Positioning performance analysis of the time sum of arrival algorithm with error features*

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The theoretical positioning accuracy of multilateration (MLAT) with the time difference of arrival (TDOA) algorithm is very high. However, there are some problems in practical applications. Here we analyze the location performance of the time sum of arrival (TSOA) algorithm from the root mean square error (*RMSE*) and geometric dilution of precision (GDOP) in additive white Gaussian noise (AWGN) environment. The TSOA localization model is constructed. Using it, the distribution of location ambiguity region is presented with 4-base stations. And then, the location performance analysis is started from the 4-base stations with calculating the *RMSE* and GDOP variation. Subsequently, when the location parameters are changed in number of base stations, base station layout and so on, the performance changing patterns of the TSOA location algorithm are shown. So, the TSOA location characteristics and performance are revealed. From the *RMSE* and GDOP state changing trend, the anti-noise performance and robustness of the TSOA localization algorithm are proved. The TSOA anti-noise performance will be used for reducing the blind-zone and the false location rate of MLAT systems.

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Multilateration (MLAT) positioning (also known as passive positioning) technology is widely used in military, civil aviation and so on. The time difference of arrival (TDOA) and the time sum of arrival (TSOA) could be used in MLAT systems. However, TDOA has more random blind areas, and error location rate is high. So, the compensation methods for positioning errors must be found. At the end of 1990s, the ellipse (or TSOA) algorithm started to be used for location and tracking targets. In 2005, the mobile targets position was calculated accurately using combination algorithm of TSOA and TDOA in the NLOS environment by Taiwan University of Science and Technology[1]. In 2009, the airport surface location test using TSOA algorithm was carried out successfully in MLAT system by ERA company[2]. Around 2010, it was preliminarily proved that targets location accuracy of the TSOA positioning algorithm was higher than that of the TDOA[3-5]. But, none of them told us the accuracy of the TSOA algorithm. Generally, the positioning accuracy of MLAT system is determined by two factors. One is geometric distribution of base stations in surveillances area, and the other is distance measurement error between base station and transponder.

In this paper, the TSOA simulation positioning system is constructed, then the influence factors about arrival time error and measurement error to the positioning accuracy are analyzed in detail. If the number of base stations exceeds a specified value, the positioning performance of TSOA algorithm could also be greatly improved.

Using the root mean square error (*RMSE*) and geometric dilution of precision (GDOP) parameter[6,7], the positioning performance of the TSOA algorithm is analyzed and evaluated with simulations[8,9]. The simulation results indicate that the positioning accuracy of TSOA is pretty robust under some conditions with related parameters changing. The anti-error performance of TSOA positioning algorithm in MLAT system is better than that of other algorithms under the same location scenes.

Generally, the TSOA positioning algorithm is also known as ellipse location algorithm[10]. The arrival time of the target replying signals (or transponder responses) is detected by the receiving base stations and collected by the master station (MS), the arrival time sum could be calculated by the MS, and the target coordinate could be calculated when the equations of arrival time sum are solved. So, the target can be tracked. In the two-dimensional plane, an ellipse is determined by two foci. When the base stations are on the two foci, the ellipse would be determined by the arrival time sum of the targets signal[11].

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If there are three receiving base stations, the three ellipses would be formed. The three ellipses intersect with each other to form only one common focus point. This point is the target location. The other intersections are called ambiguous points. These points must be removed using the operation information. At last, the target position is determined. The basic positioning principle of TSOA algorithm is shown in Fig.1.

In Fig.1, BS is the ground base station and used to receive the replying signals. MS is the ground main station and used to collect all base station signals, then calculates the time sum of arrival. The MS is a base station too.

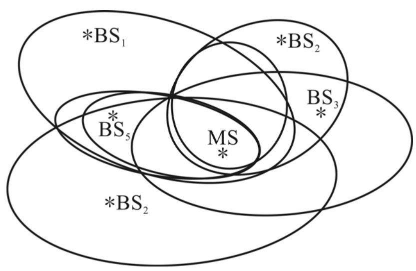


Fig.1 The positioning principle of TSOA algorithm

If (x, y) are the target position coordinates to be estimated, and (X_i, Y_i) are the known position coordinates of the No.i BS, the distance between the moving target and the No.i BS is

$$R_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2} \quad ,$$

$$R_i^2 = \left(X_i - x\right)^2 + \left(Y_i - y\right)^2 =$$

$$K_i - 2X_i x - 2Y_i y + x^2 + y^2 ,$$

$$\Delta r = r_i + r_1 , \tag{1}$$

where $K_i = X_i^2 - Y_1^2$.

Let $R_{i,1}$ indicate the distance sum, which is from the target to the base station i, and from the moving target to the main station. Then the distance sum could be expressed as follows,

$$R_{i,1} = cT_{i,1} = R_i + R_1 = \sqrt{\left(X_i - x\right)^2 + \left(Y_i - y\right)^2} +$$

$$\sqrt{\left(X_1 - x\right)^2 + \left(Y_1 - y\right)^2} , \tag{2}$$

where c is the speed of light and $T_{i,1}$ is the measured value of TSOA. Linearizing Eq.(2), we could obtain

$$R_i^2 = (R_{i,1} - R_1)^2 \ . \tag{3}$$

Eq.(3) could be rewritten as

$$R_{i,1}{}^2 - 2R_{i,1}R_1 + R_1{}^2 = K_i - 2X_i x - 2Y_i y + x^2 + y^2 . \tag{4}$$

When i=1, Eq.(1) is simplified as

$$R_1{}^2 = K_1 - 2X_1 x - 2Y_1 y + x^2 + y^2 . \tag{5}$$

With Eq.(4) minus Eq.(5), the result could be obtained:

$$R_{i,1}{}^2 - 2R_{i,1}R_1 = K_i - K_1 - 2X_{i,1}x - 2Y_{i,1}y . \tag{6}$$

In Eq.(6), $X_{i,1} = X_i - X_1$, $Y_{i,1} = Y_i - Y_1$. Consider x, y and R_1 as unknown parameters, and the nonlinear equations could be formed by Eq.(6). The solution of the equations could be used to calculate the position coordinates of moving targets.

According to the principle of TDOA[12,13] and TSOA algorithm, the distributions of ambitious points and region could be obtained and shown as Fig.2. In the figure, there are four base stations, and the base stations are in star layout. Obviously, the distributions of ambitious points regions in the TDOA and TSOA algorithms are completely different. We could find that the TSOA positioning characteristics are very distinctive.

In general, suppose the MLAT positioning system with TSOA location algorithm is operated with random noise. Without loss of generality, the time measurement error between the base stations and target satisfies the Gaussian white noise distribution model.

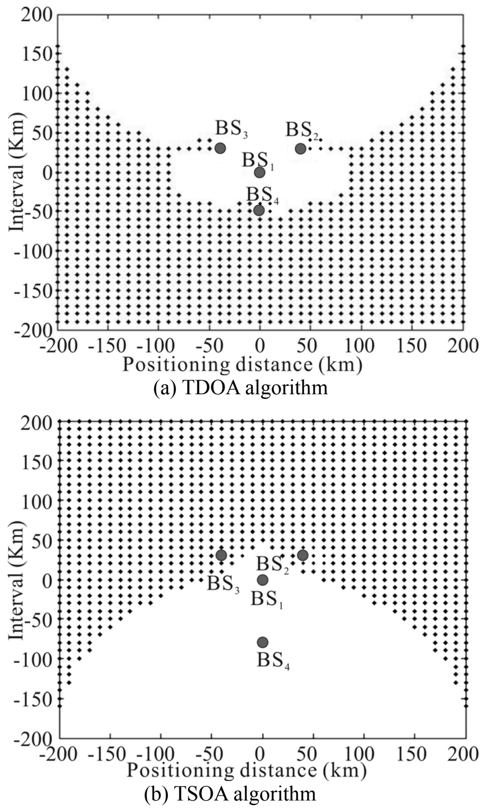


Fig.2 The ambitious regions of TDOA and TSOA

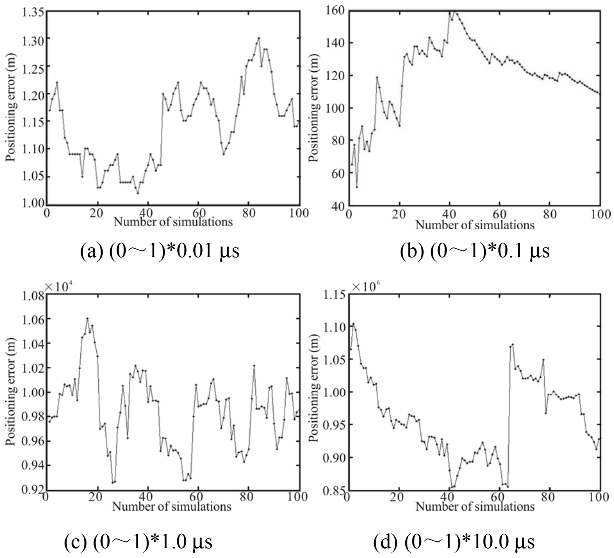
The variations of arrival time measurement error are described with the additive Gaussian white noise (AGWN) model. Eq.(6) is used to evaluate the target localization accuracy of TSOA algorithm, and the error distribution between estimated position and ideal position is analyzed by using the *RMSE* with additional random noise. The formula of *RMSE* could be expressed as

$$RMSE = \sqrt{E[(x - \hat{x})^2 + (y - \hat{y})^2]} \quad . \tag{7}$$

In order to analyze the *RMSE* of TSOA algorithm, the simulation positioning system based on TSOA algorithm is established with four base stations. That is to say, there are four base stations in the positioning model. There is one main station and three auxiliary stations in the four base stations.

The coordinate parameters of these base stations are as follows: the main station O (0, 0), the auxiliary station A (−6 000, 3 000), the auxiliary station B (−6 000, 0), the auxiliary station C (0, 3 000), and suppose the target location is T (−2 000, 4 000). The distance unit is kilometer (km). The layout structure of the four base stations is rectangle model. The given TSOA time dynamic range is 0—200 μs.

The 100 random numbers are selected as the measurement error for arrival time sum of the TSOA algorithm (in other words, the additional random noise is 100 random values, i.e., (0～1)*(0.01 μs, 0.1 μs, 1.0 μs, 10.0 μs)). These parameters are substituted into the TSOA positioning algorithm. The variations of *RMSE* with measurement errors are shown in Fig.3. The simulation results indicate that the arrival time measurement error in the form of random noise has great influence on the positioning accuracy of the TSOA algorithm.



(a) (0～1)*0.01 μs (b) (0～1)*0.1 μs

(c) (0～1)*1.0 μs (d) (0～1)*10.0 μs

Fig.3 The measurement errors of TSOA algorithm

From the vertical axis parameter variation, we could find that when the time measurement error increases, the *RMSE* of the TSOA localization algorithm increases rapidly. However, whether the arrival time error is increased or not, the *RMSE* would approach a stable value after 100 points of iterative operation. This phenomenon indicates that the performance of the TSOA location algorithm has higher robustness.

In the line of sight (LOS) location condition, suppose the range of the arrival time sum is within 0—200 μs. In this hypothetical simulation environment, referring to the actual operation situation of an airport, the four observation time errors are chosen to evaluate performance. Therefore, the target position can be calculated using the arrival time sum with random noise. So we could find that there is a deviation between the calculated position and the real position of the target in Fig.3. And the simulation results present that the closer the random noise value in the arrival time measurement error is to the given arrival time sum, the greater its impact on the positioning accuracy. As a result, the *RMSE* used for measuring positioning accuracy becomes larger. In general, the variation trend of *RMSE* is close to the center of location deviation. Therefore, increasing the number of TSOA location algorithm iterations could rapidly improve the positioning accuracy to some extent.

Here we would discuss the relationship between the location accuracy and number of base stations and geometric layout. The GDOP is a good quality index to evaluate the performance of the spatial geometry layout of the base stations.

Differential operations are performed on both sides of $\Delta r_i = r_i + r_0$ in Eq.(1), respectively, and the GDOP expression is

$$GDOP=\sqrt{\sigma^2{}_x+\sigma^2{}_y+\sigma^2{}_z}=$$
$$\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(b_{1i}b_{1j}+b_{2i}b_{2j}+b_{3i}b_{3j})\sigma_{ij}} \quad (8)$$

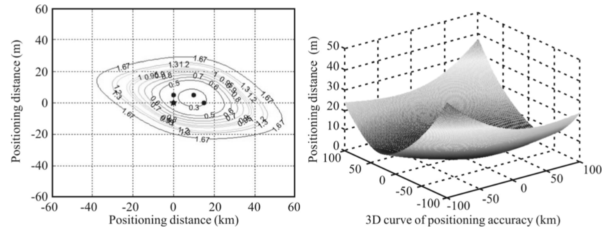
Known from Eq.(8), σ_{ij} is directly determined by the time measurement error. The b_{ix}, b_{iy}, and b_{iz} indicate the positioning errors in three directions of x, y, z in distance, which are between the target and the No.i base station. The σ_x, σ_y, and σ_z indicate the variances in three directions of x, y, z in distance between the target and all base stations. So, the GDOP is a magnification of the target positioning error in three directions of x, y, z, which reflects the relationship among the positioning error, the targets, and spatial geometric distribution. Then various factors affecting GDOP parameters could be found out, including geometric position between the target and base stations, standard deviation of the TSOA time measurement error, target height, baseline length, location of the main station, etc.

Generally, suppose the MLAT positioning system is working on the light of sight situation, and the coordinates of the four base stations are shown in Tab.1. The surveillance area is 60 km×60 km and the target altitude is 10 km. The GDOP simulation planes and spatial contour map distributions are shown in Fig.4.

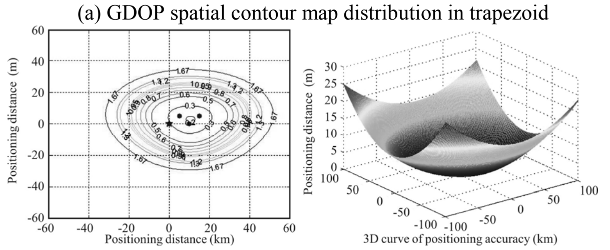
Tab.1 The coordinates of the BS in different layouts

Layout (km)	MS (km)	BS_1 (km)	BS_2 (km)	BS_3 (km)
Trapezoid	(0,0,0.02)	(0,5,0.02)	(10,5,0.02)	(15,0,0.02)
Parallelogram	(0,0,0.02)	(5,5,0.02)	(15,5,0.02)	(10,0,0.02)
Star	(0,0,0.02)	(−5,5,0.02)	(5,5,0.02)	(0,−5,0.02)
Rectangle	(0,0,0.02)	(0,5,0.02)	(15,0,0.02)	(15,5,0.02)
Line	(0,0,0.02)	(−2,−2,0.02)	(1,1,0.02)	(−1,−1,0.02)
Square	(0,0,0.02)	(0,10,0.02)	(10,0,0.02)	(10,10,0.02)

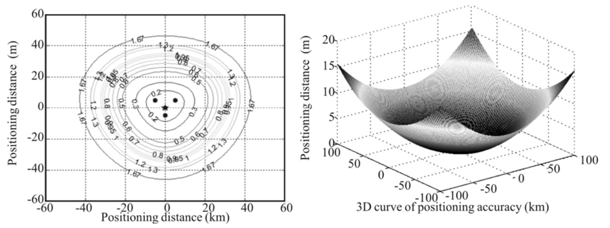
Here we focus on the coverage when the positioning accuracy is 5 m (namely, *GDOP*<1.67). The 5-m accuracy coverage is shown in Tab.2.



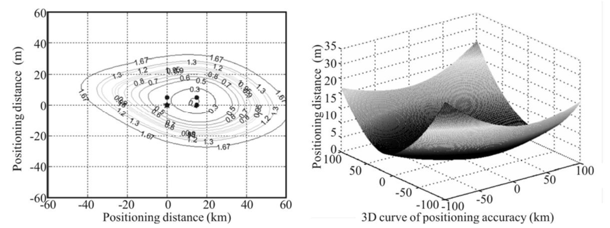
(a) GDOP spatial contour map distribution in trapezoid



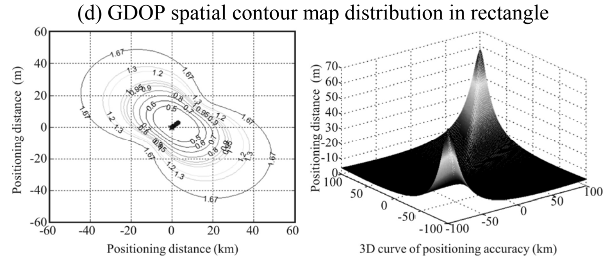
(b) GDOP spatial contour map distribution in parallelogram



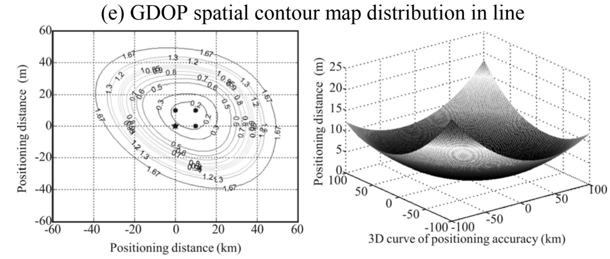
(c) GDOP spatial contour map distribution in star



(d) GDOP spatial contour map distribution in rectangle



(e) GDOP spatial contour map distribution in line



(f) GDOP spatial contour map distribution in square

Fig.4 The GDOP planes and corresponding spatial distributions of TSOA algorithm

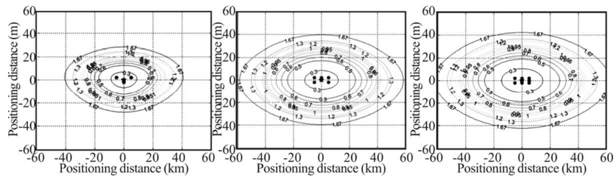
Tab.2 The 5-m accuracy coverage in different layouts

Layout models	Trapezoid	Parallelogram	Star	Rectangle	Line	Square
Coverage	33.3%	27.7%	47.2%	30.5%	25.5%	38.8%

As shown in Tab.2, the base station layout can affect location coverage. The greater the location coverage, the greater the surveillance area, and the better the positioning effect. And the closer the target is to the center of the layout, the higher the positioning accuracy. Known from Fig.4, the largest coverage can be achieved when using the star layout, while the location coverage is the worst with the line layout, but the orientation is better. Because the star layout is a good model for MLAT, its location performance is analyzed with different parameters, for example, different numbers of the base stations and different moving target heights.

Here, we suppose other conditions do not significantly affect the GDOP value, when the number of base stations or moving target heights are changed.

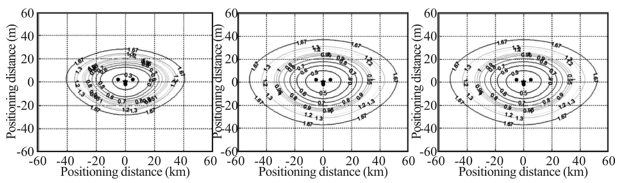
When the number of base stations is changed, the GDOP coverage pattern would change as shown in Fig.5.



(a) 4 base stations (b) 6 base stations (c) 8 base stations

Fig.5 The coverage trend with different stations

When the target flying height is changed, the GDOP overlay shape would change, too, as shown in Fig.6.



(a) H=10 km (b) H=20 km (c) H=30 km

Fig.6 The coverage trend with different target heights

In the TSOA positioning algorithm, the star distribution of base stations is a better location layout. When the other conditions are basically unchanged, the higher the number of base stations, and the higher the target flying, the better the location coverage.

In the same operating environment, the performance of the TDOA algorithm can be compared with that of the TSOA algorithm with different base stations in star layout. The results are shown in Fig.7 and Fig.8. The coordinates of the four base stations are shown in Tab.1 when using four base stations.

As shown in Fig.7, the convergence speed of *RMSE* of the TSOA algorithm is faster than that of the TDOA algorithm in the four base stations location.

When the number of base stations increases to 8, *RMSE* of the TSOA algorithm and TDOA algorithm can be calculated as shown in Fig.8. As the number of base stations increases, the amplitude of *RMSE* decreases and the speed of convergence becomes faster.

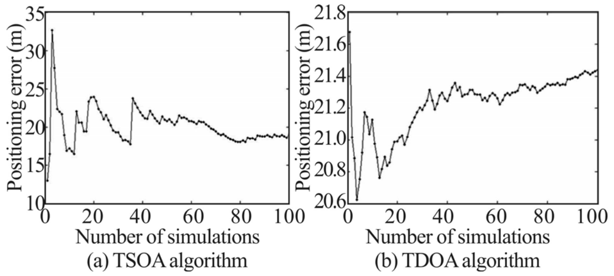


Fig.7 The *RMSE* comparison of location algorithms with four base stations

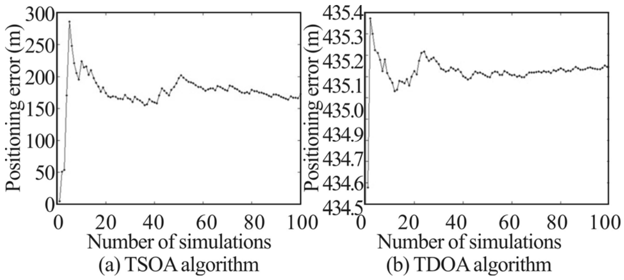


Fig.8 The *RMSE* comparison of location algorithms with eight base stations

The TSOA algorithm has lower amplitude of positioning error and better stability of location error change than the TDOA algorithm. Therefore, in the same noise environment, the TSOA algorithm is more robust than the TDOA algorithm. Different patterns of the base stations distribution will have a great impact on location performance using the same number of base stations. The star layout pattern is selected when the random noise is very difficult to reduce. In the same operation patterns, and with the star layout model, the influence of measurement error on positioning accuracy becomes less and less significant with the increase of the number of base stations, keeping the *RMSE* stable within a certain range. And the location coverage of the TSOA algorithm is more and more wide. But for the TSOA location algorithm, the *RMSE* and GDOP coverage of multiple base stations are not always increasing with the increase of the number of base stations. It is necessary to select the appropriate number of base stations and appropriate layout. Therefore, when the random noise is difficult to reduce, the TSOA location algorithm can be chosen, and the number and layout of base stations should be considered in detail.

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