Nearly deterministic quantum Fredkin gate based on weak cross-Kerr nonlinearity*

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A scheme of an optical quantum Fredkin gate is presented based on weak cross-Kerr nonlinearity. By an auxiliary coherent state with the cross-Kerr nonlinearity effect, photons can interact with each other indirectly, and a non-demolition measurement for photons can be implemented. Combined with the homodyne detection, classical feedforward, polarization beam splitters and Pauli-X operations, a controlled-path gate is constructed. Furthermore, a quantum Fredkin gate is built based on the controlled-path gate. The proposed Fredkin gate is simple in structure and feasible by current experimental technology.

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Quantum computing[1] has attracted great attention in efficiently solving problems that classical computers could not do within a reasonable amount of time. There are many different architectures for quantum computers[2,3] based on different physical systems, such as spin-based systems (quantum dot, nuclear magnetic resonance, nitrogenvacancy centers in diamond), trapped ion system and superconductor. Optical quantum system is a prominent candidate for quantum computing and communication[4,5], which has slow decoherence and easy single qubit manipulation. However, such a system has a disadvantage that it is difficult for photons to interact with each other.

E. Knill et al[6] proved that quantum logic gates can be constructed by only using linear optical elements, single photon sources and detectors. The quantum Fredkin gate is essential in quantum computing[7,8]. By using auxiliary photons, interferometers and multiple two-qubit and single-qubit gates, some quantum Fredkin gate schemes were proposed[9,10]. All the quantum logic gates based on linear optics are probabilistic, i.e., the probability of the realization of correspondent operation cannot be unit.

Fortunately, based on weak cross-Kerr nonlinearities[11-13], a nearly deterministic controlled-not (CNOT) gate and a high efficiency quantum non-demolition single photon number resolving detector were proposed by K. Nemoto et al[14,15]. So we can witness it to realize the universal quantum computation in principle. For example, as the quantum swap gate can be constructed by three CNOT gates, we can get a nearly deterministic one in theory. However, this kind of swap gate needs many weak cross-Kerr media, so the structure is very complex and the overhead is too expensive in resources. Therefore, it is necessary to construct some important multi-qubit gates directly. The first quantum Fredkin gate was proposed by Milburn[9]. In his scheme, strong cross-Kerr nonlinearities strength was required, which is infeasible with current technology. Recently, some nearly deterministic schemes were proposed based on weak cross-Kerr nonlinearities[16-19], but almost all of them are complex in structure and require more weak cross-Kerr media and coherent sates. In this paper, we propose a scheme to construct the quantum Fredkin gate by only using one weak cross-Kerr medium.

In a controlled-path gate, the paths of the target photon are determined by the control photon. Here, we use the polarization of photons as qubit, which means that the horizontally (vertically) linear polarization $|H\rangle(|V\rangle)$ is as the qubit $|0\rangle(|1\rangle)$. Assume that our control and target qubits are initially prepared as $c_0|H\rangle_c+c_1|V\rangle_c$ and $d_0|H\rangle_t+d_1|V\rangle_t$, where $|c_0|^2+|c_1|^2=1$ and $|d_0|^2+|d_1|^2=1$. Then the two-qubit states can be given as

$$|\psi\rangle=\left(c_0|H\rangle_c+c_1|V\rangle_c\right)\otimes\left(d_0|H\rangle_t+d_1|V\rangle_t\right)=$$
$$c_0d_0|H\rangle_c|H\rangle_t+c_0d_1|H\rangle_c|V\rangle_t+$$
$$c_1d_0|V\rangle_c|H\rangle_t+c_1d_1|V\rangle_c|V\rangle_t. \quad (1)$$

This controlled-path gate is depicted schematically in Fig.1. The control photon interacts with a coherent state $|\alpha\rangle$ according to the cross-Kerr nonlinearity, so that a phase shift can be induced in the coherent state. After three polarization beam splitters (PBS_1, PBS_2, PBS_3) which transmit the photon with horizontal polarization

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and reflect the photon with vertical polarization, the input state $|\psi\rangle|\alpha\rangle$ finally evolves into as follow[14]:

$$\left(c_0 d_0 |H\rangle_c |H\rangle_t + c_0 d_1 |H\rangle_c |V\rangle_t\right)|\alpha\rangle +$$
$$\left(c_1 d_0 |V\rangle_c |H\rangle_t + c_1 d_1 |V\rangle_c |V\rangle_t\right)|\alpha e^{i\theta}\rangle. \quad (2)$$

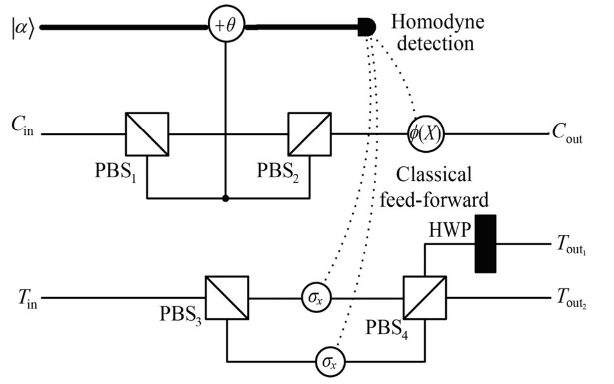

Fig.1 Schematic diagram of a controlled-path gate with weak cross-Kerr nonlinearity

After they pass through cross-Kerr media, a general X homodyne detection is performed on the coherent state. If the measurement outcome indicates zero phase shift, σ_x operation and a phase shift through a classical feedforward are not performed. Otherwise, the measurement outcome indicates phase shift with θ, and the classical feedforward is performed. The two-photon state can be projected into the following state:

$$c_0 |H\rangle_c \left(d_0 |H\rangle_t + d_1 |V\rangle_t\right)_{T_{out_2}} +$$
$$c_1 |V\rangle_c \left(d_0 |V\rangle_t + d_1 |H\rangle_t\right)_{T_{out_1}}. \quad (3)$$

After passing through PBS_4 and a 45° tilted half wave plate (HWP) whose action is given as

$$\begin{cases} |H\rangle \to |V\rangle \\ |V\rangle \to |H\rangle \end{cases}, \quad (4)$$

the output state can evolve into

$$c_0 |H\rangle_c \left(d_0 |H\rangle_t + d_1 |V\rangle_t\right)_{T_{out_2}} +$$
$$c_1 |V\rangle_c \left(d_0 |H\rangle_t + d_1 |V\rangle_t\right)_{T_{out_1}}. \quad (5)$$

It can be seen from Eq. (5) that the target photon can be output through the port T_{out_2} if the control photon is in the state $|H\rangle$, while the target photon can be output through the port T_{out_1} if the control photon is in the state $|V\rangle$. In this scheme, the X homodyne measurement can distinguish between two scenarios of phase shifts (0 and $+\theta$) of the coherent state, which are embodied by two different measurement outcome functions[14] of $f(x, \alpha)$ and $f(x, \alpha\cos\theta)$. However, the Gaussian functions of $f(x, \alpha)$ and $f(x, \alpha\cos\theta)$ partially overlap, so error exists in this scheme. The error probability is given by

$$P_{error} = erfc\left(X_d / 2\sqrt{2}\right)/2, \quad (6)$$

where $X_d=2\alpha(1-\cos\theta)$ is the distance of the peaks of the two Gaussian functions. When $X_d=10.5$, the error probability will be $P_{error}\approx 10^{-4}$. Therefore, we construct a nearly deterministic controlled-path gate based on the weak cross-Kerr nonlinearity. After a controlled-path gate, the target photon can be simultaneously in two different spatial modes depending on the polarizations of the control photon. Therefore, an operation on the condition of the control photon's polarizations can be directly performed on the spatial modes of the target.

A Fredkin gate is also called a controlled-swap gate, which is one of the most important three-qubit controlled unitary gates. The two input states go through if the control photon is in the state $|H\rangle$, the two states of the input photons are swapped if the control photon is in the state $|V\rangle$. Therefore, the unitary matrix of the quantum Fredkin gate can be written as

$$U_{Fredkin} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

Fig.2 shows a schematic diagram of a quantum Fredkin gate based on the weak cross-Kerr nonlinearity. To show the work procedure of this scheme, we consider an arbitrary input state given as

$$a_1 |H\rangle_c |H\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_2 |H\rangle_c |H\rangle_{t_{in_1}} |V\rangle_{t_{in_2}} +$$
$$a_3 |H\rangle_c |V\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_4 |H\rangle_c |V\rangle_{t_{in_1}} |V\rangle_{t_{in_2}} +$$
$$a_5 |V\rangle_c |H\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_6 |V\rangle_c |H\rangle_{t_{in_1}} |V\rangle_{t_{in_2}} +$$
$$a_7 |V\rangle_c |V\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_8 |V\rangle_c |V\rangle_{t_{in_1}} |V\rangle_{t_{in_2}}, \quad (8)$$

where the subscript c and t_{inj} (j=1,2) denote the control qubit and the target qubits, respectively, and the complex coefficients a_i (i=1, 2,…, 8) satisfy the normalized condition $\sum_{i=1}^{8} a_i^2 = 1$.

The control photon interacts with a coherent state $|\alpha\rangle$ by the cross-Kerr nonlinearity, so that a phase shift can be induced in the coherent state. After passing through PBS_1 and PBS_2, the system state evolves as

$$(a_1 |H\rangle_c |H\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_2 |H\rangle_c |H\rangle_{t_{in_1}} |V\rangle_{t_{in_2}} +$$
$$a_3 |H\rangle_c |V\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_4 |H\rangle_c |V\rangle_{t_{in_1}} |V\rangle_{t_{in_2}})|\alpha\rangle +$$
$$(a_5 |V\rangle_c |H\rangle_{t_{in_1}} |H\rangle_{t_{in_2}} + a_6 |V\rangle_c |H\rangle_{t_{in_1}} |V\rangle_{t_{in_2}} +$$

$$a_7\left|V\right\rangle_c\left|V\right\rangle_{t_{\mathrm{in}_1}}\left|H\right\rangle_{t_{\mathrm{in}_2}}+a_8\left|V\right\rangle_c\left|V\right\rangle_{t_{\mathrm{in}_1}}\left|V\right\rangle_{t_{\mathrm{in}_2}})\left|\alpha\mathrm{e}^{\mathrm{i}\theta}\right\rangle. \quad (9)$$

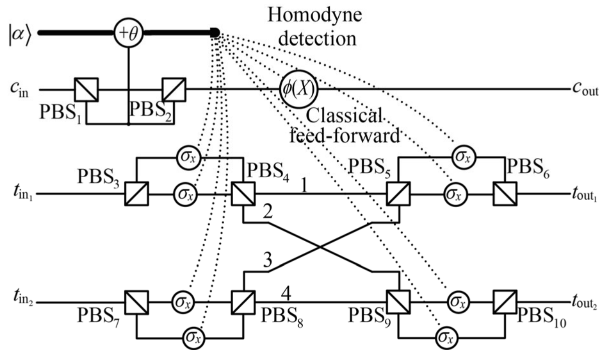

Fig.2 Schematic diagram of a nearly deterministic quantum Fredkin gate based on the weak cross-Kerr nonlinearity

Then a general X homodyne detection is performed on the coherent state. When the measurement outcome indicates zero phase shift, total eight σ_x operations and a phase shifting are not performed. In this case, the state of the control photon is projected to $\left|H\right\rangle$, and the states of the two target photons are not exchanged and in path 1 and 4, respectively. Otherwise, the measurement outcome indicates θ phase shift, and the classical feedforward is performed. In this case, the state of the control photon is projected to $\left|V\right\rangle$, the states of the two target photons are swapped, and the two photons are in path 2 and 3, respectively.

So after passing through PBS_3, PBS_4, PBS_7, PBS_8 and σ_x operations, the state of the three photons will evolve into:

$$\left|H\right\rangle_c\left(a_1\left|H\right\rangle_1\left|H\right\rangle_4+a_2\left|H\right\rangle_1\left|V\right\rangle_4+a_3\left|V\right\rangle_1\left|H\right\rangle_4+a_4\left|V\right\rangle_1\left|V\right\rangle_4\right)+\left|V\right\rangle_c\left(a_5\left|V\right\rangle_2\left|V\right\rangle_3+a_6\left|H\right\rangle_2\left|V\right\rangle_3+a_7\left|V\right\rangle_2\left|H\right\rangle_3+a_8\left|H\right\rangle_2\left|H\right\rangle_3\right). \quad (10)$$

After passing through PBS_5, PBS_6, PBS_9, PBS_{10} and σ_x operations, the output state evolves into:

$$\left|H\right\rangle_c\left(a_1\left|H\right\rangle_{t_{\mathrm{out}_1}}\left|H\right\rangle_{t_{\mathrm{out}_2}}+a_2\left|H\right\rangle_{t_{\mathrm{out}_1}}\left|V\right\rangle_{t_{\mathrm{out}_2}}+a_3\left|V\right\rangle_{t_{\mathrm{out}_1}}\left|H\right\rangle_{t_{\mathrm{out}_2}}+a_4\left|V\right\rangle_{t_{\mathrm{out}_1}}\left|V\right\rangle_{t_{\mathrm{out}_2}}\right)+\left|V\right\rangle_c\left(a_5\left|H\right\rangle_{t_{\mathrm{out}_1}}\left|H\right\rangle_{t_{\mathrm{out}_2}}+a_6\left|V\right\rangle_{t_{\mathrm{out}_1}}\left|H\right\rangle_{t_{\mathrm{out}_2}}+a_7\left|H\right\rangle_{t_{\mathrm{out}_1}}\left|V\right\rangle_{t_{\mathrm{out}_2}}+a_8\left|V\right\rangle_{t_{\mathrm{out}_1}}\left|V\right\rangle_{t_{\mathrm{out}_2}}\right). \quad (11)$$

We can see from Eq.(11) that a quantum Fredkin gate is constructed. As depicted in controlled-path gate, the error probability in this Fredkin gate is extremely low when the amplitude of the coherent state is big enough. So it is feasible to construct this nearly deterministic Fredkin gate based on cross-Kerr nonlinearity with more feedforward operations. Since the weak nonlinearity strength is an extremely small θ, it is worth mentioning that we should use a larger probe field according to Ref.[14]. Anyway, it shows the potential power of the weak (but not tiny) cross Kerr nonlinearities.

To summarize, we present a nearly deterministic Fredkin gate based on weak cross-Kerr nonlinearity. We must ensure that we have received classical feedforwards before the target photons arrive. We can add some time delay units at the input port of target photons as one solution if necessary. In conclusion, fewer resources are required, and the structures of the two gates are very simple. They are feasible with current technology since the weak cross-Kerr nonlinearities can be generated with electromagnetically induced transparencies (EIT) and the intensity of Kerr nonlinearity can be improved via the non-local atomic interaction in atomic ensembles.

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