An improved Prandtl-Ishlinskii model for compensating rate-dependent hysteresis in fast steering mirror system*

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To solve the rate-dependent hysteresis compensation problem in fast steering mirror (FSM) systems, an improved Prandtl-Ishlinskii (P-I) model is proposed in this paper. The proposed model is formulated by employing a linear density function into the STOP operator. By this way, the proposed model has a relatively simple mathematic format, which can be applied to compensate the rate-dependent hysteresis directly. Adaptive differential evolution algorithm is utilized to obtain the accurate parameters of the proposed model. A fast steering mirror control system is established to demonstrate the validity and feasibility of the improved P-I model. Comparative experiments with different input signals are performed and analyzed, and the results show that the proposed model not only suppresses the rate-dependent hysteresis effectively, but also obtains high tracking precision.

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In order to improve the precision of fast steering mirror (FSM) system[1-4] based on piezoelectric actuator (PZA)[5-7], an effective compensation strategy for hysteresis is particularly important. Prandtl-Ishlinskii (P-I) model is one of the most popular hysteresis compensation models for its unique property and simple implementation[8,9]. However, when the operating frequency varies with time, the compensation accuracy is worse by using traditional P-I model[10,11]. To overcome the limitation, Mei-Ju Yang et al[10] presented an inverse modified rate-dependent Prandtl–Ishlinskii (MRPI) model by employing dynamic envelope functions into the PLAY operators. Mohammad Al Janaideh et al[12] improved the P-I model with a variable threshold, which can describe the rate-dependent hysteresis and displacement amplitude within 500 Hz. Another rate-dependent P-I model by adding a quadratic polynomial to the classical P-I model is proposed in Ref.[13]. Sining Liu et al[8] investigated the wiping out and congruency properties of the generalized P-I model proposed by Mohammad as a supplement. Mohammad et al[14] proposed an inverse rate-dependent P-I model with the same format, which is accurate but mathematically complex. However, these compensation methods are focused on improving precision of the PZA obviously, and they have not been introduced to improve an FSM control system.

From above analysis, the compensation methods are based on PLAY operator, and the complex inverse rate-dependent model is required when it comes to hysteresis compensation. Therefore, to compensate the rate-dependent hysteresis directly and effectively, an improved P-I model combining STOP operator with a modified density function is proposed in this paper. Experimental results in FSM control system are also provided to further validate the proposed model.

The PLAY operator and STOP operator are the basic operators of P-I model, which are shown in Fig.1. Researches show that the relationship between the displacement output and the voltage input of a piezoelectric material can be expressed by PLAY operator. From Fig.1, it is clear that the STOP operator is an inverse of the PLAY operator. Therefore, the STOP operator can be applied to describe the relationship between the expected displacement and the required input voltage and then compensate the nonlinearity of a hysteresis material, which is expressed as

$$\begin{cases} y_i(0)=\min\left(r_i,\max\left(-r_i,x(t_k+T)-x(t_k)-y_0\right)\right) \\ y_i(t_k+T)=\min\left(r_i,\max\left(-r_i,x(t_k+T)-x(t_k)-y_i(t_k)\right)\right) \end{cases},\quad(1)$$

where $x(t_k)$ is the control input in the time of t_k, $y_i(t_k)$ is

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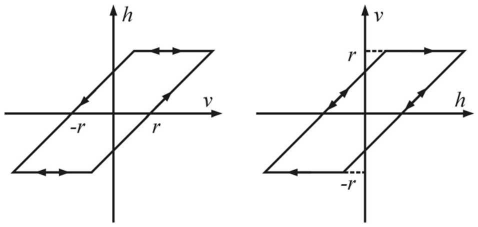
the output of the operator in the time of t_k, r_i is the threshold, y_0 is the initial value of the operator, and T is the sampling time.



(a) PLAY operator (b) STOP operator

Fig.1 The basic operators of P-I model

In practical system, considering the input voltage is positive, Eq.(1) should be modified to a unilateral one, which can be described as

$$\begin{cases} y_i(0)=\min\left(r_i,\max\left(0,x\left(t_k+T\right)-x\left(t_k\right)-y_0\right)\right) \\ y_i\left(t_k+T\right)=\min\left(r_i,\max\left(0,x\left(t_k+T\right)-x\left(t_k\right)-y_i\left(t_k\right)\right)\right) \end{cases}. \quad (2)$$

The normalization processing of the input and the output is carried out by Eq.(2), and then the threshold can be defined as

$$r_i=\frac{i}{n},\left(i=1,2,3,\cdots,n\right), \quad (3)$$

where n is the quantity of the operators.

For rate-dependent hysteresis, the hysteresis increases with the rate of control input, thus making it difficult to obtain perfect compensation performance. Therefore, a linear density function is proposed to compensate the hysteresis and improve the tracking accuracy. The modified density function is expressed as

$$w_i'=a_i+b_i\,|\,v\,|, \quad (4)$$

where w_i' is the modified density function, and $w_i'>0$. a_i and b_i are the parameters to be identified. v is the velocity of the input, which can be described as

$$v=\frac{d_k-d_{k-1}}{T}, \quad (5)$$

where d_k and d_{k-1} are respectively the inputs in the time of t_k and t_{k-1}, respectively. Then, the rate-dependent P-I model based on STOP operator (RPIS) can be described as

$$H(t_k)=\sum_i^n w_i' y_i(t_k), \quad (6)$$

where $H(t_k)$ is the output of the model.

The adaptive differential evolution (ADE) algorithm proposed by Cheng[15] is applied to accurately fit parameters of proton exchange membrane fuel cell (PEMFC) model. In this paper, ADE[15] is chosen to obtain the accurate parameters of P-I model. The input signal of identification adopts a sinusoid signal, and the hysteresis will be immediately reflected on the output displacement. Therefore, the hysteresis curves between the input voltage and output displacement under different frequencies are shown in Fig.2. Obviously, the proposed method can fit the actual hysteresis well under sinusoidal signals with different frequencies. The identified parameters of RPIS are shown in Tab.1.

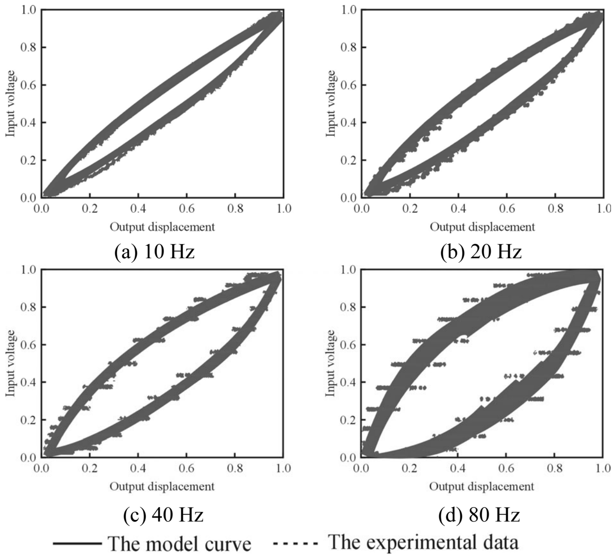


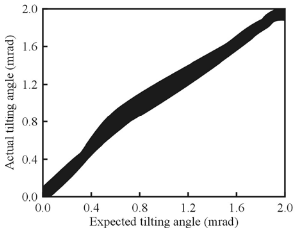
Fig.2 The identification results of RPIS model

Tab.1 The identified parameters of the RPIS model

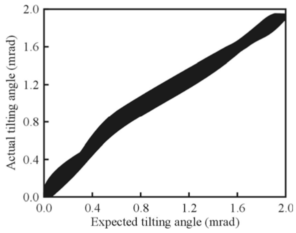
Parameter	Value	Parameter	Value
a_1	0.059 7	b_1	0.014 2
a_2	0.099 3	b_2	0.007 8
a_3	0.115 5	b_3	0.004 5
a_4	0.018 5	b_4	0.000 0
a_5	0.022 8	b_5	0.001 2
a_6	0.023 6	b_6	0.000 1
a_7	0.081 5	b_7	0.006 4
a_8	0.000 3	b_8	0.000 0
a_9	0.014 3	b_9	−0.000 1
a_{10}	0.828 9	b_{10}	−0.009 8

In order to demonstrate the compensation performance of hysteresis, the off-line compensation under different frequencies with RPIS model is shown in Fig.3. Obviously, the nonlinear hysteresis is restrained greatly, which shows the effectiveness of the proposed model.

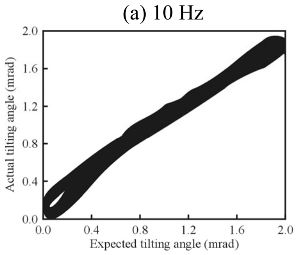
To further verify the validity and feasibility of the proposed RPIS model, the experimental setup is established and shown in Fig.4. The experimental system consists of a self-designed FSM, a PZT controller (E-500), a self-designed digital signal processing system and a computer. Under the experimental conditions, the tilting angle of the mirror is measured by self-designed grating encoder and sent to the DSP controller. The required voltage is obtained by the RPIS model, and then transformed and enlarged to an analog signal. The PZT controller obtains the analog signal and regulates the FSM towards the desired reference trajectories. To verify the effectiveness of STOP operator and the necessity of the linear density function, the experimental results, such as no compensation and compensation with PIS model, are also given for comparison, where PIS represents the RPIS model without a linear function. The identified parameters of the PIS model are summarized as: w_1=0.202 57, w_2=0.107 17, w_3=0.095 23, w_4=0.018 69, w_5=0.026 61, w_6=0.023 55, w_7=0.077 19, w_8=0.000 24, w_9=0.014 37, w_{10}=0.821 74.



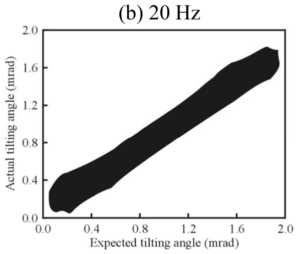
(a) 10 Hz



(b) 20 Hz



(c) 40 Hz



(d) 80 Hz

Fig.3 The off-line compensation for hysteresis under different frequencies with RPIS model

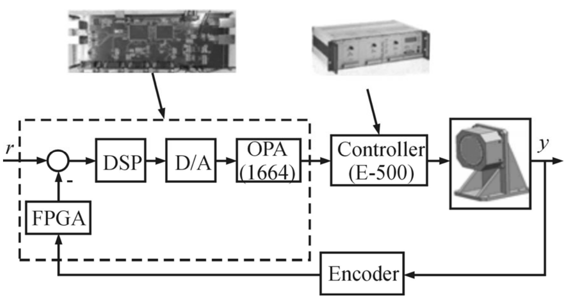
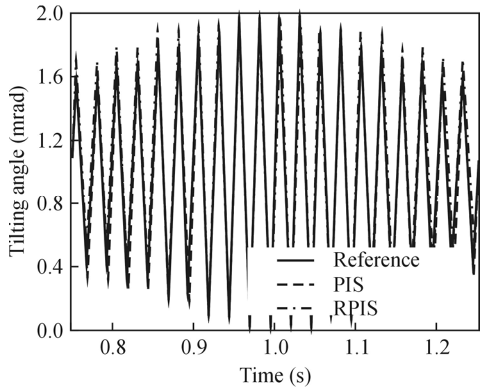
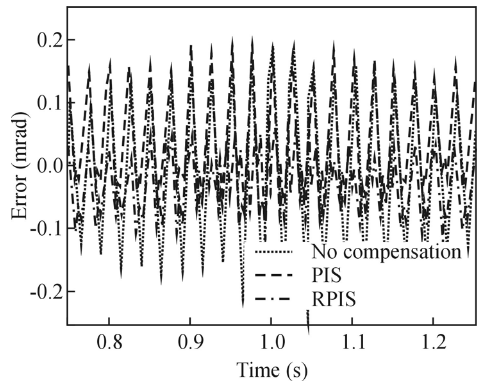


Fig.4 The experimental setup

To check the tracking performance of RPIS model for hysteresis compensation, Fig.5 gives the comparison results of tracking performance in the open loop control system for reference $r=1+k\sin(80\pi t)$ (mrad) ($0.6\le k\le 1$). Obviously, with the proposed model, the actual displacement follows the required displacement better. With the PIS model, the maximum tracking error is 0.18 mrad, and the tracking precision is improved by over 21%. With the RPIS model, the maximum tracking error is 0.11 mrad, and the tracking precision is improved by over 52%. Therefore, the RPIS model obtains better tracking performance.



(a) Tilting angle

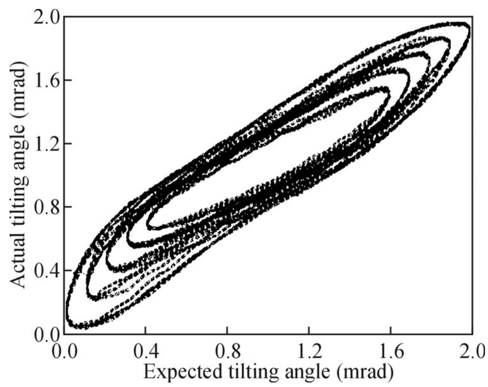


(b) Error

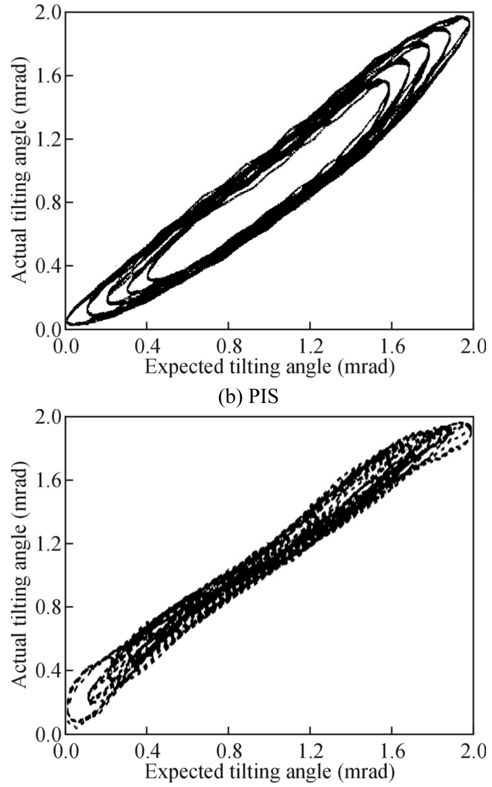
Fig.5 The comparison of tracking performance under sinusoid signal with variable amplitude

In addition, to show the on-line compensation for hysteresis, Fig.6 gives the comparison results of nonlinear hysteresis for reference $r=1+k\sin(80\pi t)$ (mrad) ($0.6\le k\le 1$). Without compensation, the nonlinear hysteresis increases with the increase of amplitude. With the PIS model, the nonlinear hysteresis can be improved more or less, which indicates that the PIS model can be applied to compensate the hysteresis directly. For the RPIS model, the nonlinear hysteresis can be restrained greatly, which demonstrates the feasibility of the proposed model for rate-dependent hysteresis compensation.

Fig.5 and Fig.6 give the comparative results under sinusoid signal with variable amplitude at 40 Hz. In order to further verify the validity of the RPIS model in variable



(a) No compensation

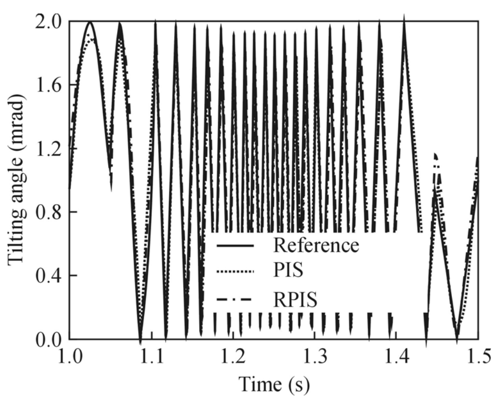


(c) RPIS

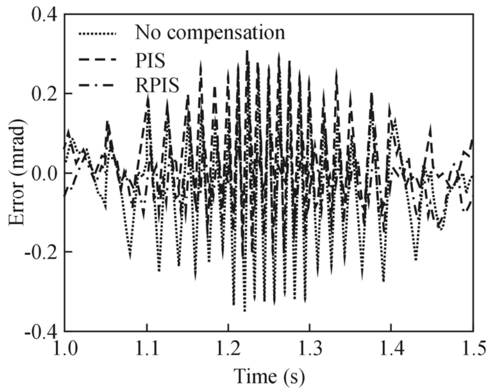
Fig.6 The comparison of hysteresis loop under sinusoid signal with variable amplitude

frequency, Fig.7 gives the tracking performance for reference r=1+sin(2πft) (mrad) (10≤f≤80). Without compensation, the tracking error increases with the increase of frequency, and the maximum tracking error is 0.35 mrad. With the PIS model, the maximum tracking error is 0.3 mrad. With the RPIS model, the tracking error is stable under different frequencies, which is 0.12 mrad. Obviously, the comparative results of tracking errors indicate that the RPIS model can compensate the rate-dependent hysteresis directly and effectively.

A novel strategy for hysteresis compensation is proposed in this paper to improve the precision of the FSM system. Comparative results are provided and discussed under sinusoid inputs with different amplitudes and frequencies. The RPIS model can be applied to compensate rate-dependent hysteresis directly in real time without calculating the complex inverse model. The RPIS model is introduced to FSM control system, and the ADE algorithm is applied to identify the parameters of the P-I model. The performance of the proposed model is verified by the tracking precision in an open loop system, and the effective compensation method for hysteresis can improve the open loop tracking precision greatly.



(a) Tilting angle



(b) Error

Fig.7 The comparison of tracking performance under sinusoid signal with variable frequency

References

[1] Jiang Z H, Wang T F and Sun T, Applied Mechanics & Materials **336-338**, 1809 (2013).

[2] Li M, Lin S and Li S M, Optoelectronics Letters **8**, 297 (2012).

[3] Yong-jun Xiao, Yan-fei Liu and Ran Dong, Optoelectronics Letters **7**, 451 (2011).

[4] Lu Y, Fan D and Zhang Z, Optik-International Journal for Light and Electron Optics **124**, 2443 (2013).

[5] Yeom T, Simon T W and Zhang M, Sensors & Actuators A Physical **176**, 99 (2012).

[6] Yang M J, Gu G Y and Zhu L M, Sensors & Actuators A Physical **189**, 254 (2012).

[7] Kijanka P, Packo P and Staszewski W J, Journal of Intelligent Material Systems & Structures **27**, 337 (2016).

[8] Liu S and Su C Y, Smart Materials & Structures **20**, 225 (2011).

[9] Sining Liu and Chun-Yi Su, A Modified Generalized Prandtl-Ishlinskii Model and Its Inverse for Hysteresis Compensation, American Control Conference, 4759 (2013).

[10] Yang M J, Li C X and Gu G Y, Smart Materials & Structures **24**, 125006 (2015).

[11] Li W and Chen X, Sensors & Actuators A Physical **199**, 89 (2013).

[12] Janaideh M A, Rakheja S and Su C Y, Mechatronics **19**, 656 (2009).

[13] Gan J, Zhang X and Wu H, Review of Scientific Instruments **87**, 212 (2016).

[14] Al Janaideh M and Krejci P, IEEE/ASME Transactions on Mechatronics **18**, 1498 (2013).

[15] Cheng J and Zhang G, International Journal of Electrical Power & Energy Systems **62**, 189 (2014).