

Performance analysis of multiuser diversity scheduling schemes in FSO communication system*

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Taking into consideration the aperture averaging, the system performance of a point-to-multipoint free space optical (FSO) system for various multiuser diversity scheduling schemes is studied over exponentiated Weibull (EW) fading channels. The selection principles of greedy scheduling (GS), selective multiuser diversity scheduling (SMDS), proportional fair scheduling (PFS) and selective multiuser diversity scheduling with exponential rule (SMDS-ER) schemes are introduced and compared on the basis of time-varying behavior of turbulence channel fading in the present system. The analytical average capacity expressions for the GS and SMDS schemes are derived, respectively. Then, the relative capacity simulations for PFS and SMDS-ER schemes are also provided over EW fading channels with the binary phase shift keying (BPSK) modulation. The results show that the GS scheme obtains the maximum average capacity at the cost of the fairness of users. The SMDS-ER receives the minimum capacity, but it guarantees the fairness of users. The SMDS and PFS schemes can get balance between capacity and fairness. This study can be used for FSO system design.

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Free space optical (FSO) communication, which has lots of advantages, such as unlicensed spectrum, excellent security and low cost, has drawn considerable attention^[1,2]. Besides, it is a promising solution for the "last mile" problem, compared with the traditional wireless communication^[3]. Nevertheless, the system performance is restricted to serious atmospheric related issues, especially when the distance between source and destination is longer than 1 km. As a major limiting factor, atmospheric turbulence, which results from the pressure of atmosphere and inhomogeneities in temperature, will lead to random fluctuations of amplitude and phase of light intensity^[4]. In order to assess the performance of FSO system in different atmospheric turbulence regimes, some statistical fading models, such as log-normal (LN), K and gamma-gamma (G-G) distributions, have been presented with good achievements. Traditionally, LN and K distributions are often valid in weak and strong turbulence regimes, respectively. G-G model is commonly adopted for all turbulence regimes at a point receiver. However, for larger receiver apertures under moderate-to-strong conditions, G-G distribution does not hold very well^[5]. Recently, a generalized turbulence-induced fading model, named exponentiated Weibull (EW) distribution, has been proposed by R. Barrios and F. Dios to model the distribution of the irradiance. The

studies in Refs.[6] and [7] showed that EW distribution provides the fantabulous fitting between simulation and experiment data under different aperture averaging conditions with weak-to-strong turbulence strengths. As is well-known, for the atmospheric turbulence fading, diversity is one promising fading mitigation technique. Commonly, there are several diversity techniques adopted to improve the FSO system performance^[8]. One is the wavelength diversity, which is inoperative in FSO system because the effect of atmospheric turbulence remains nearly the same for all wavelengths. Time diversity is considered to be inefficient and time-consuming due to bit interleaving and coding. Spatial diversity, which employs multiple laser transmitters/receivers, is also an alternative solution to relieve the impact of atmospheric turbulence. However, the deployments of multiple transmitters and receivers usually bring about a dramatic increase in cost and complexity. As a novel technique, multiuser diversity (MD) is extended from a radio frequency system to a point-to-multipoint turbulent FSO communication system^[9]. This system is comprised of a central node with K apertures and K users, where the central node launches the same data to all users and aims to serve only one selected user at each time slot.

Motivated by the above analysis, MD is extended to FSO

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system with the consideration of EW distribution in this paper. On the basis of aperture averaging, the channel capacities of greedy scheduling (GS), selective multiuser diversity scheduling (SMDS), proportional fair scheduling (PFS) and selective multiuser diversity scheduling with exponential rule (SMDS-ER) schemes are obtained over EW fading channels with binary phase shift keying (BPSK) modulation. The system performance is analyzed with different turbulence strengths, receiver aperture sizes and user numbers.

Fig.1 shows the FSO communication system with MD scheme, in which a source node is equipped with K independent users and K optical transmit apertures. Each aperture at central node is directed to the optical receiver. It is assumed that the distances between the central node and users are the same, and the channel state of each user is independent and identically distributed.

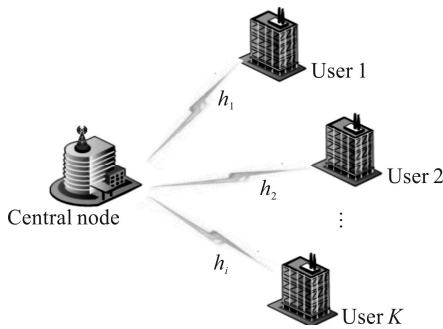


Fig.1 A point-to-multipoint FSO communication system

The MD-FSO system employs intensity modulated/direct detection (IM/DD) with BPSK subcarrier intensity modulation. The received electrical signal of i -th channel at m -th time slot can be given as^[5,9]

$$y_{i,m} = h_{i,m} R x_{i,m} + n_{i,m}, \quad i \in (1, k), \quad (1)$$

where $h_{i,m}$ represents the channel gain, R denotes the responsivity of photodetector, $x_{i,m}$ is the input signal with average transmitted optical power P_t , and $n_{i,m}$ represents the signal-independent additive white Gaussian noise (AWGN) with zero mean and variance s_n^2 . Thus, the instantaneous electrical signal-to-noise ratio (SNR) $\gamma_{i,m}$ of each link at m th time slot is given as

$$g_{i,m} = \bar{g}_{i,m} h_{i,m}^2 = R^2 P_t^2 h_{i,m}^2 / 2s_n^2, \quad (2)$$

where $\bar{g}_{i,m}$ is the average SNR, and $\bar{g}_{i,m} = R^2 P_t^2 / 2s_n^2$.

The EW distribution is used to model atmospheric turbulence, and the probability density function (PDF) f_{EW} of the channel gain $h_{i,m}$ by this model can be expressed as^[6,7]

$$f_{EW}(h_{i,m}) = \frac{a_{i,m} b_{i,m} \alpha_{i,m} \beta_{i,m}^{-1}}{h_{i,m} \Gamma(\alpha_{i,m}) \Gamma(\beta_{i,m})} \exp\left\{-\frac{\alpha_{i,m}}{\beta_{i,m}} \frac{1}{h_{i,m}}\right\} \left[1 - \exp\left\{-\frac{\alpha_{i,m}}{\beta_{i,m}} \frac{1}{h_{i,m}}\right\}\right]^{\alpha_{i,m}-1}, \quad (3)$$

where $\alpha_{i,m} > 0$ denotes a shape parameter, which depends on receiver aperture size, $\beta_{i,m} > 0$ is a shape parameter related to the scintillation index, and $\eta_{i,m} > 0$ is a scale parameter.

The PDF with regard to $\gamma_{i,m}$ of i th channel at m th time slot can be derived by the relationship of $f_{g_{i,m}}(g) = f_{EW}(\sqrt{g/\bar{g}_{i,m}}) / 2\sqrt{g\bar{g}_{i,m}}$ as

$$f_{g_{i,m}}(g) = \frac{a_{i,m} b_{i,m}}{2\bar{g}_{i,m} h_{i,m}^{b_{i,m}}} \left(\sqrt{g/\bar{g}_{i,m}}\right)^{b_{i,m}-2} \exp\left\{-\frac{\alpha_{i,m} \sqrt{g/\bar{g}_{i,m}}}{h_{i,m}}\right\} \left[1 - \exp\left\{-\frac{\alpha_{i,m} \sqrt{g/\bar{g}_{i,m}}}{h_{i,m}}\right\}\right]^{\alpha_{i,m}-1}, \quad (4)$$

and the corresponding CDF can be written as

$$F_{g_{i,m}}(g) = 1 - \exp\left\{-\frac{\alpha_{i,m} \sqrt{g/\bar{g}_{i,m}}}{h_{i,m}}\right\} \left[1 - \exp\left\{-\frac{\alpha_{i,m} \sqrt{g/\bar{g}_{i,m}}}{h_{i,m}}\right\}\right]^{\alpha_{i,m}-1}. \quad (5)$$

For the investigated system with MD scheme, it is assumed that the turbulence strengths are the same for all the channels, i.e., $\alpha_{i,m} = \alpha$, $\beta_{i,m} = \beta$, $\eta_{i,m} = \eta$, and all the links have the same average SNR, i.e. $\bar{g}_{i,m} = \bar{g}$. Besides, the destination knows the channel state information (CSI) at the beginning of each time slot, and feeds it back to the source of system^[10,11].

In a system with greedy scheduling (GS) scheme, where the central node serves the user with the best channel condition at any time slot, the instantaneous capacity of i th channel at m th time slot is defined on the basis of Shannon formula as

$$C_{i,m} = \log_2(1 + g_{i,m}). \quad (6)$$

Let us define a random variable γ_{GS} based on GS scheme, and γ_{GS} can be expressed as^[9]

$$g_{GS} = \max(g_{i,m})_{i=1}^K. \quad (7)$$

The CDF of γ_{GS} can be obtained as

$$F_{GS}(g) = \prod_{i=1}^K F_{g_{i,m}}(g), \quad (8)$$

and the PDF can be deduced by differentiating Eq.(8) with regard to γ as follows

$$f_{GS}(g) = K \prod_{i=1}^{K-1} F_{g_{i,m}}(g) f_{g_{i,m}}(g). \quad (9)$$

For the MD-FSO system with GS scheme, the instantaneous capacity $C_{ins,m}$ of m th time slot can be given by

$$C_{ins,m} = \log_2 \left[1 + \max(g_{i,m})_{i=1}^K \right]. \quad (10)$$

The average capacity C_{GS} can be obtained from Eqs.(6), (9) and (10) as

$$C_{GS} = E\{C_{ins,m}\} = \frac{1}{\ln 2} \int_0^\infty \ln(1+g) f_{GS}(g) dg = \frac{abK}{2 \ln 2 \bar{g}^b} \int_0^\infty \ln(1+g) \left(\sqrt{g/\bar{g}}\right)^{b-2} \exp\left\{-\frac{\alpha \sqrt{g/\bar{g}}}{h}\right\} \left[1 - \exp\left\{-\frac{\alpha \sqrt{g/\bar{g}}}{h}\right\}\right]^{\alpha K-1} dg. \quad (11)$$

If the variable change $x = \sqrt{g/\bar{g}}/h$ is carried out, Eq.(11) will take the form as

$$C_{GS} = \frac{abK}{\ln 2}$$

$$\int_0^\infty \ln(1 + \bar{g}^2 x^2) x^{b-1} \exp(-x^b) dx - \exp(-x^b) \int_0^\infty x^{aK-1} dx \quad (12)$$

With the help of Meijer's G-function and generating function of the second kind Stirling number^[12], Eq.(12) can be further simplified as

$$C_{GS} = \frac{abK}{2 \ln 2} \sum_{n=m}^{\infty} \frac{a^n}{n!} (-1)^{2m+n-q} \frac{\Gamma(\frac{p}{2}) \Gamma(\frac{b}{2}) \Gamma(\frac{p}{2} + \frac{b}{2})}{\Gamma(\frac{p}{2}) \Gamma(\frac{b}{2}) \Gamma(\frac{p}{2} + \frac{b}{2})} \frac{\Gamma(\frac{p}{2}) \Gamma(\frac{b}{2}) \Gamma(\frac{p}{2} + \frac{b}{2})}{\Gamma(\frac{p}{2}) \Gamma(\frac{b}{2}) \Gamma(\frac{p}{2} + \frac{b}{2})} \quad (13)$$

where $\Delta(a, b) = b/a, (b+1)/a, \dots, (b+a-1)/a$, and $l/p = \beta/2$. Here, l and p are both positive numbers.

In this section, a practically feasible approach of SMDS scheme^[13] is used to make up the drawback of GS scheme that the central node should have vast knowledge of system configuration. The SMDS performs as follows.

- 1) Let us define an optimum threshold γ_{th} according to outage probability ε given by $\varepsilon = P(g_i < g_{th}, i \in \{1, \dots, K\}) = [F_{g_{th}}(g_{th})]^K$. Thus, γ_{th} can be computed as $g_{th} = \bar{g}^2 [-\ln(1 - \varepsilon^{1/aK})]^{2/b}$.
- 2) If the instantaneous SNR in i th channel at m th time slot satisfies $\gamma_{i,m} \geq \gamma_{th}$, the corresponding user is allowed to feedback its CSI to the central node. $g_m^s @ \{g_{i,m}; g_{i,m} > g_{th}\}$ can be denoted as a set of selected instantaneous SNRs and $I_m^s @ \{i; i \in \{1, \dots, K\} \text{ and } g_{i,m} > g_{th}\}$ is defined as the index set of selected users.
- 3) If $I_m^s = \emptyset$, the scheduler selects a random user to communicate, i.e., $k_{m,SMDS}^* = \text{argrand } g_m$, here, $g_m @ \{g_{1,m}, g_{2,m}, \dots, g_{K,m}\}$.
- 4) If $I_m^s \neq \emptyset$, the scheduler selects the user with maximum instantaneous SNR at m th time slot as the best one, i.e., $k_{m,SMDS}^* = \text{arg max}_{i \in I_m^s} g_m^s$.

Thus, the variable $k_{m,SMDS}^*$ can be written as

$$k_{m,SMDS}^* = \begin{cases} \text{max}_{i \in I_m^s} g_m^s, & \text{if } I_m^s \neq \emptyset \\ \text{rand } g_m, & \text{if } I_m^s = \emptyset \end{cases} \quad (14)$$

and the CDF of $k_{m,SMDS}^*$ can be obtained as

$$F_{k_{m,SMDS}^*}(x) = P\{k_{m,SMDS}^* \leq x\} = P\left\{\bigcap_{i=1}^K g_i < g_{th} \mid \bigcup_{i=1}^K g_i \geq g_{th}\right\} = \frac{F_{g_{th}}(x)^K - \prod_{i=1}^K F_{g_{th}}(g_{th})}{1 - \prod_{i=1}^K F_{g_{th}}(g_{th})} \quad (15)$$

Similar to Eqs.(8) and (9), the corresponding PDF can be written as

$$f_{k_{m,SMDS}^*}(x) = f_{g_{th}}(x) \frac{d}{dx} F_{g_{th}}(x)^K + \sum_{i=1}^{K-1} f_{g_{th}}(g_{th}) \frac{d}{dx} F_{g_{th}}(x)^{K-i}$$

$$f_{g_{th}}(x) \frac{d}{dx} F_{g_{th}}(x)^K - F_{g_{th}}(g_{th}) \frac{d}{dx} F_{g_{th}}(x)^{K-1} \quad (16)$$

The average capacity C_{SMDS} of SMDS scheme can be expressed as

$$C_{SMDS} = \frac{1}{\ln 2} \int_0^\infty \ln(1+x) f_{k_{m,SMDS}^*}(x) dx, \quad (17)$$

and this equation can be numerically calculated by Romberg integration method.

Compared with GS and SMDS schemes, PFS scheme^[14], which is real time and sensitive to time delay, provides a balance between the average capacity and user fairness. In PFS system, users compete for resources according to their channel conditions according to their own average channel observations. The scheduler selects the best user at m th time slot as

$$i_m^* = \text{arg max}_{i \in \{1, \dots, K\}} (R_{i,m} / \bar{R}_{i,m}), \quad (18)$$

where $R_{i,m}$ is the instantaneous capacity of i th channel at m th time slot, and $\bar{R}_{i,m}$ is the average capacity of user i before m th time slot. The average capacity of each channel can be updated on the basis of

$$\bar{R}_{i,m+1} = \begin{cases} (1-1/t_c) \bar{R}_{i,m} + (1/t_c) R_{i,m}, & i = i_m^* \\ \bar{R}_{i,m}, & i \neq i_m^* \end{cases} \quad (19)$$

where t_c represents a sliding window and determines the trade-off between capacity and latency. When t_c tends to ∞ , average capacity can be updated by $\bar{R}_{i,m+1} = \bar{R}_{i,m} = \bar{R}_{i,0}$, where $\bar{R}_{i,0}$ is a chosen initial value, and then the PFS becomes the GS.

Based on the findings of exponential rule, a modified SMDS scheme, named SMDS-ER, is studied to provide user fairness and achieve the maximum throughput with the latency taken into account. In this algorithm, the scheduler selects the best user at m th time slot as^[15]

$$i_m^* = \text{arg max}_{i \in \{1, \dots, K\}} v_i \frac{R_{i,m}}{\bar{R}_{i,m}} \exp\left(-\frac{D_{i,m}}{\bar{D}_m}\right) \quad (20)$$

where v_i is the weighting coefficient for i th user, and let us set $v_i = 1$ in a homogeneous FSO system environment with the same turbulence condition. s is dependent on the number of users and turbulence strength. $D_{i,m}$ denotes the latency observed by i th user, and it is assumed that i th user has no service until m th time slot. \bar{D}_m represents the average latency observed by $K-1$ users at m th time slot, and can be defined as

$$\bar{D}_m @ \frac{1}{K-1} \sum_{i=1, i \neq i_m^*}^K D_{i,m} \quad (21)$$

The analytical average capacity results for GS and SMDS schemes are obtained from Eqs.(13) and (17). Generating functions of the second kind Stirling number and Romberg integration method are adopted to calculate the equations. Without loss of generality, it is assumed that the distance of each link in a selected system is equal. The parameters (α, β, η) employed to describe the atmospheric turbulence are extracted from the best PDF fitting in Refs.[6] and [7].

Fig.2 shows the average capacity of a point-to-multipoint

EW-distributed system on the basis of GS scheme under different user conditions with two aperture sizes of 100 mm and 200 mm. For weak and strong turbulence conditions, the corresponding Rytov variance values, which are often used to describe turbulence strength, are equal to 0.32 and 15.97. Romberg integration method is applied to verify the validity of the theoretical derivation. As seen, the average capacity increases with the increase of K for different turbulence strengths. For example, in the strong regime with aperture of 200 mm, at a given SNR of 5 dB, the average capacity values of the studied system are approximately 3.5 bit/s/Hz and 1.5 bit/s/Hz for $K=10$ and 5, respectively. This is because the increase of users offers a higher possibility of selecting a user with better channel state. Besides, the average capacity performance gets improved with the increase of turbulence strengths. This is due to that stronger turbulence strength brings more fluctuations in phase and amplitudes, which provides a better change to obtain the best user. Moreover, the average capacity performance is significantly improved by increasing the aperture size because aperture averaging can mitigate the variances of the power and intensity, which has been confirmed in previous work^[8].

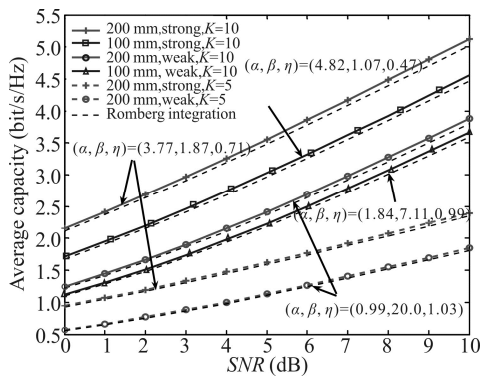


Fig.2 Average capacity performance versus SNR for an MD FSO system with GS scheme under weak and strong turbulence conditions for different K and aperture sizes

The average capacity against SNR of the SMDS algorithm is shown in Fig.3 with the help of Romberg integration under weak and strong turbulence conditions with different values of outage probability, user and aperture size. It can be found from Fig.3, the slopes here are almost the same for different curves, which shows that the average capacity gradient change with SNR is fixed in an FSO system based on SMDS scheme. Similar to the conclusions in Fig.2, the average capacity increases with the increase of users, turbulence strengths and aperture sizes. In addition, it can be also found that the average capacity performance of lower outage probability is obviously better than that of higher outage probability. For instance, for a 200 mm aperture under strong turbulence condition with 50 users in Fig.3(b), when the SNR is equal to 5 dB, the average capacities for outage probability of 0.2 and 0.5 are approximately 4.5 bit/s/Hz and 3 bit/s/Hz, respectively. Comparing Fig.3(b) with Fig.3(a), it can be seen that in the strong regime, the effect of the increasing users and outage probabilities are more appar-

ent, when aperture size increases. For example, for the outage probability of 0.2 under strong turbulence condition, at a given SNR of 10 dB, the average capacity difference of $K=5$ and $K=50$ is about 0.6 bit/s/Hz for a receiver with aperture size of 50 mm, but that is 1.1 bit/s/Hz for the aperture size of 200 mm.

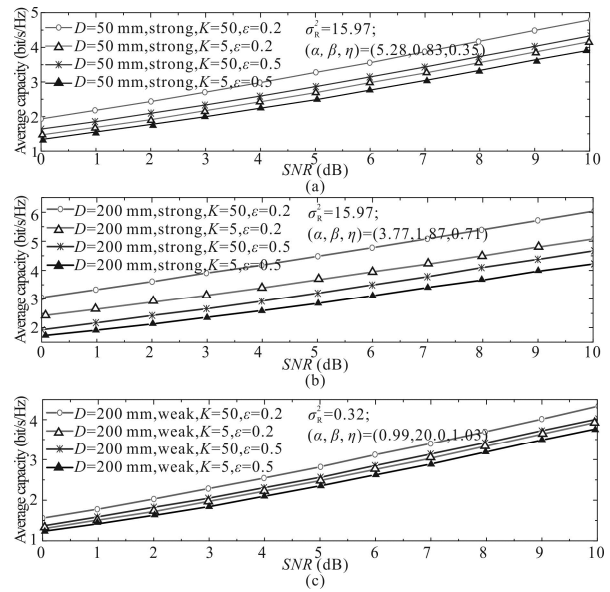


Fig.3 Average capacity performance versus SNR for an MD FSO system with SMDS scheme under weak and strong turbulence conditions for different outage probabilities, users and aperture sizes

Fig.4 shows the relative capacity, which is normalized to an AWGN channel, of PFS and SMDS-ER schemes over EW fading channels under weak and strong turbulence conditions. The parameters of $\epsilon=0.2$ and 0.5 , $t_c=10$ and 100 , $D=50$ and 200 mm are selected to avoid entanglement. As can be seen, the relative capacity reaches to a saturated value as the number of users increases. At a given number of users, the relative capacity increases with the increase of turbulence strength and aperture size, or the decrease of outage probability. Besides, the relative capacity improvement induced by the increase of turbulence strength for a larger aperture is more obvious, compared with that for a smaller aperture. For instance, at given $K=15$ and $t_c=100$, the relative capacity is increased by 15% from weak to strong turbulence strength for 200 mm receiver aperture while it is increased by 9.8% for 50 mm receiver aperture. It can be also found from Fig.4 that the relative capacity performance is improved with the increase of t_c . It is because the increasing sliding window t_c is very effective to the average capacity of each user. It can also be concluded that with the increase of users, the relative channel capacity growth reduces gradually, and this phenomenon is more obvious in weak turbulence situation for both scheduling schemes.

Fig.5 illustrates the relative capacity of the GS, SMDS, PFS and SMDS-ER algorithms of the studied systems over EW fading channels under strong turbulence condition. The parameters of $\epsilon=0.5$, $t_c=100$ and $D=50$ mm are selected to avoid entanglement. As seen, the GS scheme has the maximum relative

capacity in these four algorithms, for the reason that the priority of GS algorithm is decided by the present channel state without taking fairness into account. The relative capacity of SMDR scheme is slightly worse than that of GS scheme resulting from the introduction of outage possibility. Compared with GS and SMDR schemes, the PFS provides user fairness at the cost of capacity. In addition, it is demonstrated that with considering latency and outage possibility, the SMDR-ER scheme achieves the minimum capacity but pays more attention to better fairness.

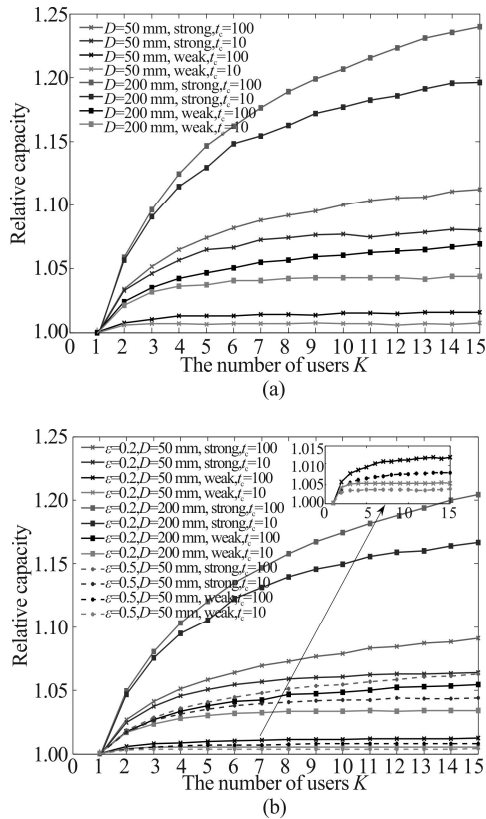


Fig.4 Relative capacity against the number of users for an MD-FSO system with (a) PFS and (b) SMDS-ER algorithms in weak and strong turbulence regimes for different ϵ , t_c and D

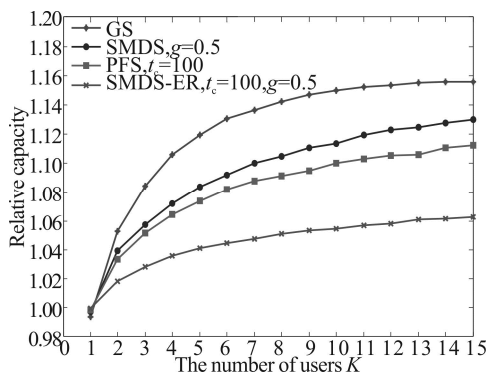


Fig.5 Relative capacity of GS, SMDS, PFS and SMDS-ER schemes versus the number of users for $\epsilon=0.5$, $t_c=100$ and $D=50$ mm under strong turbulence condition

In summary, the performance of an MD-FSO communication system based on different schemes is analyzed over EW distribution in weak and strong regimes. The theoretical capacities of GS and SMDR schemes are studied, respectively, with the help of the second kind Stirling number and Romberg integration method. The relative capacity performance of PFS and SMDS-ER schemes is demonstrated as well. It is concluded that the capacity performance can be improved with the increase of apertures sizes, turbulence strengths and sliding windows or the decrease of outage possibilities over EW fading channels. The study shows that the GS scheme achieves the maximum capacity among various scheduling algorithms at the cost of user fairness, while SMDS-ER receives the minimum capacity, and guarantees the fairness. The PFS achieves better balance between capacity and user fairness. The design of MD-FSO system should be based on the purpose of communication and channel conditions. This work is applicable for multiuser diversity FSO system.

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