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A code-aided carrier synchronization algorithm based on improved nonbinary low-density parity-check codes*

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In order to further improve the carrier synchronization estimation range and accuracy at low signal-to-noise ratio (*SNR*), this paper proposes a code-aided carrier synchronization algorithm based on improved nonbinary low-density parity-check (NB-LDPC) codes to study the polarization-division-multiplexing coherent optical orthogonal frequency division multiplexing (PDM-CO-OFDM) system performance in the cases of quadrature phase shift keying (QPSK) and 16 quadrature amplitude modulation (16-QAM) modes. The simulation results indicate that this algorithm can enlarge frequency and phase offset estimation ranges and enhance accuracy of the system greatly, and the bit error rate (*BER*) performance of the system is improved effectively compared with that of the system employing traditional NB-LDPC code-aided carrier synchronization algorithm.
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In coherent optical orthogonal frequency division multiplexing (CO-OFDM) system, carrier frequency offset (CFO) is generated because of the frequency deviation between emitting laser and local oscillator laser. CFO not only influences symbol decisions, but also causes inter-carrier interference (ICI), resulting in serious decline of system performance. It is interesting to note that the CO-OFDM system is very sensitive to phase noise (PHN) caused by the linewidth of the emitting laser or the local oscillator laser, which can deteriorate the system performance. Furthermore, the PHN can also result in the common phase error (CPE) of demodulation data and ICI. The CPE will cause the phase rotation of all demodulation sub-carriers, which can lead to decision error. Considering the ICI could reduce optical signal-to-noise ratio (*OSNR*) of the effective system, the system performance will declines, and even the system cannot work properly[1-4]. Therefore, it would be significant to further study the carrier synchronization technology in CO-OFDM system.

In a polarization-division-multiplexing (PDM) CO-OFDM system, the traditional carrier synchronization algorithm is very difficult to work in low *SNR* environment, so the decoder cannot operate normally. Because of the excellent performance of low-density parity-check (LDPC) code which is close to the Shannon limit, code-aided (CA) carrier synchronization algorithm has become a hot and key direction recently[5-9]. Ref.[10] proposed an improved polarity decision phase detection algorithm based on decoding soft information, where the carrier frequency offset and phase offset estimation ranges are small. Ref.[11] proposed a simplified pilot joint carrier CA synchronization algorithm based on maximum likelihood (ML) criterion, which cannot be applied to the system with limited bandwidth. Ref.[12] proposed an ML carrier estimation method based on equal interval pilot symbols. However, it has a small synchronization range.

Under the condition of low *SNR*, the large frequency offset and phase offset make the synchronizer and decoder unable to converge, while the existing CA carrier synchronization algorithms are difficult to meet the requirements of estimation range and accuracy simultaneously. In order to solve this problem, this paper employs an improved nonbinary LDPC (NB-LDPC) CA carrier synchronization algorithm with low complexity to PDM-CO-OFDM system to realize the frequency and phase offset estimation in low *SNR* conditions, based on joint detection and decoding method.

In the PDM-CO-OFDM system, it is assumed that the ideal window synchronization is completed, then a fast Fourier transform (FFT) is performed on the received OFDM signal sampling value and we can get the following formula

$$\boldsymbol{r}_{ki} = \boldsymbol{c}_{ki} \cdot \exp\left[\mathrm{j}\varphi_k\right] \cdot \boldsymbol{H}_k + \boldsymbol{\zeta}_{ki} = \boldsymbol{c}_{ki} \cdot \exp\left[\mathrm{j}\left(2\pi k \Delta f T + \theta\right)\right] \cdot \boldsymbol{H}_k + \boldsymbol{\zeta}_{ki}, \tag{1}$$

where $\boldsymbol{c}_{ki} = [c_{ki}^x \quad c_{ki}^y]^{\mathrm{T}}$ and $\boldsymbol{r}_{ki} = [r_{ki}^x \quad r_{ki}^y]^{\mathrm{T}}$ represent the

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transmitted and received signals on the first k sub-carriers of the OFDM symbol of i, respectively. $\boldsymbol{H}_k$ is the matrix of channel frequency domain transfer function, reflecting the channel linear response of PDM effect. Δf and θ are frequency offset of the receiver and the CPE, respectively, and $\boldsymbol{\zeta}_{ki}=[\zeta_{ki}^{x} \quad \zeta_{ki}^{y}]^{\mathrm{T}}$ is a random noise with two polarization components. When the number of sub-carriers of the OFDM signals becomes large enough, $\boldsymbol{\zeta}_{ki}$ approximates to Gaussian noise. In the first CO-OFDM symbol of i, the Gaussian white noises of sub-carriers are different, but the phase shift is the same[13].

The NB-LDPC CA carrier synchronization algorithm can be briefly expressed as follows: according to the system model of posterior probability message estimation carrier synchronization parameter of NB-LDPC decoding, the NB-LDPC decoder is used to realize the estimation and compensation of synchronization parameters by deciding the posterior mean of feedback on each iteration, then the Costas loop is utilized to estimate the partial information of each symbol, and the next iteration is carried out after the compensation for the phase offset of signal vector. Detailed implementation steps are given below.

The two polarization directions in PDM system should be carried by frequency offset and phase offset estimation respectively, and the x polarization direction is taken as an example. Ignoring the multiplicative factor independent of c and φ to be estimated, the joint conditional probability density function of the receiving symbol sequence can be simplified as

$$P(\boldsymbol{r}\mid \boldsymbol{c};\varphi)=\prod_{k=0}^{K-1}\left\{\exp\left[\mathrm{Re}\left(\boldsymbol{r}_{ki}\boldsymbol{c}_{ki}^{*}\mathrm{e}^{-\mathrm{j}\varphi_k}\right)\right]\right\}, \tag{2}$$

where the superscript * represents the conjugate operation. The log likelihood function of the synchronization parameter is given as follows

$$LL(\varphi)\cong \mathrm{Re}\left\{\sum_{k=0}^{K-1}\boldsymbol{r}_{ki}\cdot \boldsymbol{A}_k^{*}(\boldsymbol{r},\varphi)\cdot\exp(-\mathrm{j}\varphi_k)\right\}, \tag{3}$$

where $\boldsymbol{A}_k(\boldsymbol{r},\varphi)$ is the mean value of all the possible symbols on the modulation constellation points to the posterior probability, and it is expressed as

$$\boldsymbol{A}_k(\boldsymbol{r},\varphi)=\sum_{m=0}^{M-1}p_{m|\boldsymbol{r}}(k)\cdot s_m, \tag{4}$$

where s_m is the value of the first m points on modulation constellations (m=0, 1,…M−1, M is the number of constellation points), $p_{m|\boldsymbol{r}}(k)$ is the posterior probability of transmitted symbols of the first k sub-carriers, and $p_{m|\boldsymbol{r}}(k)=p_{\boldsymbol{r}}[\boldsymbol{c}_{ki}=s_m|\boldsymbol{r}]$.

In order to reduce the complexity of the existing CA carrier synchronization algorithms and enlarge the synchronization range, we firstly use posterior means $\boldsymbol{A}_k(\boldsymbol{r},\varphi^{(n)})$ to eliminate the modulation information of signals on each iteration, that is

$$y_k^{(n)}=\boldsymbol{r}_k\cdot \boldsymbol{A}_k^{*}\left(\boldsymbol{r},\varphi^{(n)}\right)=$$
$$\exp\left[\mathrm{j}\left(2\pi k\Delta fT+\theta\right)\right]+\boldsymbol{\zeta}_{ki}\cdot \boldsymbol{A}_k^{*}\left(\boldsymbol{r},\varphi^{(n)}\right). \tag{5}$$

Then we adopt the frequency offset estimation based on multi-delay correlation function and obtain the following formula combined with Eq.(5)

$$\Delta \hat{f}^{(n)}T=1/\left[\pi(D+1)\right]\times$$
$$\arg\left\{\sum_{d=1}^{D}1/(K-d)\cdot\sum_{k=0}^{K-d-1}\left(y_k^{(n)}\right)^{*}\cdot y_{k+d}^{(n)}\right\}, \tag{6}$$

where D=K/2 is the average correlation step length. Then the next iteration is carried out after the compensation for the phase offset of signal vector by using the Costas loop. The basic idea of Costas loop iterative phase estimation is to make the derivative of the log likelihood function to be zero[14], and the derivative of Eq.(3) is

$$LL^{'}(\varphi)=\mathrm{d}\ln\left[p(\boldsymbol{r};\varphi)\right]/\mathrm{d}\varphi\cong$$
$$\sum_{k=0}^{K-1}\mathrm{Im}\left[\boldsymbol{A}_k^{*}(\boldsymbol{r},\varphi)\boldsymbol{r}_{ki}\exp(-\mathrm{j}\varphi_k)\right] \quad . \tag{7}$$

When the frequency offset estimator is able to work properly, the frequency offset Δf is small enough and φ_k is mainly determined by θ considering the signal vector after a certain amount of frequency offset compensation. In order to make Eq.(7) to be zero, the system updates the phase estimation value per each symbol interval and Eq.(8) can be used to iterate, taking the Costas loop structure based on back propagation into account.

$$\hat{\theta}_{k+1}=\hat{\theta}_k+\lambda e_k=\hat{\theta}_k+\lambda\,\mathrm{Im}\left[\boldsymbol{A}_k^{*}(\boldsymbol{r},\varphi)\boldsymbol{r}_k\exp(-\mathrm{j}\varphi_k)\right], \tag{8}$$

where λ is the loop filter gain, and e_k is the output of phase error detector, which is decided by the derivative $LL'(\theta)$ of the log likelihood function.

As shown in Fig.1, at the transmitter of the transmission process, the transmitted symbols are first encoded by NB-LDPC codes, and then loaded onto the sub-carriers by 16QAM modulation. The time-domain OFDM signals are obtained by inverse fast Fourier transform (IFFT). Subsequently, the OFDM signals are converted to analog electrical signals that can be loaded into the Mach-Zehnder modulator (MZM) by digital-to-analog converter (DAC). The optical carrier wave is divided into two parts by polarizing beam splitter (PBS), which includes horizontal and vertical polarization components. Such signals are generated in two branches and then they form a polarization multiplexing transmission signal by means of a polarizing beam combiner (PBC). Finally, the signals get into a standard single mode fiber (SMF) for transmission. At the end of receiver, the polarization- and phase-diverse coherent receiver with a local oscillator laser is used to treat electrical signals for both polarization states. The received optical signals are sampled by an analog to digital converter (ADC). Following the unit of dispersion compensation and timing synchronization, the signals are fed into the channel estimation unit for PMD damage and other linear injury compensation after the FFT processing. The improved NB-LDPC CA carrier synchronization algorithm can achieve frequency offset, and phase noise compensation is eventually used to recover the original binary data.

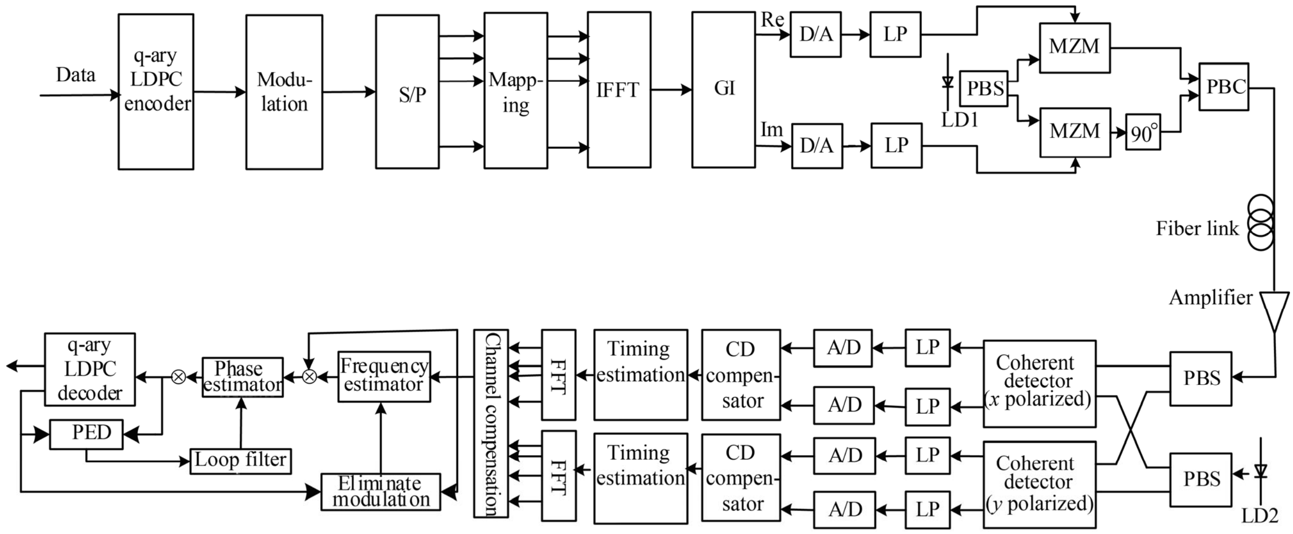
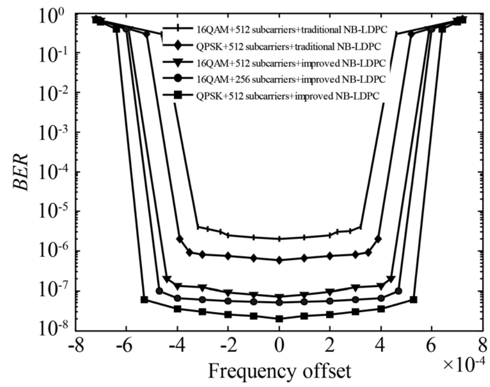


Fig.1 Diagram of the improved PDM-CO-OFDM system based on NB-LDPC CA carrier synchronization

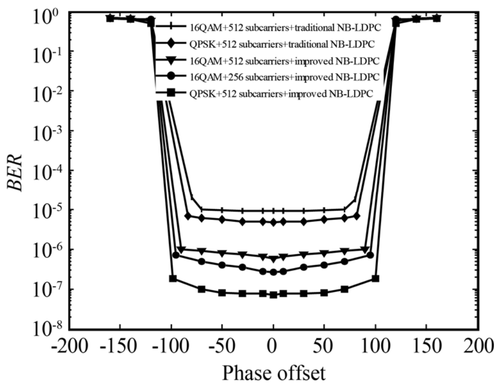
In the simulation system, the rate of single carrier symbol is set to be 30 G baud and modulation type is QPSK and 16QAM. The points of IFFT and number of sub-carriers are 1 024 and 512, respectively. The linewidth of laser is 100 kHz and the transmission distance of SMF is 4 600 km. Moreover, the number of iterations of NB-LDPC decoding is set to be 25 and the range of *OSNR* is set from 12 dB to 18 dB. Forward error correction (FEC) technology is also adopted at the receiver. In order to prove the effectiveness of our improved NB-LDPC CA carrier synchronization algorithm in reducing the impact of phase offset and frequency offset on the OFDM system, we construct a GF(4) NB-LDPC code by means of permutation polynomials method in Ref.[15], in which the code length is 5 970 and the encoding rate is 0.875. The degree distribution of check matrix $\boldsymbol{H}$ is (3, 20), and the quadratic permutation polynomial and linear permutation polynomial are $f(x)=68x+315x^2(\text{mod}5\ 970\times3)$ and $g(x)=3x(\text{mod}4)$, respectively. Furthermore, we select the CA carrier synchronization algorithm based on traditional NB-LDPC coding[16] for comparison in simulation and obtain the frequency and phase estimation ranges, the mean square error (*MSE*) of estimation and *BER* graphs of the received signals, which are analyzed for the system transmission performance.

In order to verify the effectiveness of our improved NB-LDPC CA carrier synchronization algorithm in enlarging the estimation ranges of the system frequency and phase, the phase offset θ and normalized frequency offset ΔfT of the system are set to be 30° and 1×10^{-4}, respectively. The results in Fig.2(a) demonstrate the frequency estimation ranges with θ=30° under different modulation modes and different numbers of sub-carriers. As shown in Fig.2(a), the estimated frequency range of the system based on the improved NB-LDPC CA carrier synchronization algorithm is larger than that of the system based on the traditional NB-LDPC CA carrier synchronization algorithm. And the improved system can be adapted to different sub-carriers. When the modulation mode is QPSK and the number of sub-carriers is 512, the estimated frequency offset range of the system based on the improved NB-LDPC CA carrier synchronization algorithm is approximately (-5.3×10^{-4}, 5.3×10^{-4}). The phase estimation ranges are shown in Fig.2(b) with $\Delta fT=1\times10^{-4}$. We can see that when the modulation mode is QPSK and the number of sub-carriers is 512, the estimated phase offset range of the system based on the improved NB-LDPC CA carrier synchronization algorithm is approximately (−100°, 100°), which is larger than that of the system based on the traditional NB-LDPC CA carrier synchronization algorithm. Thus, the proposed algorithm still has a large synchronization range under the condition of low *SNR*.

The measured *MSE* curves of frequency offset estimation under the QPSK and 16QAM modulations at θ=30° are plotted in Fig.3(a). As a comparison, Fig.3(a) also gives the modified Cramer Rao bound (MCRB). When the *OSNR* is less than 15 dB, the *MSE* of frequency offset estimation has a certain distance away from MCRB. However, when the *OSNR* is greater than 15 dB, the *MSE* curves are declined for both QPSK and 16QAM modulations. Furthermore, the frequency estimation error based on the improved NB-LDPC CA carrier synchronization algorithm is gradually close to MCRB, and the accuracy of frequency offset estimation is higher compared with the traditional NB-LDPC CA carrier synchronization algorithm. Fig.3(b) demonstrates the *MSE* curves based on phase offset estimation at ΔfT=1×10^{-4}. Similarly, when the *OSNR* is larger than 16 dB, the phase estimation error based on the improved NB-LDPC CA carrier synchronization algorithm has little difference from MCRB. Thus it can be seen that the improved NB-LDPC CA carrier synchronization algorithm has a great advantage to improve the accuracy of phase offset estimation in PDM-CO-OFDM system.

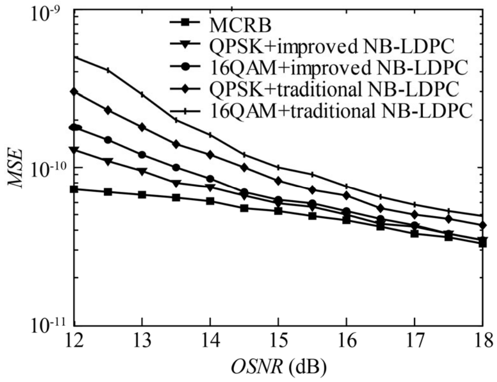


(a)

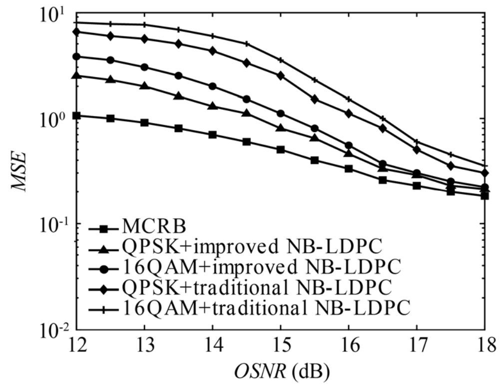


(b)

Fig.2 (a) Range of frequency offset estimation; (b) Range of phase offset estimation



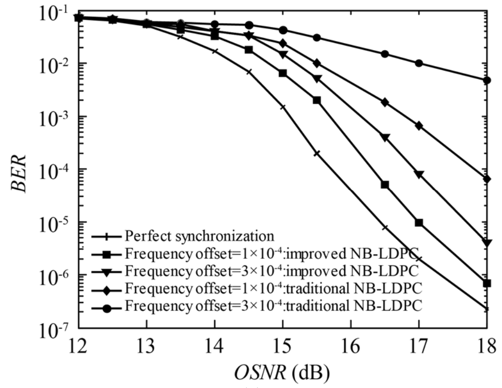
(a)



(b)

Fig.3 (a) *MSE* of frequency offset estimation; (b) *MSE* of phase offset estimation

The *BER* curves of frequency offset estimation based on two kinds of CA carrier synchronization algorithms for different frequency offsets are shown in Fig.4(a). We can see that the effect of *BER* on the system performance varies for different frequency offsets. When the frequency offset is small, the system *BER* performance is close to ideal synchronization, while the system loss becomes apparent as the frequency offset increases. Based on our data, compared with the system based on traditional NB-LDPC CA carrier synchronization algorithm, when the system employs the improved NB-LDPC CA carrier synchronization algorithm at ΔfT=1×10^{-4}, the *OSNR* performance is improved by 1.56 dB at the *BER* of 10^{-4}. The decoding performance of the traditional NB-LDPC CA carrier synchronization algorithm has deteriorated rapidly, while the improved NB-LDPC CA carrier synchronization algorithm can still work effectively when ΔfT is equal to 1×10^{-4}. Fig.4(b) gives the *BER* performance based on NB-LDPC coding under different phase offsets. When the system employs the improved NB-LDPC CA carrier synchronization algorithm at θ=30°, the *OSNR* performance is improved by 1.28 dB at the *BER* of 10^{-4} and it is more close to the ideal synchronization with comparison to the system based on traditional NB-LDPC CA carrier synchronization algorithm. Therefore, we can draw a conclusion that the improved NB-LDPC CA carrier synchronization algorithm can effectively improve the bit error performance of the PDM-CO-OFDM system.



(a)

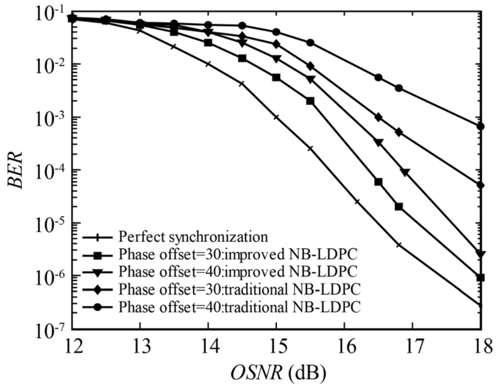


Fig.4 (a) *BER* of frequency offset estimation; (b) *BER* of phase offset estimation

In conclusion, using an improved NB-LDPC code-aided carrier synchronization algorithm in PDM-CO-OFDM system, this paper studies synchronization range, *MSE* and *BER* performance of the OFDM system by simulations. According to the *BER* curves, the improved NB-LDPC CA carrier synchronization algorithm can enlarge the synchronization range and improve the system *BER* performance compared with the traditional algorithm under low *SNR* conditions. By means of *MSE* diagrams, it is discovered that the estimation accuracy of OFDM system can be enhanced effectively and the error of synchronization estimation becomes very small with MCRB by adopting the improved algorithm. Thus, the improved NB-LDPC code-aided carrier synchronization algorithm can meet the requirements of carrier synchronization estimation range and estimation accuracy of PDM-CO-OFDM system, and it can effectively improve the *BER* performance on the condition of low *SNR*.

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