Outage performance of multihop free-space optical communication system over exponentiated Weibull fading channels with nonzero boresight pointing errors*

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The outage performance of the multihop free-space optical (FSO) communication system with decode-and-forward (DF) protocol is studied by considering the joint effects of nonzero boresight pointing errors and atmospheric turbulence modeled by exponentiated Weibull (EW) distribution. The closed-form analytical expression of outage probability is derived, and the results are validated through Monte Carlo simulation. Furthermore, the detailed analysis is provided to evaluate the impacts of turbulence strength, receiver aperture size, boresight displacement, beamwidth and number of relays on the outage performance for the studied system.

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Recently, free-space optical (FSO) communication has been widely concerned owing to its several attractive advantages, such as unlicensed spectrum, high bandwidth capacity, innate security and robustness to electromagnetic interference[1,2]. However, the application of FSO systems is hampered by atmospheric turbulence and pointing errors (misalignment)[3-5]. Several statistical models of atmospheric turbulence, including log-normal (LN) and Gamma-Gamma (G-G), have been proposed to estimate the FSO performance properly. Recently, a novel fading model known as exponentiated Weibull (EW) distribution has been proposed for Gaussian beams with aperture averaging effect considered in Ref.[6].

Another major degrading issue is the misalignment between transmitter and receiver induced by the vibration and sway of buildings, which is also known as pointing errors[7]. Recently, a novel and generalized pointing error model has been developed[8], which includes both boresight and jitter for more practical scenarios. Then, the error rate and outage performance of FSO system were investigated for LN and G-G fading channels based on this model in Ref.[8]. In Ref.[9], the average bit error rate (*ABER*) and outage probability of FSO system were analyzed by combining the EW distribution and the nonzero boresight pointing error model. Actually, these two works are both on the basis of point-to-point (PP) communication system.

Multihop relaying[10-12], as a typical relay configuration, is very effective to mitigate the performance degradation due to turbulence-induced fading. However, no work has been reported on the outage performance for multihop FSO system over EW distributed turbulence channel considering nonzero boresight pointing errors yet, to the best of our knowledge.

In this paper, the end-to-end outage probability of multihop relay-assisted FSO communication system with DF relaying protocol over a composite channel model consisting of nonzero boresight pointing errors and EW-distributed turbulence fading is presented in this paper. On the basis of the composite probability density function (PDF) and cumulative distribution function (CDF) of the aggregated channel gain, the outage probability for a PP link is derived. Then, the analytical end-to-end outage probability expressions with binary pulse position modulation (BPPM) for the multihop FSO system are obtained and verified by Monte Carlo (MC) simulation.

As is shown in Fig.1, the source terminal S transmits the signal to the destination terminal D through $M-1$ non-regenerative relays T_1, T_2,…, T_{M-1}. All the PP links are assumed as non-identical and independently distributed. In each PP link, the transmitted signal x based on BPPM suffers from pointing errors, atmospheric turbulence and additive white Gaussian noise (AWGN), thus the received electrical signal y is written as

$$y = RIx + w , \tag{1}$$

where R is the detector responsivity, w represents the AWGN with variance σ_n^2, I is the channel gain which is regarded as $I=I^aI^pI^l$, where I^a refers to the turbulence-induced fading, I^p indicates the fading due to pointing errors, and I^l reflects the path loss which is deterministic. Considering the intensity modulation and direct detection (IM/DD), the instantaneous electrical

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signal-to-noise ratio (*SNR*) at the corresponding receiver is defined as

$$\phi = \bar{\phi} I^2 = \frac{R^2 P_{\mathrm{t}}^2 I^2}{2\sigma_n^2}, \tag{2}$$

where P_{t} is the average transmitted optical power of x, and $\bar{\phi} = (RP_{\mathrm{t}})^2 / (2\sigma_n^2)$ is defined as the average electrical *SNR* according to Ref.[8].

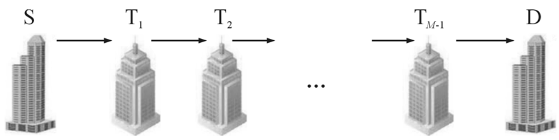

Fig.1 Schematic diagram of a multihop FSO communication system

The turbulence-induced fading is modeled by EW distribution, and the PDF of I^a can be given as

$$f_{I^a}\left(I^a\right) = \frac{\alpha\beta}{\eta}\left(\frac{I^a}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{I^a}{\eta}\right)^{\beta}\right] \times \left\{1-\exp\left[-\left(\frac{I^a}{\eta}\right)^{\beta}\right]\right\}^{\alpha-1}, I^a > 0, \tag{3}$$

where α>0 and β>0 are shape parameters, and η>0 is a scale parameter. The values of these parameters used in this paper are all set according to Ref.[13].

As for the nonzero boresight pointing error model, the PDF of I^p in Ref.[8] is given as

$$f_{I^p}(I^p) = \frac{\rho^2}{A_0^{\rho^2}} \exp\left(-\frac{s^2}{2\sigma_{\mathrm{s}}^2}\right)(I^p)^{\rho^2-1} \times I_0\left(\frac{s}{\sigma_{\mathrm{s}}^2}\sqrt{-\frac{\omega_{\mathrm{Zeq}}^2}{2}\ln\frac{I^p}{A_0}}\right), 0 \le I^p \le A_0, \tag{4}$$

where A_0 defines the pointing loss which is the part of the collected optical power when the difference between the centers of beam and detector equals zero, $\rho=\omega_{\mathrm{Zeq}}/2\sigma_{\mathrm{s}}$ is the ratio between the equivalent beamwidth ω_{Zeq} and jitter standard deviation σ_{s}, and s is the boresight displacement[8]. Moreover, $I_0(\cdot)$ indicates the modified Bessel function of the first kind with order zero.

According to Ref.[8], the PDF of the channel gain I can be calculated as

$$f_I\left(I\right) = \int f_{I|I^a}\left(I \mid I^a\right) f_{I^a}\left(I^a\right) \mathrm{d}I^a. \tag{5}$$

Considering I^l=1, the conditional PDF is given as

$$f_{I|I^a}\left(I \mid I^a\right) = \frac{1}{I^a} f_{I^p}\left(\frac{I}{I^a}\right) = \frac{\rho^2}{A_0^{\rho^2} I^a}\left(\frac{I}{I^a}\right)^{\rho^2-1} \times \exp\left(-\frac{s^2}{2\sigma_{\mathrm{s}}^2}\right) \times I_0\left(\frac{s}{\sigma_{\mathrm{s}}^2}\sqrt{-\frac{\omega_{\mathrm{Zeq}}^2}{2}\ln\frac{I}{A_0 I^a}}\right). \tag{6}$$

Substituting Eqs.(3) and (6) into Eq.(5), the PDF of I can be achieved as

$$f_I\left(I\right) = \frac{\alpha\rho^2 I^{\beta-1}}{(\eta A_0)^{\beta}} \exp\left(-\frac{s^2}{2\sigma_{\mathrm{s}}^2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k \Gamma(\alpha)}{k!\,\Gamma(\alpha-k)} \times \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{s^2\rho^2}{2\sigma_{\mathrm{s}}^2\beta}\right)^j G_{j+1,j+2}^{j+2,0}\left[\frac{(1+k)I^{\beta}}{(\eta A_0)^{\beta}}\middle|\begin{matrix}\kappa_1\\ \kappa_2\end{matrix}\right], \tag{7}$$

where $\kappa_1 = \underbrace{\rho^2/\beta,\ldots,\rho^2/\beta}_{(j+1)\text{terms}}$, $\kappa_2 = 0,\underbrace{\rho^2/\beta-1,\ldots,\rho^2/\beta-1}_{(j+1)\text{terms}}$.

Then, by using Eq.(07.34.21.0003.01) in Ref.[14], the CDF of I can be expressed as

$$F_I\left(I\right) = \frac{\alpha\rho^2}{\beta} \exp\left(-\frac{s^2}{2\sigma_{\mathrm{s}}^2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k \Gamma(\alpha)}{(k+1)!\,\Gamma(\alpha-k)} \times \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{s^2\rho^2}{2\sigma_{\mathrm{s}}^2\beta}\right)^j G_{j+2,j+3}^{j+2,1}\left[\frac{(1+k)I^{\beta}}{(\eta A_0)^{\beta}}\middle|\begin{matrix}\zeta_1\\ \zeta_2\end{matrix}\right], \tag{8}$$

where $\zeta_1 = 1,\underbrace{\rho^2/\beta+1,\ldots,\rho^2/\beta+1}_{(j+1)\text{terms}}$, $\zeta_2 = 1,\underbrace{\rho^2/\beta,\ldots,\rho^2/\beta}_{(j+1)\text{terms}},0$.

Considering a special case of s=0 in which the boresight displacement is neglected, the CDF of I is simplified as

$$F_I\left(I\right) = \frac{\alpha\rho^2}{\beta} \times \sum_{k=0}^{\infty}\frac{(-1)^k \Gamma(\alpha)}{(k+1)!\,\Gamma(\alpha-k)} \times G_{2,3}^{2,1}\left[\frac{(1+k)I^{\beta}}{(\eta A_0)^{\beta}}\middle|\begin{matrix}1,\rho^2/\beta+1\\ 1,\rho^2/\beta,0\end{matrix}\right]. \tag{9}$$

Outage probability, as an important metric to evaluate the communication system, is defined as the probability that the instantaneous *SNR* falls lower than a specified threshold. Based on the outage performance of each PP link, the outage probability of a multihop relay-assisted FSO communication system can be obtained as

$$P_{\mathrm{out}}(\varphi_{\mathrm{th}}) = P[\min(\varphi_v) \le \varphi_{\mathrm{th}}] = 1-\prod_{v=1}^{M}[1-P(\varphi_v \le \varphi_{\mathrm{th}})] = 1-\prod_{v=1}^{M}[1-P(I_v \le \sqrt{\varphi_{\mathrm{th}}/\bar{\varphi}_v})] = 1-\prod_{v=1}^{M}[1-P_{\mathrm{out}}^{v}(\varphi_{\mathrm{th}})], \tag{10}$$

where $P_{\mathrm{out}}^{v}(\varphi_{\mathrm{th}})$ is the outage probability of the vth (v=1, …, M) hop and can be written as

$$P_{\mathrm{out}}^{v}(\varphi_{\mathrm{th}}) = F\left(\sqrt{\frac{\varphi_{\mathrm{th}}}{\bar{\varphi}_v}}\right) = \frac{\alpha_v\rho_v^2}{\beta_v}\exp\left(-\frac{s_v^2}{2\sigma_{s_v}^2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k \Gamma(\alpha_v)}{(k+1)!\,\Gamma(\alpha_v-k)} \times \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{s_v^2\gamma_v^2}{2\sigma_{s_v}^2\beta_v}\right)^j G_{j+2,j+3}^{j+2,1}\left[(1+k)\left(\frac{\sqrt{\varphi_{\mathrm{th}}/\bar{\varphi}_v}}{\eta_v A_{0_v}}\right)^{\beta_v}\middle|\begin{matrix}\zeta_1\\ \zeta_2\end{matrix}\right]. \tag{11}$$

The subscript v=1, …, M in Eq.(11) indicates the corresponding PP link (vth hop) in the studied system. Substituting Eq.(11) into Eq.(10), the result for the case with nonzero boresight included can be further achieved as

$$P_{\text{out}} = 1-\prod_{v=1}^{M}\left\{1-\frac{\alpha_v\rho_v^2}{\beta_v}\exp(-\frac{s_v^2}{2\sigma_{s_v}^2})\times \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!}\frac{\Gamma(\alpha_v)}{\Gamma(\alpha_v-k)}\times\sum_{j=0}^{\infty}\frac{1}{j!}(\frac{s_v^2\gamma_v^2}{2\sigma_{s_v}^2\beta_v})^j\times G_{j+2,j+3}^{j+2,1}\left[(1+k)(\frac{\sqrt{\frac{\varphi_{\text{th}}}{\overline{\varphi_v}}}}{\eta_v A_{0_v}})^{\beta_v}\left|\begin{matrix}\zeta_1\\ \zeta_2\end{matrix}\right.\right]\right\}. \quad (12)$$

Letting s=0, the outage probability of the multihop relay-assisted system in the absence of boresight displacement can be obtained as

$$P_{\text{out}} = 1-\prod_{v=1}^{M}\left\{1-\frac{\alpha_v\rho_v^2}{\beta_v}\times\sum_{k=0}^{\infty}\frac{(-1)^k\Gamma(\alpha_v)}{(k+1)!\Gamma(\alpha_v-k)}\times G_{2,3}^{2,1}\left[(1+k)(\frac{\sqrt{\varphi_{\text{th}}/\overline{\varphi_v}}}{\eta_v A_{0_v}})^{\beta_u}\left|\begin{matrix}1,\frac{\rho_v^2}{\beta_v}+1\\ 1,\frac{\rho_v^2}{\beta_v},0\end{matrix}\right.\right]\right\}. \quad (13)$$

On the basis of the previously obtained analytical expressions of end-to-end outage probability as well as the MC simulation results, we discuss the outage performance of the multi-hop FSO communication system. The inverse CDF method and acceptance-rejection method are used to obtain the random values from the atmospheric turbulence and nonzero boresight pointing error fading models, respectively, in the MC simulation. The equal average *SNR* at each terminal is assumed, and the fixed hop length is supposed to be 1 km. The link parameters are shown in Tab.1.

Tab.1 System parameter settings

Parameter	Symbol	Value
Optical wavelength	λ	780 nm
Receiver responsivity	R	0.5 A/W
SNR threshold	φ_{th}	1 dB
Noise standard deviation	σ_n	10^{-7} A/Hz
Beamwidth	ω_Z	1.5 m
Jitter standard deviation	σ_s	0.3 m

In moderate turbulence regime, the outage probability of the mutihop relay-assisted system against the average transmit power P_t with different number of hops M is depicted in Fig.2. The receiving aperture size (d) of 200 mm is adopted. The close agreements of the analytical results with MC simulation verify the correctness of our model. It is obviously seen that the outage performance can be enhanced by increasing the transmit power. Besides, the outage probability increases as the number of hops increases. Meanwhile, the total link distance also increases. Furthermore, the difference of end-to-end outage probability between the two adjacent M is less obvious as the number of hops increases.

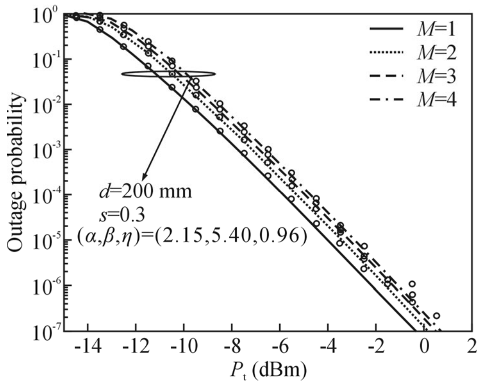

Fig.2 End-to-end outage probability in moderate turbulence regime with different number of hops *M* as boresight displacement *s*=0.3 m

The outage probability of the studied FSO systems with two and five hops (M=2 and 5) in moderate turbulence regime for different boresight displacement is shown in Fig.3. Three different boresight displacements (s=0, 0.2 m and 0.4 m) are adopted to assess the impact of the boresight error. It is found that the outage performance can be seriously deteriorated by a larger boresight displacement (s) for fixed number of hops. This is because that with the increase of s, the collected energy of the receiver decreases. Moreover, the degradation of the outage performance by increasing number of hops M is almost the same no matter how the boresight displacement value changes. Specifically, as the outage probability equals 10^{-7}, the differences of the average transmit power between M=2 and M=5 are all about 1 dB for s=0, 0.2 m and 0.4 m.

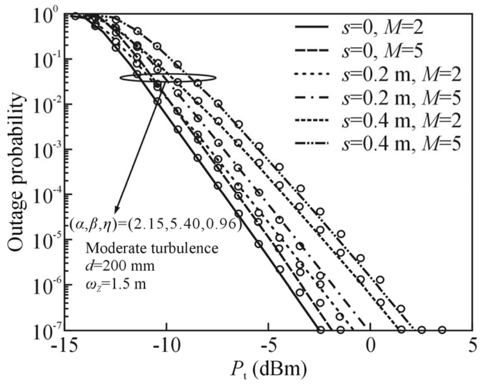

Fig.3 End-to-end outage probability in moderate turbulence regime with different boresight displacements and number of hops

Fig.4 illustrates the outage performance of the studied FSO system with two and five hops (M=2 and 5) in moderate and strong turbulence regimes with receiving aperture sizes of 100 mm and 200 mm. As expected, the outage performance worsens as the strength of turbulence increases. But the outage probability can be reduced by increasing the receiver aperture size, which is due to the aperture averaging effect. It is also found that in the presence of nonzero boresight pointing errors, the aperture averaging effect is more evident in strong turbulence regime than that in moderate turbulence regime. This is because that aperture averaging is more effective to reduce the irradiance fluctuations as the strength of the turbulence increases. Furthermore, the performance improvement owing to the increase of receiving aperture size can compensate for the performance loss due to the broadening of the total transmission distance. Specifically, the outage performance of longer transmission distance (M=5) with larger aperture (d=200 mm) is much better than that of with only two hops and aperture size of 100 mm.

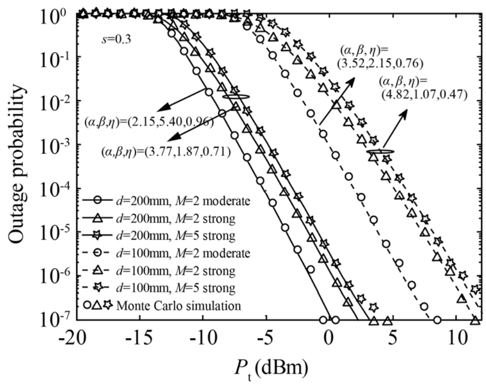

Fig.4 End-to-end outage probability of the studied FSO system with different turbulence conditions, aperture sizes and number of hops as s=0.3 m

Under strong turbulence condition, the outage probability against the average transmit power P_t of the multihop FSO system for different number of hops and beamwidths with receiving aperture size of 200 mm is presented in Fig.5. As is shown, to achieve the $ABER$ lower than 10^{-2}, the wider beamwidth works better than the narrower one. Moreover, this superiority is more and more obvious with the increase of transmit power. Furthermore, it is clearly seen that the degradation because of the increase of M (the total transmission distance is also increased) can be mitigated by widening the beamwidth. For instance, the outage probability of longer transmission distance (M=5) with wider beamwidth (ω_Z =1.5 m) is much lower than that with two hops and beamwidth of 1 m.

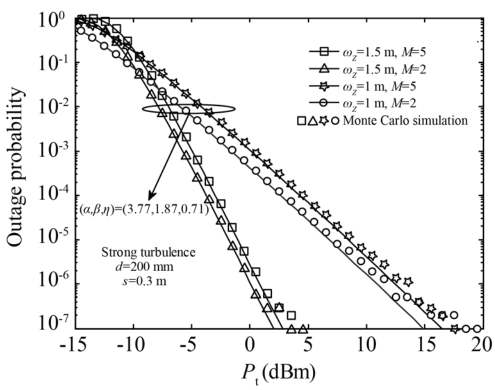

Fig.5 End-to-end outage probability of the studied system in strong turbulence regime with different beamwidths and number of hops as s=0.3 m

In this paper, the end-to-end outage probability of multihop FSO system utilizing DF relay transmission over EW fading channel with nonzero pointing errors is studied. The results show that the outage probability increases with the increase of the number of hops or boresight displacement. The deterioration induced by increasing the number of hops always keeps constant for different boresight displacements. The impairment of the outage performance due to increasing number of hops can be mitigated by increasing the receiving aperture size and widening the beamwidth. This work is beneficial for the design of the relay-assisted FSO system.

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