-to-noise ratio (SNR) and pose a serious limitation on the achievable temperature/strain measurement accuracy. Provided that the laser frequency and the physical state of sensing fiber remain invariant over the measurement time, the amplitude fluctuations of Rayleigh scattering signals remain stable. Thus, unlike the thermal noise and shot noise of a detector, CRN can not be reduced by signal averaging. It has been reduced by changing the optical frequency of a laser and integrating a large number of independent backscattered signals or increasing the laser bandwidth<sup>[6,9,12]</sup>. In Ref.[9], the temperature coefficients of Brillouin frequency shift (BFS) and change in relative self-heterodyne detection signal power were calibrated, and CRN reduction was achieved by using broad-band laser experimentally. However, the stimulated Brillouin scattering (SBS) threshold improvement, system performance enhancement

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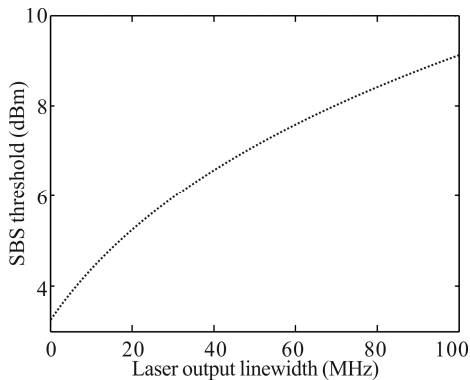
and measurement time reduction are not studied in the self-heterodyne detection BOTDR system with broad-band laser.

In this paper, a broad-band laser is used in a self-heterodyne detection BOTDR system to achieve high-accuracy temperature measurement. The theoretical analysis and formula derivation are performed in the self-heterodyne detection system with broad-band laser. The observed SBS threshold enhancement obtained by employing broad-band laser results to be about 3 dB, accordant with the theoretical analysis. Simultaneously, the root-mean-square (RMS) error of temperature for the 70-m-long heated fiber is reduced from 1.3 °C to 0.7 °C, and the measurement time is shortened to a third at least.

SBS is a nonlinear process that the interactions of the pump and Stokes lights reinforce an acoustic wave through electrostriction, leading to the Stokes light amplification<sup>[13]</sup>. However, with the increase of the incident optical power, SBS can lead to interference, distortion and excess attenuation of the optical signals, resulting in the system SNR reduction and measurement accuracy degradation for large energy fluctuations. The SBS threshold  $P_{th}$  for continue wave can be written as

$$P_{th} = 21 \frac{KA_{eff}}{g_B L_{eff}} \frac{\alpha}{\epsilon} + \frac{D\nu_L}{D\nu_B} \frac{\ddot{\phi}}{\dot{\phi}}, \quad (1)$$

where  $K$  is the polarization factor,  $A_{eff}$  is the effective core area of optical fiber,  $g_B$  is Brillouin gain coefficient,  $D\nu_L$  and  $D\nu_B$  are the laser linewidth and Brillouin natural linewidth, and  $L_{eff}=(1-e^{-\alpha L})/\alpha$  is the effective interaction length, here  $\alpha$  and  $L$  are the attenuation coefficient and optical fiber length. For a 25-km-long standard single-mode fiber, with the parameters of  $K=3/2$ ,  $A_{eff}=50 \text{ mm}^2$ ,  $g_B=5 \times 10^{-11} \text{ m/W}$  and  $\alpha=0.2 \text{ dB/km}$ <sup>[13]</sup> for modeling, the SBS thresholds for different laser output linewidths can be obtained from Eq.(1), and their relationship is shown in Fig.1. With the broadening of the laser linewidth, the SBS threshold can be improved effectively.



**Fig.1 The relationship between SBS threshold and laser linewidth**

Generally, the distributed feedback semiconductor laser (DFB-LD) works in the state of single longitudinal mode. However, due to the distribution for population inversion in a band of the DFB-LD, the laser output is composed of equally frequency spaced and amplitude

time-variant spectral lines in the single longitudinal mode spectrum<sup>[14]</sup>. Provided that  $2N+1$  spectral lines are included in the broad-band laser and the intensities of Rayleigh and Brillouin scattering signals generated by the modulated pulse propagating in the fiber are constant, the electrical fields of the two signals are

$$E_R(t) = \sum_{q=-N}^N E_{Rq} \exp[i(2p\nu_{Rq}t + f_{Rq})], \quad (2)$$

$$E_B(t) = E_S(t) + E_{AS}(t) = \sum_{q=-N}^N \{ E_{Sq} \exp\{i[2p(\nu_0 + qD\nu - \nu_{Bq})t + f_{Sq}]\} + E_{ASq} \exp\{i[2p(\nu_0 + qD\nu + \nu_{Bq})t + f_{ASq}]\} \}, \quad (3)$$

where  $E_{Rq}$  and  $f_{Rq}$  are the field intensity and phase of Rayleigh scattering light generated by the  $q$ th spectral line, respectively,  $\nu_{Rq}$  is the frequency of  $q$ th Rayleigh scattering light that equals to the frequency of the  $q$ th probe light,  $E_{Sq}$  and  $E_{ASq}$  are the field intensities of Stokes and anti-Stokes lights generated by the  $q$ th spectral line,  $\nu_0$  is the center frequency of the laser output spectrum,  $\nu_{Bq}$  is BFS of the  $q$ th spectral line,  $D\nu$  is the frequency interval of spectral lines,  $f_{Sq}$  and  $f_{ASq}$  are the phases of  $q$ th Stokes and anti-Stokes lights, respectively.

Considering that  $E_{Sq}$  equals to  $E_{ASq}$  for spontaneous Brillouin scattering, here the self-heterodyne detection signal of Rayleigh and Stokes lights can be used to analyze heterodyne detection signal of Rayleigh and Brillouin lights. It is assumed that Rayleigh and Stokes lights have aligned polarization, when these two lights are mixed up together in the self-heterodyne detection system, the intensity of superposed signal can be detected by a photodetector (PD), and the self-heterodyne detection signal can be obtained

$$i(t) = R \{ \overline{E}_R(t) + \overline{E}_S(t) \} \{ E_R(t) + E_S(t) \} = R \sum_{q=-N}^N \{ \overline{E}_{Rq} \exp[-i(2p\nu_{Rq}t + f_{Rq})] + E_{Sq} \exp\{-i[2p(\nu_0 + qD\nu - \nu_{Bq})t + f_{Sq}]\} \} \sum_{q=-N}^N \{ E_{Rq} \exp[i(2p\nu_{Rq}t + f_{Rq})] + E_{Sq} \exp\{i[2p(\nu_0 + qD\nu - \nu_{Bq})t + f_{Sq}]\} \} = i_R(t) + i_S(t) + i_{RS}(t), \quad (4)$$

where  $R$  is the responsivity of a detector,  $\overline{\phantom{x}}$  represents the conjugate,  $i_R(t)$  and  $i_S(t)$  are the photocurrent items produced by Rayleigh and Stokes lights, and  $i_{RS}(t)$  is the cross photocurrent item of the two lights. Due to the characteristics of narrow linewidth and strong coherence of single spectral line, and the independence and randomness of the phases for different spectral lines, a single-frequency signal carries the information of BFS and Brillouin intensity obtained by self-heterodyne detection signal of Rayleigh light and Stokes light generated from the same spectral line, and the heterodyne detection signals obtained by Rayleigh and Stokes lights from different spectral lines perform as the background noise in the form of combination frequencies. The items of the direct current, sum-frequency and doubling-frequency in Eq.(4) can be filtered by the band-pass filter, and a summing

AC signal current at the different frequencies between Rayleigh scattering and Stokes light is

$$i_{AC}(t) = 2R \sum_{q=-N}^N \sqrt{P_{Rq}(t)P_{Sq}(t)} \cos[2\pi\nu_{Bq}t + f_{RSq}(t)], \quad (5)$$

where  $P_{Rq}(t)$  and  $P_{Sq}(t)$  are the powers of Rayleigh and Stokes lights generated by the  $q$ th spectral line at moment  $t$ , respectively,  $f_{RSq}(t)$  is the phase difference between the  $q$ th Rayleigh and Stokes lights, and it fluctuates within  $[0, 2\pi]$ , following the uniform random distribution. Although the dependence of the BFS on probe light wavelength is 7 MHz/nm around 1 550 nm<sup>[8]</sup>, the broadening due to different Brillouin frequency shifts of the probe lights is basically symmetrical, and the center of the superposed Brillouin scattering spectrum (BSS) remains unchanged. Therefore, for the broad-band laser with a linewidth much narrower than 1 nm, the impact of the broadening on the measured superposed BSS can be neglected.

For the sake of analysis, supposed that the probe pulse light of  $2N+1$  spectral lines with equal power is used, and the negligible differences among  $P_{Rq}(t)$ ,  $P_{Sq}(t)$  and  $\nu_{Bq}$  are all ignored, the mean-square signal photocurrent is

$$\langle i_{AC}^2(t) \rangle = 2R^2 \sum_{p,q=-N}^N \left\langle \sum_{p'=q}^N \cos(f_{RSp}(t) - f_{RSq}(t)) \right\rangle \ddot{y} \ddot{b} \quad (6)$$

$$P_R(t)P_S(t) = 2(2N+1)R^2 P_R(t)P_S(t) .$$

Therefore, the SNR  $R_{SN}$  of self-heterodyne detection BOTDR system employing broad-band laser with  $2N+1$  spectral lines can be expressed as

$$R_{SN} = \frac{2(2N+1)R^2 P_R(t)P_S(t)}{4kT\mathcal{D}f/R_L + 2eR(2N+1)P_R(t)\mathcal{D}f + \langle i_{CRNb}^2 \rangle} , \quad (7)$$

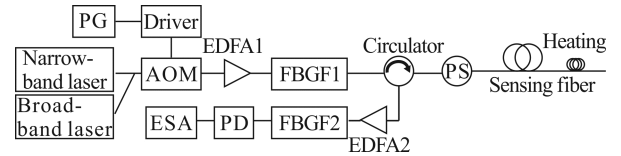
where the first and the second items in the denominator are the thermal and the shot noises of detector, respectively, the third term is CRN that caused by the interference between the lights backscattered at different positions along the fiber, and the coherent fading has a  $\mathcal{D}f/(\mathcal{D}\nu_b \mathcal{D}z)$  dependence<sup>[11]</sup>, here  $\mathcal{D}f$  is the electrical bandwidth of PD,  $\mathcal{D}\nu_b$  is the bandwidth of the broad-band laser, and  $\mathcal{D}z$  is the spatial resolution of the self-heterodyne BOTDR system.  $k$  is Boltzmann constant,  $T$  is thermodynamic temperature,  $e$  is elementary charge, and  $R_L$  is load resistance. Compared with that with the narrow-band laser, the SNR improvement of the self-heterodyne detection system with broad-band laser is

$$SNRI = \frac{(2N+1) \langle 4kT\mathcal{D}f/R_L + 2eRP_R(t)\mathcal{D}f + \langle i_{CRNb}^2 \rangle \rangle}{4kT\mathcal{D}f/R_L + 2eR(2N+1)P_R(t)\mathcal{D}f + \langle i_{CRNb}^2 \rangle} , \quad (8)$$

where  $\langle i_{CRNb}^2 \rangle$  is in proportional to  $\mathcal{D}f \mathcal{X}_g / (\mathcal{D}\nu_b \mathcal{D}z)$ , here  $\nu_g$  is the velocity of optical radiation in the fiber, and  $\mathcal{D}\nu_b$  is the bandwidth of the narrow-band laser.

To verify the system performance improvement with broad-band laser, an experimental setup of self-

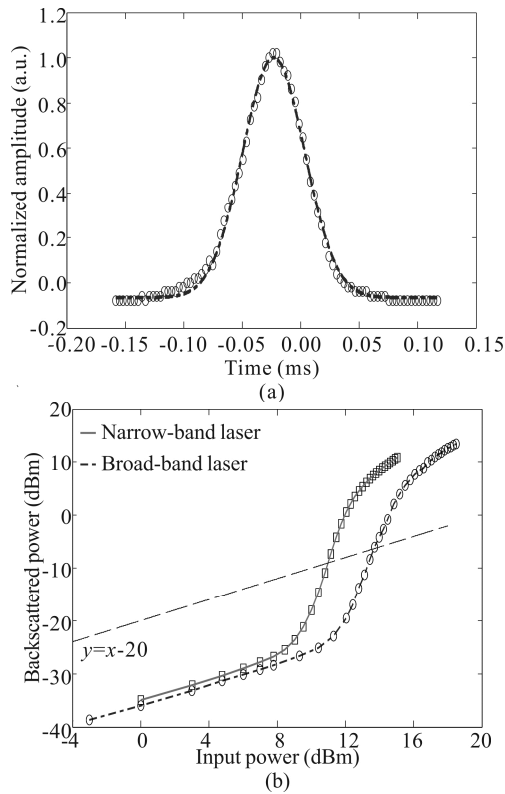
heterodyne detection BOTDR system is shown in Fig.2. The laser output was modulated by an acousto-optic modulator (AOM) driven by pulse generator (PG) to generate pulse sequence with a repetition rate of 10 kHz. The pulse width and peak power of the optical pulse were 130 ns and 27 dBm, respectively. The modulated pulse was amplified by an erbium doped fiber amplifier 1 (EDFA1) and amplified spontaneous emission noise filtered by fiber Bragg grating filter 1 (FBGF1), and then launched into the sensing fiber to generate the backscattered Rayleigh and spontaneous Brillouin scattering signals for producing the beat signal. The beat signal was detected by a PD with bandwidth of 11.9 GHz and the electrical spectrum analyzer (ESA) with resolution bandwidth of 8 MHz operated in the ‘zero-span’ mode to obtain the power traces along the sensing fiber at different beat frequencies. Each trace was averaged for 5 000 times. The length of sensing fiber was about 9.5 km, and measurements were performed at room temperature of 28 °C. The 70-m-long optical fiber at the end was wound with no tension to avoid any strain and heated by a thermostatic waterbath. To reduce the polarization-induced fading noise and increase the fiber input power, polarization scrambler (PS) was used at the fiber input. For comparison, the peak powers were the same for two systems employing the narrow-band and broad-band lasers.



**Fig.2 Experimental setup of the self-heterodyne detection BOTDR system**

Two main laser components of a narrow-band laser with 1.86 MHz (TL-2020-C-102A, Santur CO., Ltd.) at 3 dB and a broad-band laser with 55.23 MHz (OSTDFB-1550-SM, Fby Photoelectric Technology CO., Ltd.) at 3 dB were applied to the self-heterodyne detection BOTDR system. The wavelength difference between these two lasers is 2.34 pm. To understand the spectrum characteristics of the broad-band laser, the spectrum can be measured by an optical spectrum analyzer (OSA) with a resolution bandwidth of 26 MHz and a free spectral range of 10 GHz (corresponding to 6.77 ms observed on the oscilloscope), and it is shown in Fig.3(a). Comparing with the theoretical analysis of a broad-band laser, the fine spectral lines included in the laser output spectrum can not be observed, this is attributed to the low bandwidth resolution of the OSA. To compare the influence of linewidth on SBS threshold, the backscattered powers are monitored by power meter for these two lasers, as shown in Fig.3(b). The SBS threshold is obtained with the criterions of 1% Stokes conversion (the straight line in Fig.3(b)), it can be improved by

about 3 dB with the broad-band laser, and the result is in agreement with the theoretical analysis basically. And this indicates that the fiber input power can be increased effectively for the improvement of the system SNR.



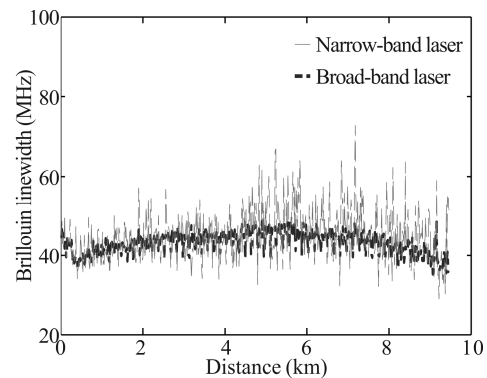
**Fig.3 (a) Measured laser output spectrum of the broad-band laser and (b) SBS threshold of two lasers**

To compare the influence of laser linewidth on the performance of self-heterodyne detection BOTDR sensing system, the spectrum parameters are obtained by fitting the measured spectra with a Lorentzian curve in a spatial resolution. To verify the shortness of measurement time with broad-band laser, the comparison of all spectrum parameters are made between the narrow-band laser measurements for three times in three different days independently and the broad-band laser measurement only for one time, and the output frequencies of the narrow-band laser at each time are different slightly, and the phases of the backscattered signals change. And these three independent self-heterodyne detection signals for the narrow-band laser are integrated.

The Brillouin linewidths are shown in Fig.4, the greater volatility of the fitting linewidths observed for the narrow-band laser mainly comes from the greater amplitude fluctuations of Rayleigh scattering, and the averaged linewidths of the total fiber for two lasers are both approximately 43 MHz. The RMS errors of the linewidths for the narrow-band and the broad-band lasers are 6.9 MHz and 2.7 MHz, respectively, and the experimental result shows that the measurement accuracy

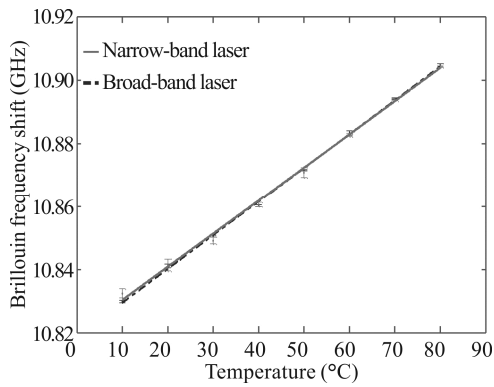
mainly be improved effectively for the broad-band laser with greatly reduced CRN. As the Rayleigh and Brillouin scattering signals generated from a single space resolution with the same features of frequency drift and chirp, and high coherence of the two signals is kept, the spectral broadening due to the frequency instability of the laser can be eliminated by employing the self-heterodyne detection of Rayleigh and Brillouin scattering that the reduction of measurement accuracy at the fiber end can be avoided properly. The comparative study of Brillouin linewidths for these two lasers is not mentioned in Ref.[9].

It would be specially mentioned that the self-heterodyne detection Brillouin spectrum for the broad-band laser composed of independent spectral lines is the superposed Brillouin spectrum from different spectral lines. Therefore, for the broad-band laser with linewidth of 55.23 MHz, the linewidth of the superposed Brillouin spectrum can not be broadened as shown in Fig.4 for the dependence of the BFS on the wavelength to be 7 MHz/nm.



**Fig.4 Self-heterodyne detection Brillouin linewidths along 9.5-km-long sensing fiber**

To realize the fiber temperature measurement based on the center frequency change of the self-heterodyne detection power spectrum, it is necessary to calibrate the temperature coefficients of the BFS for two lasers. The 70-m-long fiber at the end of 9.5-km-long fiber was heated by the thermostatic waterbath, and temperature setup is adjusted from 10 °C to 80 °C changing with 10 °C step. The linear fitting results of the measured BFS changing with the temperature variation are depicted in Fig.5. The temperature coefficients of the BFS are both about 1.07 MHz/°C for these two lasers, and the coefficient for the broad-band laser is also provided in Ref.[9]. But beyond that, the changes of BFS for two lasers at different temperatures are given in Fig.5. As shown in Fig.5, the larger deviation at a given temperature for the narrow-band laser observed mainly results from the greater CRN, which indicates that the broad-band laser is more beneficial for improving system performance indicators of the self-heterodyne detection BOTDR.

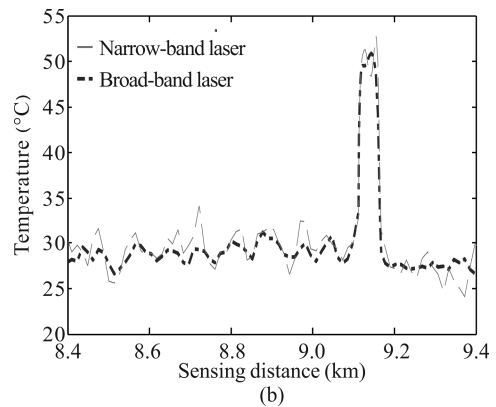
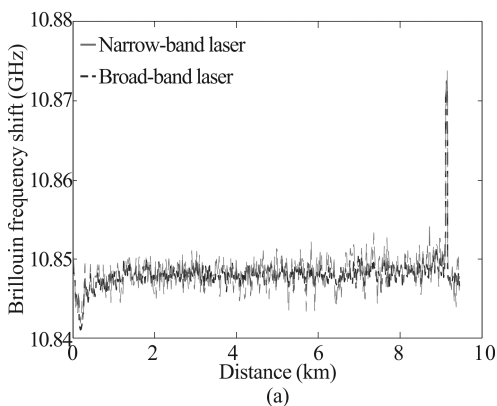


**Fig.5 The relationship of BFS and fiber temperature**

When the 70-m-long fiber in the thermostatic waterbath was heated to 50 °C, the BFS measured by these two systems with narrow-band and broad-band lasers are shown in Fig.6. Fig.6(a) shows the BFS curves of the total fiber. Although the BFS of the unheated section measured by these two systems are both approximately 10.848 GHz, the RMS errors of the BFS obtained by the self-heterodyne detection BOTDR systems with narrow-band and broad-band lasers are 1.6 MHz and 0.9 MHz, respectively. This is attributed to the CRN of the self-heterodyne detection Brillouin signals. As the temperature of the heated fiber rises, the intensity of the self-heterodyne detection signal increases, and the heterodyne detection Brillouin linewidth narrows<sup>[15]</sup>. In optical fiber Brillouin sensing system, the minimum detectable BFS  $d\nu_B$  is proportion to the Brillouin linewidth  $D\nu_B$ , and is can be shown as<sup>[16]</sup>

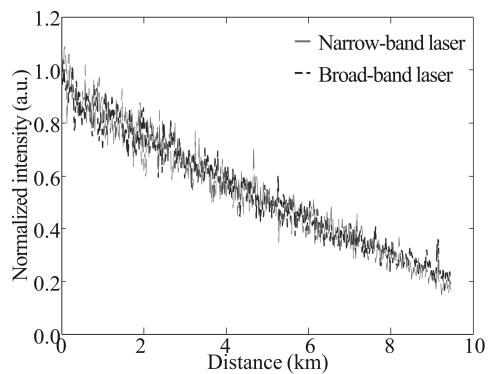
$$d\nu_B = \frac{D\nu_B}{\sqrt{2}(R_{SN})^{1/4}}, \quad (9)$$

which indicates that narrower Brillouin spectrum and higher SNR represent higher frequency resolution. Therefore, the frequency fluctuation for the heated fiber is smaller than that for the unheated fiber in the room temperature, and the RMS errors of the BFS for the 70-m-long heated section are 1.4 MHz and 0.8 MHz, and thus the temperature errors are separately 1.3 °C and 0.7 °C. The experimental results show that the fluctuations of BFS and temperature induced by CRN can be reduced effectively with the employment of broad-band laser, and the measurement time is saved, realizing high accuracy and rapid measurement of temperature.



**Fig.6 (a) BFS and (b) temperature obtained by the self-heterodyne BOTDR systems with narrow-band and broad-band lasers**

The averaged normalized intensity of the self-heterodyne detection Brillouin signals of three measurements for narrow-band laser and the normalized intensity of one measurement for broad-band laser are shown in Fig.7. The RMS errors of the intensities for the unheated fiber with the narrow-band and broad-band lasers are 0.051 and 0.043, and those for the heated fiber are 0.045 and 0.021, respectively. Compared with using the narrow-band laser, the use of a broad-band laser in the self-heterodyne detection BOTDR system will benefit not only reducing the effect of laser frequency instability and CRN for Brillouin linewidth measurement, but also reducing CRN and improving accuracy for Brillouin frequency/intensity measurement that is required for performing high accuracy measurement of temperature. Moreover, the measurement time can be shortened to a third at least with the broad-band laser.



**Fig.7 Normalized self-heterodyne detection Brillouin traces along 9.5-km-long sensing fiber**

In conclusion, in the BOTDR system based on the self-heterodyne detection of Rayleigh and Brillouin scattering, SNR is not only limited by the electronic noise, but also can be greatly degraded by the CRN. With the broad-band laser, the SNR is approximately equivalent to that of the integrated three independent Brillouin signals for the narrow-band laser. And the RMS errors of the linewidths and BFS for the broad-band laser are about

4.2 MHz and 0.7 MHz less than that for the narrow-band laser. The experimental results show that CRN can be reduced obviously and the sensing performance is improved efficiently by employing the broad-band laser, and the measurement time can be saved.

## References

- [1] D. Culverhouse, F. Farahi, C. N. Pannell and D. A. Jackson, *Electronics Letters* **25**, 913 (1989).
- [2] T. Wei, *Journal of Optoelectronics·Laser* **28**, 492 (2017). (in Chinese)
- [3] X. Feng, W. J. Wu, X. Y. Li, X. W. Zhang and J. Zhou, *Smart Structures and Systems* **15**, 245 (2015).
- [4] H. Y. Xia, M. J. Shangguan, G. L. Shentu, Ch. Wang, J. W. Qiu, M. Y. Zheng, X. P. Xie, X. K. Dou, Q. Zhang and J. W. Pan, *Optics Communications* **381**, 37 (2016).
- [5] K. Nishiguchi, Li Che-Hsien, A. Guzik and K. Kishida, *Sensors***14**, 4731 (2014).
- [6] H. H. Kee, G. P. Lees and T. P. Newson, *Optics Letters* **25**, 695 (2000).
- [7] S. Kim, H. Kwon, I. Yang, S. Lee, J. Kim and S. Kang, *Sensors* **13**, 15452 (2013).
- [8] C. L. Li, Y. G. Lu and X. P. Zhang, *Electronics Letters* **48**, 1139 (2012).
- [9] X. J. Li, Y. Q. Li, Y. J. He and L. X. Zhang, *Acta Photonica Sinica* **45**, 1106004 (2016). (in Chinese)
- [10] Y. Q. Li, X. J. Li, H. B. Fan, Q. An and L. X. Zhang, *Optoelectronics Letters* **13**, 0414 (2017).
- [11] K. De Souza, *Measurement Science and Technology* **17**, 1065 (2006).
- [12] Y. Q. Li, L. X. Zhang, H. B. Fan and H. Li, *Optoelectronics Letters* **14**, 84 (2018).
- [13] S. L. Hu, L. W. Li, X. K. Yi and F. Teng, *IEEE Photonics Technology Letters* **29**, 330 (2017).
- [14] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, England: Cambridge University Press, 900 (1995).
- [15] M. Nikles, L. Thevenaz and P. A. Robert, *Journal of Lightwave Technology* **15**, 1842 (1997).
- [16] Y. Q. Li, X. J. Li and Q. An, *Acta Optica Sinica* **36**, 1006001 (2016). (in Chinese)